Implementation with Missing Data

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Main Question

- How should planners tackle the problem of designing mechanisms
  - with **missing choice data**

    (i.e., when they do not know all choices of all individuals)
  - to decentralize desired collective goals?

Motivations

The lack of data on individuals' choices is **natural** as missing data is a fact of life:

- Monitoring and storage of individuals' revealed preferences are costly.

An Incomplete Literature Review

- Maskin (1999), Moore and Repullo (1990), Dutta and Sen (1991);
- Jackson (1991);
- Bergemann and Morris (2005), (2008), (2009), and (2011);
- Eliaz (2002);
- Barlo and Dalkıran (2009), Korpela (2012), de Clippel (2014), Hayashi et al. (2020), Barlo and Dalkıran (2021, 2022).

A Suitable Setting – I

- A *planner*, a fresh CEO or an appointed trustee, is to run a firm, and
- depending on the *state of the firm* that she does not observe, she is to choose one of the following *alternatives*: expansion, prudence, or contraction.
- The chiefs of *finance* and *marketing* observe the firm's state, be it (S)trong, (N)ormal, or (W)eak, and their own *preferences* contingent on firm's states.
- The planner needs to *implement* a given goal contingent on firm's states by extracting the CFO's and the CMO's information.

A Suitable Setting – II

- In the classical setting, the planner and the individuals are fully informed of how *payoff states* (chiefs' preference profiles) are associated with *firm's states*.

- In our model, the planner and the individuals do not fully know this association, but have partial information about it. This constitutes the **missing choice data**:

  From **past data** on accounting records and meeting minutes, they are partially informed about how chiefs' preferences correspond to firm's states:

  - Last quarter, when the firm's state was **normal**, the *CFO* strictly preferred *prudence* to *contraction*, while
  - the *CMO* strictly preferred the *prudence* to *expansion*,
  - and there is no further information pertaining to the firm's normal state.

A Suitable Setting – III

- In the classical setting, the planner and the individuals are fully informed of how *payoff states* (chiefs' preference profiles) are associated with *firm's states*.

- In our model, the planner and the individuals do not fully know this association, but have partial information about it. This constitutes the **missing choice data**:

  From past data on accounting records and meeting minutes, they are partially informed about how chiefs' preferences correspond to firm's states:

  - So, at the firm's **normal** state, the *CEO* and the *CMO* do not know how the *CFO* ranks *expansion* compared to *contraction* and *prudence*, and
  - the *CEO* and the *CFO* do not know how the *CMO* ranks *contraction* compared to *expansion* and *prudence*.

A Suitable Setting – IV

- In the classical setting, the planner and the individuals are fully informed of how *payoff states* (chiefs' preference profiles) are associated with *firm's states*.
- In our model, the planner and the individuals do not fully know this association, but have partial information about it. This constitutes the **missing choice data**.
- The missing choice data is **publicly observable**.
- When can the planner **implement a given goal** for the firm via a mechanism by using **only the incomplete public choice data** and by refraining from relying on
    - chiefs' assessments about the other's possible preferences (types), and
    - chiefs' knowledge of their own types?

Our Contributions

- We *formalize* such implementation problems with missing data,
- *propose* a suitable notion of equilibrium along with resulting concepts of (full) implementation,
- *obtain* necessary conditions that are sufficient in economic environments,
- *establish* that more information enriches implementation opportunities,
- *analyze* the implementability of a suitable efficiency notion.

An Example: Missing Choice Data

	(S)trong		*(N)ormal*		*(W)eak*	
	CFO	*CMO*	*CFO*	*CMO*	*CFO*	*CMO*
$\{c, e, p\}$	$\{e\}$				$\{p\}$	$\{p\}$
$\{c, e\}$						
$\{c, p\}$	$\{p\}$		$\{p\}$			
$\{e, p\}$		$\{e\}$		$\{p\}$		

There are three alternatives $X = \{c, e, p\}$, and the CFO and the CMO observe the *state of the firm* $\Theta = \{S, N, W\}$ along with their own preferences (strict rankings).

- $\pi^* : \Theta \to \Omega$ identifies the *true association* between Θ and Ω where Ω denotes the *payoff states* and equals the set of all strict ranking profiles.
- The planner and the individuals do not know the true association but observe the above *incomplete public choice data*.

An Example: Inferences from the Incomplete Choice Data

	(S)trong		*(N)ormal*		*(W)eak*	
	CFO	*CMO*	*CFO*	*CMO*	*CFO*	*CMO*
$\{c, e, p\}$	$\{e\}$				$\{p\}$	$\{p\}$
$\{c, e\}$						
$\{c, p\}$	$\{p\}$		$\{p\}$			
$\{e, p\}$		$\{e\}$		$\{p\}$		

Rationality implies the following inferences about individuals' preferences from the incomplete public choice data: At firm's state S,

- the preferences of the CFO must be s.t. $e \; P_{CFO} \; p \; P_{CFO} \; c$ (denoted by epc);
- the preferences of the CMO is an element in $\{cep, ecp, epc\}$.

An Example: Inferences from the Incomplete Choice Data

	(S)trong		*(N)ormal*		*(W)eak*	
	CFO	*CMO*	*CFO*	*CMO*	*CFO*	*CMO*
$\{c, e, p\}$	$\{e\}$				$\{p\}$	$\{p\}$
$\{c, e\}$						
$\{c, p\}$	$\{p\}$		$\{p\}$			
$\{e, p\}$		$\{e\}$		$\{p\}$		

Rationality implies the following inferences about individuals' preferences from the incomplete public choice data: At firm's state N,

- the preferences of the CFO must be in $\{epc, pce, pec\}$;
- the preferences of the CMO is in $\{cpe, pce, pec\}$.

An Example: Inferences from the Incomplete Choice Data

	(S)trong		*(N)ormal*		*(W)eak*	
	CFO	*CMO*	*CFO*	*CMO*	*CFO*	*CMO*
$\{c, e, p\}$	$\{e\}$				$\{p\}$	$\{p\}$
$\{c, e\}$						
$\{c, p\}$	$\{p\}$		$\{p\}$			
$\{e, p\}$		$\{e\}$		$\{p\}$		

Rationality implies the following inferences about individuals' preferences from the incomplete public choice data: At firm's state W,

- the preferences of the CFO must be in $\{pce, pec\}$;
- the preferences of the CMO is in $\{pce, pec\}$.

An Example: The Inference Correspondence

	(S)trong		*(N)ormal*		*(W)eak*	
	CFO	*CMO*	*CFO*	*CMO*	*CFO*	*CMO*
$\{c, e, p\}$	$\{e\}$				$\{p\}$	$\{p\}$
$\{c, e\}$						
$\{c, p\}$	$\{p\}$		$\{p\}$			
$\{e, p\}$		$\{e\}$		$\{p\}$		

Inferences from the incomplete public choice data generate

- the *inference correspondence*, $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, where
- $\mathcal{K}(\theta) \subset \Omega$ is the set of ranking profiles **compatible with the public choice data**.
- We require the following: for all $\theta \in \Theta$
  - $\pi^*(\theta) \in \mathcal{K}(\theta)$ (i.e., the truth must be compatible with the public choice data), and
  - $\mathcal{K}(\theta) \neq \emptyset$ (a natural regularity condition).

An Example: The Full Data

	(S)trong		*(N)ormal*		*(W)eak*	
	CFO	*CMO*	*CFO*	*CMO*	*CFO*	*CMO*
$\{c, e, p\}$	$\{e\}$				$\{p\}$	$\{p\}$
$\{c, e\}$						
$\{c, p\}$	$\{p\}$		$\{p\}$			
$\{e, p\}$		$\{e\}$		$\{p\}$		

Inferences from the incomplete public choice data generate

- the *inference correspondence*, $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, where
- $\mathcal{K}(\theta) \subset \Omega$ is the set of ranking profiles **compatible with the public choice data**.
- The data is **complete** when $\mathcal{K}(\theta) = \{\pi^*(\theta)\}$ for all $\theta \in \Theta$.
  - This corresponds to the **standard** case (see Maskin (1999) or de Clippel (2014)).

An Example: The Partially Informed Planner

	(S)trong		*(N)ormal*		*(W)eak*	
	CFO	*CMO*	*CFO*	*CMO*	*CFO*	*CMO*
$\{c, e, p\}$	$\{e\}$				$\{p\}$	$\{p\}$
$\{c, e\}$						
$\{c, p\}$	$\{p\}$		$\{p\}$			
$\{e, p\}$		$\{e\}$		$\{p\}$		

Inferences from the incomplete public choice data generate

- the *inference correspondence*, $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, where
- $\mathcal{K}(\theta) \subset \Omega$ is the set of ranking profiles **compatible with the public choice data**.
- The data is **complete** when $\mathcal{K}(\theta) = \{\pi^*(\theta)\}$ for all $\theta \in \Theta$.
- In all other cases, the planner and the individuals are **partially informed**.

An Example: The Inferences in the Example

	(S)trong		*(N)ormal*		*(W)eak*	
	CFO	*CMO*	*CFO*	*CMO*	*CFO*	*CMO*
$\{c, e, p\}$	$\{e\}$				$\{p\}$	$\{p\}$
$\{c, e\}$						
$\{c, p\}$	$\{p\}$		$\{p\}$			
$\{e, p\}$		$\{e\}$		$\{p\}$		

Inferences from the incomplete public choice data generate

- At S: $\mathcal{K}(S) = \{\{epc\} \times \{cep, ecp, epc\}\}$.
- At N: $\mathcal{K}(N) = \{\{epc, pce, pec\} \times \{cpe, pce, pec\}\}$.
- At W: $\mathcal{K}(W) = \{\{pce, pec\} \times \{pce, pec\}\}$.

An Example: Agents' Inferences

	(S)trong		*(N)ormal*		*(W)eak*	
	CFO	*CMO*	*CFO*	*CMO*	*CFO*	*CMO*
$\{c, e, p\}$	$\{e\}$				$\{p\}$	$\{p\}$
$\{c, e\}$						
$\{c, p\}$	$\{p\}$		$\{p\}$			
$\{e, p\}$		$\{e\}$		$\{p\}$		

Agent's observe **their own type** (ranking), the **incomplete public choice data**, and **firm's realized state** **but not other individuals' types**. Thus,

- at S the CMO observing his type say cep infers that the payoff state equals (epc, cep);
- at N the CFO observing his type say pce infers that the realized payoff state must be in $\{(pce, cpe), (pce, pce), (pce, pec)\}$;
- ...

An Example: Inferences

	(S)trong		*(N)ormal*		*(W)eak*	
	CFO	*CMO*	*CFO*	*CMO*	*CFO*	*CMO*
$\{c, e, p\}$	$\{e\}$				$\{p\}$	$\{p\}$
$\{c, e\}$						
$\{c, p\}$	$\{p\}$		$\{p\}$			
$\{e, p\}$		$\{e\}$		$\{p\}$		

Agent's observe **their own type** (ranking), the **incomplete public choice data**, and **firm's realized state** **but not other individuals' types**.

The planner observes **only** the **incomplete public choice data**; so, can make inferences only based on the inference correspondence $\mathcal{K} : \Theta \to \Omega$.

An Example: The Social Choice Correspondence

	(S)trong		*(N)ormal*		*(W)eak*	
	CFO	*CMO*	*CFO*	*CMO*	*CFO*	*CMO*
$\{c, e, p\}$	$\{e\}$				$\{p\}$	$\{p\}$
$\{c, e\}$						
$\{c, p\}$	$\{p\}$		$\{p\}$			
$\{e, p\}$		$\{e\}$		$\{p\}$		

The social choice correspondence (SCC) $f : \Theta \twoheadrightarrow X$ is *exogenously given*. Here,

- we consider a *plausible* SCC: $f(S) = \{e\}$ and $f(\theta) = \{p\}$ for all $\theta \neq S$.
- For any θ, $f(\theta)$ equals the set of *reliably* Pareto efficient alternatives at θ:

  $f(\theta) = \bigcap_{\omega \in \mathcal{K}(\theta)} PO(\omega)$, where $PO(\omega) \equiv \{x \in X \mid \nexists y \in X \text{ with } yP_i^{\omega}x, \forall i \in N\}$.

  $x \in f(\theta)$ implies that no matter what the true ranking profile is, it must be PO.

▸ Details

The Model – I

- The environment is of *incomplete information*.
- X is the set of *alternatives* and the set of its non-empty subsets is $\mathcal{X}$.
- Ω_i is the set of possible *preferences* (**payoff types**) of individual $i \in N$.
- $\Omega = \times_{i \in N} \Omega_i$ denotes the set of **payoff states** (type profiles).
- Θ is the **states of the economy**.
- $f : \Theta \to \mathcal{X}$ is a given **social choice correspondence** (SCC).

The Model – II

Inferences from the Incomplete Public Choice Data

- The **inference correspondence** is $\mathcal{K} : \Theta \twoheadrightarrow \Omega$ s.t. $\mathcal{K}(\theta) \equiv \times_{i \in N} \mathcal{K}_i(\theta)$ for all θ,

- $\mathcal{K}(\theta) \subset \Omega$ is the set of payoff states that are **compatible with the public choice data** at $\theta \in \Theta$:
  - if the publicly observable choice of $i \in N$ at θ from $S \in \mathcal{X}$ with $x, y \in S$ contains x, then it is publicly known that $x R_i^{\omega} y$ for all $\omega \in \mathcal{K}(\theta)$.

- $\pi_i^* : \Theta \to \Omega_i$ captures the *true association* between Θ and Ω_i such that for all θ, $\pi^*(\theta) \equiv \times_{i \in N} \pi_i^*(\theta)$ is in $\mathcal{K}(\theta)$.

Information/Knowledge Requirements

The **information and knowledge requirements** of our model are:

(i) the **planner** knows N, X, Ω, Θ, and $f : \Theta \to \mathcal{X}$; and

(ii) each **individual** i knows N, X, Ω, Θ, $f : \Theta \to \mathcal{X}$, and

- the *realized state of the economy* $\theta \in \Theta$ and
- *i's true realized type* $\pi_i^*(\theta) \in \mathcal{K}_i(\theta)$ *at* θ; and

(iii) items (i), (ii), and $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, **inferences compatible with the public choice data**, are **common knowledge** among the individuals and the planner.

Mechanisms

A mechanism $\mu = (M, g)$ consists of

- **messages**, $M_i \neq \emptyset$, and **outcome function**, $g : M \to X$ with $M \equiv \times_{i \in N} M_i$.
- given $m_{-i} \in M_{-i} \equiv \times_{j \neq i} M_j$, the **opportunity set** of i in μ for m_{-i} is

$$O_i^{\mu}(m_{-i}) \equiv g(M_i, m_{-i}) = \{g(m_i, m_{-i}) \mid m_i \in M_i\}.$$

Inferences from the incomplete public choice data, $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, enable

- predictions about individuals' strategic behavior in mechanism μ and
- determination of whether or not μ "implements" a given SCC.

“Equilibrium” at Firm’s State N

In our example, consider the following mechanism, **firm’s state N**, and recall that

$$\mathcal{K}(N) = \{\{epc, pce, pec\} \times \{cpe, pce, pec\}\} \text{ and } f(N) = \{p\}.$$

		CMO		
		L	M	R
CFO	U	p	e	c
	M	e	p	ⓟ
	D	c	p	p

The planner infers that

- M is a best response of the CFO to the CMO choosing R, for all CFO’s rankings in $\mathcal{K}_{CFO}(N)$.
- R is a best response of the CMO to the CFO choosing M, for all CMO’s rankings in $\mathcal{K}_{CMO}(N)$.

“Equilibrium” at Firm’s State N

In our example, consider the following mechanism, **firm’s state N**, and recall that

$$\mathcal{K}(N) = \{\{epc, pce, pec\} \times \{cpe, pce, pec\}\} \text{ and } f(N) = \{p\}.$$

$$\begin{array}{cc|ccc} & & & \text{CMO} & \\ & & L & M & R \\ \hline & U & p & e & c \\ \text{CFO} & M & e & p & \textcircled{p} \\ & D & c & p & p \end{array}$$

The planner infers that

- At N, profile (M, R) is a Nash equilibrium (NE) at all payoff states in $\mathcal{K}(N)$.
- We argue that at N, the planner may **rely** on (M, R) being an “equilibrium”
    - even if the planner and the individuals are unsure of the true ranking profile associated with N.

Nash Equilirium

- Given $\mu = (M, g)$, $m^* \in M$ is a **Nash equilibrium** (NE) of μ at payoff state (ranking profile) $\omega \in \Omega$ if

$$g(m^*) \in \bigcap_{i \in N} C_i^{\omega}(O_i^{\mu}(m_{-i}^*)),$$

where for any non-empty $S \subset X$, $C_i^{\omega}(S) \equiv \{x \in S \mid x R_i^{\omega} y, \forall y \in S\}$.

- Our environment is of incomplete information where the planner and the individuals are unsure of the payoff state associated with the state of the economy.

- Thus, the use of **NE is not plausible** in our setting.

Reliable Nash Equilirium

- The **planner** needs to consider individuals' behavior in every possible ranking profile compatible with the incomplete public choice data
    - to make reliable strategic predictions and ensure outcomes consonant with the desired goal.
- If the **individuals** correlate their behavior only on the public choice data, then they do not have incentives to find out others' true preferences.
- These lead us to the notion of **reliable Nash equilibrium**.

Reliable Nash Equilirium

Definition

Given a mechanism μ, and the inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, $m^* \in M$ is a **reliable Nash equilibrium** (RNE) of μ at state of the economy θ if

$$g(m^*) \in \bigcap_{i \in N,\ \omega \in \mathcal{K}(\theta)} C_i^{\omega}(O_i^{\mu}(m_{-i}^*)).$$

Reliable Nash Equilirium

Definition

Given a mechanism μ, and the inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, $m^* \in M$ is a **reliable Nash equilibrium** (RNE) of μ at state of the economy θ if

$$g(m^*) \in \bigcap_{i \in N,\ \omega \in \mathcal{K}(\theta)} C_i^{\omega}(O_i^{\mu}(m_{-i}^*)).$$

- A profile of RNE taken across the states of the economy is equivalent to
  - an *ex-post correlated equilibrium* (ECE),
    - i.e., an ex-post equilibrium using the states of the economy as a correlation device,
  - in which *each individual's behavior depends only on the public choice data*.

▸ ECE ▸ Back

Reliable Nash Equilirium

Definition

Given a mechanism μ, and the inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, $m^* \in M$ is a **reliable Nash equilibrium** (RNE) of μ at state of the economy θ if

$$g(m^*) \in \bigcap_{i \in N,\ \omega \in \mathcal{K}(\theta)} C_i^{\omega}(O_i^{\mu}(m_{-i}^*)).$$

The RNE provides the following **robustness properties**:

(*i*) It uses **no probabilistic information**, **no belief updating**, and **no common prior assumption**; it is **belief-free**, and the equilibrium behavior features the **ex-post no-regret property**.

(*ii*) The RNE refrains from using individuals' private information and **relies only on the public choice data**.

Implementation in Reliable Nash Equilirium

Definition

Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$ and an SCC $f : \Theta \to \mathcal{X}$, a **mechanism μ implements f in RNE** if for all $\theta \in \Theta$, $f(\theta) = RNE^{\mu}(\theta)$ where $RNE^{\mu}(\theta) \equiv \{g(m^*) \in X \mid m^* \text{ is an RNE at } \theta\}$. That is,

(i) for all $\theta \in \Theta$ and all $x \in f(\theta)$, there exists $m^x \in M$ such that $g(m^x) = x$ and

$$g(m^x) \in \bigcap_{i \in N,\ \omega \in \mathcal{K}(\theta)} C_i^{\omega}(O_i^{\mu}(m_{-i}^x)), \text{ and}$$

(ii) if $m^* \in M$ is such that $g(m^*) \in \bigcap_{i \in N,\ \omega \in \mathcal{K}(\theta)} C_i^{\omega}(O_i^{\mu}(m_{-i}^*))$ for some $\theta \in \Theta$, then $g(m^*) \in f(\theta)$.

Implementation in Reliable Nash Equilirium

Implementation in RNE sustains RNE's **robustness properties**. Thus,

- individuals do not have incentives to change their prescribed behavior even if they were to learn others' payoff types, and
- outcomes of mechanisms implementing the given SCC in RNE are verifiable using only the public information and hence
- vindications based on individuals' private information are not needed.
- That is why such mechanisms **preserve privacy**.
  - See mechanism design with **privacy-aware individuals** (Nissim et al. (2012), Pai and Roth (2013), and Chen et al. (2016), among others).

Our Example - Implementation in RNE - S

State of the economy: S	State of the economy: N	State of the economy: W
$f(S) = \{e\}$	$f(N) = \{p\}$	$f(W) = \{p\}$
$\mathcal{K}(S) : \begin{array}{c} \{epc\} \\ \times \\ \{cep, ecp, epc\} \end{array}$	$\mathcal{K}(N) : \begin{array}{c} \{epc, pce, pec\} \\ \times \\ \{cpe, pce, pec\} \end{array}$	$\mathcal{K}(W) : \begin{array}{c} \{pce, pec\} \\ \times \\ \{pce, pec\} \end{array}$
$\begin{array}{c\|ccc} & L & M & R \\ \hline U & p & e & c \\ M & \textcircled{e} & p & p \\ D & c & p & p \end{array}$	$\begin{array}{c\|ccc} & L & M & R \\ \hline U & p & e & c \\ M & e & p & \textcircled{p} \\ D & c & p & p \end{array}$	$\begin{array}{c\|ccc} & L & M & R \\ \hline U & \textcircled{p} & e & c \\ M & e & p & p \\ D & c & p & p \end{array}$
RNE: (M, L) Outcomes: $\{e\}$	RNE: (M, R) Outcomes: $\{p\}$	RNE: (U, L) Outcomes: $\{p\}$

S : (M, L) is an RNE because $g(M, L) = e \in C^{\omega}_{CFO}(\{c, e, p\}) \cap C^{\omega}_{CMO}(\{e, p\})$ for all $\omega \in \mathcal{K}(S)$,
(U, L) is not an RNE as $g(U, L) = p \notin C^{\omega}_{CFO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(S)$,
(D, L) is not an RNE as $g(D, L) = c \notin C^{\omega}_{CFO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(S)$,
(M, M) is not an RNE as $g(M, M) = p \notin C^{\omega}_{CFO}(\{e, p\})$ for all $\omega \in \mathcal{K}(S)$,
(D, M) is not an RNE as $g(D, M) = p \notin C^{\omega}_{CFO}(\{e, p\})$ for all $\omega \in \mathcal{K}(S)$,
(U, R) is not an RNE as $g(U, R) = c \notin C^{\omega}_{CFO}(\{c, p\})$ for all $\omega \in \mathcal{K}(S)$,
(M, R) is not an RNE as $g(M, R) = p \notin C^{\omega}_{CMO}(\{e, p\})$ for all $\omega \in \mathcal{K}(S)$,
(D, R) is not an RNE as $g(D, R) = p \notin C^{\omega}_{CMO}(\{c, p\})$ with $\omega \in \{epc\} \times \{cep, ecp\} \subset \mathcal{K}(S)$.

Our Example - Implementation in RNE - N

State of the economy: S	State of the economy: N	State of the economy: W
$f(S) = \{e\}$	$f(N) = \{p\}$	$f(W) = \{p\}$
$\mathcal{K}(S) : \begin{array}{c} \{epc\} \\ \times \\ \{cep, ecp, epc\} \end{array}$	$\mathcal{K}(N) : \begin{array}{c} \{epc, pce, pec\} \\ \times \\ \{cpe, pce, pec\} \end{array}$	$\mathcal{K}(W) : \begin{array}{c} \{pce, pec\} \\ \times \\ \{pce, pec\} \end{array}$
$\begin{array}{c\|ccc} & L & M & R \\ \hline U & p & e & c \\ M & \textcircled{e} & p & p \\ D & c & p & p \end{array}$	$\begin{array}{c\|ccc} & L & M & R \\ \hline U & p & e & c \\ M & e & p & \textcircled{p} \\ D & c & p & p \end{array}$	$\begin{array}{c\|ccc} & L & M & R \\ \hline U & \textcircled{p} & e & c \\ M & e & p & p \\ D & c & p & p \end{array}$
RNE: (M, L) Outcomes: $\{e\}$	RNE: (M, R) Outcomes: $\{p\}$	RNE: (U, L) Outcomes: $\{p\}$

N : (M, R) is an RNE because $g(M, R) = p \in C^{\omega}_{CFO}(\{c, p\}) \cap C^{\omega}_{CMO}(\{e, p\})$ for all $\omega \in \mathcal{K}(N)$,
(M, L) is not an RNE as $g(M, L) = e \notin C^{\omega}_{CMO}(\{e, p\})$ for all $\omega \in \mathcal{K}(N)$,
(D, L) is not an RNE as $g(D, L) = c \notin C^{\omega}_{CFO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(N)$,
(U, M) is not an RNE as $g(U, M) = e \notin C^{\omega}_{CMO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(N)$,
(U, R) is not an RNE as $g(U, R) = c \notin C^{\omega}_{CFO}(\{c, p\})$ for all $\omega \in \mathcal{K}(N)$,

Our Example - Implementation in RNE - W

State of the economy: S	State of the economy: N	State of the economy: W
$f(S) = \{e\}$	$f(N) = \{p\}$	$f(W) = \{p\}$
$\mathcal{K}(S) : \begin{array}{c} \{epc\} \\ \times \\ \{cep, ecp, epc\} \end{array}$	$\mathcal{K}(N) : \begin{array}{c} \{epc, pce, pec\} \\ \times \\ \{cpe, pce, pec\} \end{array}$	$\mathcal{K}(W) : \begin{array}{c} \{pce, pec\} \\ \times \\ \{pce, pec\} \end{array}$
$\begin{array}{c\|ccc} & L & M & R \\ \hline U & p & e & c \\ M & \textcircled{e} & p & p \\ D & c & p & p \end{array}$	$\begin{array}{c\|ccc} & L & M & R \\ \hline U & p & e & c \\ M & e & p & \textcircled{p} \\ D & c & p & p \end{array}$	$\begin{array}{c\|ccc} & L & M & R \\ \hline U & \textcircled{p} & e & c \\ M & e & p & p \\ D & c & p & p \end{array}$
RNE: (M, L) Outcomes: $\{e\}$	RNE: (M, R) Outcomes: $\{p\}$	RNE: (U, L) Outcomes: $\{p\}$

W : (U, L) is an RNE because $g(M, R) = p \in C^{\omega}_{CFO}(\{c, e, p\}) \cap C^{\omega}_{CMO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(W)$,
(M, L) is not an RNE as $g(M, L) = e \notin C^{\omega}_{CFO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(W)$,
(D, L) is not an RNE as $g(D, L) = c \notin C^{\omega}_{CFO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(W)$,
(U, M) is not an RNE as $g(U, M) = e \notin C^{\omega}_{CMO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(W)$,
(U, R) is not an RNE as $g(U, R) = c \notin C^{\omega}_{CMO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(W)$,

Our Example Revisited - Implementation in RNE

These show that, the following mechanism fully implements the SCC f in RNE:

		CMO		
		L	M	R
CFO	U	p	e	c
	M	e	p	p
	D	c	p	$\boxed{p}$

There is a "danger" that emerges at S:

- (D, R) is an NE at $\hat{\omega} = (epc, epc) \in \mathcal{K}(S)$ and $g(D, R) = p \notin f(S) = \{e\}$.
- So, (D, R) is an NE at $\hat{\omega}$ that is compatible with the public choice data.
- Thus, there may be an ECE sustaining outcome p at payoff state $\hat{\omega}$, resulting in an alternative that is not f-optimal at S.
- The planner may seek to prevent the occurrence of such incidents.

Safe Implementation in Reliable Nash Equilirium

Definition

Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, we say that an SCC $f : \Theta \rightarrow \mathcal{X}$ is **safely implementable in reliable Nash equilibrium** by a mechanism $\mu = (M, g)$ if

(*i*) $f(\theta) \subset RNE^{\mu}(\theta)$ for all $\theta \in \Theta$; and

(*ii*) if $m^* \in M$ and $\theta \in \Theta$ are such that $g(m^*) \in \bigcap_{i \in N} C_i^{\omega}(O_i^{\mu}(m_{-i}^*))$ for some $\omega \in \mathcal{K}(\theta)$, then $g(m^*) \in f(\theta)$.

Condition (*ii*) says that if an action profile is an NE of μ at some ω compatible with θ, then it must result in an f-optimal alternative at θ.

Our Example Revisited - Safe Implementation in RNE

We show that, the following mechanism safely implements the SCC f in RNE:

		CMO		
		L	M	R
CFO	U	p	e	c
	M	e	p	p
	D	c	p	c

The outcome of (D, R) is changed from p to c:

- The "**danger**" that emerges at S **is eliminated**.
- Individuals' opportunity sets and the RNE **do not** change.
- In particular, (M, L) continues to be an RNE at S.
- (D, R) is not an NE at any $\omega \in \mathcal{K}(S)$ because CFO ranks p strictly above c.

▸ Details

Implementation via RNE uses Public Choice Data

- RNE uses inferences drawn from the public choice data and demands that
  - the equilibrium behavior of every individual does not depend on his private information.

▸ Reliable Nash Equilibrium

- What if the planner contemplates individual *i*'s behavior to depend on his privately observed type with or without considering others' types?

- This leads to the ex-post correlated formulation (Bergemann and Morris, 2008) and the Bayes correlated formulation (Bergemann and Morris, 2016).

▸ Ex-Post Correlated Equilibrium ▸ Bayes Correlated Equilibrium

Necessity of (Safe) Implementation in RNE

Definition

Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$ and an SCC $f : \Theta \to \mathcal{X}$, a profile of sets $\mathbf{S} := (S_i(x, \theta))_{i \in N,\ \theta \in \Theta,\ x \in f(\theta)}$ is **reliably-consistent** with f if

(*i*) for all $\theta \in \Theta$ and all $x \in f(\theta)$, $x \in \bigcap_{i \in N,\ \omega \in \mathcal{K}(\theta)} C_i^{\omega}(S_i(x, \theta))$; and

(*ii*) $x \in f(\theta)$ and $x \notin f(\tilde{\theta})$ implies that there are $j \in N$ and $\tilde{\omega} \in \mathcal{K}(\tilde{\theta})$ such that $x \notin C_j^{\tilde{\omega}}(S_j(x, \theta))$.

Moreover, a profile of sets $\mathbf{S} := (S_i(x, \theta))_{i \in N,\ \theta \in \Theta,\ x \in f(\theta)}$ is **safely-consistent** with f if (*i*) and the following hold:

(*iii*) $x \in f(\theta)$ and $x \notin f(\tilde{\theta})$ implies that for all $\tilde{\omega} \in \mathcal{K}(\tilde{\theta})$ there is $j \in N$ with $x \notin C_j^{\tilde{\omega}}(S_j(x, \theta))$.

Relation to Maskin monotonicity

The Necessity Result

Theorem (Theorem 1)

Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$ *and an SCC* $f : \Theta \to \mathcal{X}$,

(*i*) *if* f *is* **implementable in RNE**, *then there is a profile of sets that is* **reliably-consistent** *with* f; *and*

(*ii*) *if* f *is* **safely implementable in RNE**, *then there is a profile of sets that is* **safely-consistent** *with* f.

▸ Proof of Theorem 1

An Implication of Necessity

Theorem (Theorem 2)

Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$ *and an SCC* $f : \Theta \to \mathcal{X}$*, if there exists a profile of sets that is*

(*i*) *reliably-consistent with* f *and* $\mathcal{K}(\tilde{\theta}) \subset \mathcal{K}(\theta)$ *for some* $\theta, \tilde{\theta} \in \Theta$*, then* $f(\theta) \subset f(\tilde{\theta})$*;*

(*ii*) *safely-consistent with* f *and* $\mathcal{K}(\theta) \cap \mathcal{K}(\tilde{\theta}) \neq \emptyset$ *for some* $\theta, \tilde{\theta} \in \Theta$*, then* $f(\theta) = f(\tilde{\theta})$*.*

More information enriches implementation opportunities.

▸ Proof of Theorem 2

Implications of Information on Implementation

Suppose $\tilde{\theta}$ is a state of the economy at which the planner is **completely ignorant** of the payoff states, i.e., $\mathcal{K}(\tilde{\theta}) = \Omega$. Then, we have the following:

- Any SCC **implementable in RNE** must be such that $f(\tilde{\theta}) \subset \bigcap_{\theta \in \Theta} f(\theta)$.
- Any SCC that is **safely implementable in RNE** must be **constant**.
- Suppose f is singleton-valued. If f is either **implementable in RNE** or **safely implementable in RNE**, then f is **constant**.

Sufficiency

We employ the following in our sufficiency result:

Definition

Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, the environment is

(*i*) **economic** if for all $x \in X$ and for all $\theta \in \Theta$, there exist $i, j \in N$ with $i \neq j$, $\omega \in \mathcal{K}(\theta)$, and $y^i, y^j \in X$ such that $y^i P_i^\omega x$ and $y^j P_j^\omega x$; and

(*ii*) **strictly economic** if for all $x \in X$, all $\theta \in \Theta$, and all $\omega \in \mathcal{K}(\theta)$, there exist $i, j \in N$ with $i \neq j$ and $y^i, y^j \in X$ such that $y^i P_i^\omega x$ and $y^j P_j^\omega x$.

Sufficiency Result

Theorem (Theorem 3)

Let $\#N \geq 3$*. Given an inference correspondence* $\mathcal{K} : \Theta \twoheadrightarrow \Omega$ *and an SCC* $f : \Theta \to \mathcal{X}$*, if there exists a profile of sets that is*

(*i*) **reliably-consistent** *with* f *and the environment is* **economic**, *then* f *is* **implementable in RNE**; *and*

(*ii*) **safely-consistent** *with* f *and the environment is* **strictly economic**, *then* f *is* **safely implementable in RNE**.

▸ Proof of Theorem 3

Reliable Pareto Optimality / Reliable Efficiency

For any i, ω, x, let $L_i^{\omega}(x) \equiv \{y \mid xR_i^{\omega}y\}$. Then, given an inference correspondence $\mathcal{K}$,

- **reliable Pareto optimal SCC** at θ is $RPO(\theta) \equiv \{x \in X \mid x \in \cap_{\omega \in \mathcal{K}(\theta)} PO(\omega)\}$

  where, for any $\omega \in \Omega$, $PO(\omega) \equiv \{x \in X \mid \nexists y \in X \text{ such that } yP_i^{\omega}x, \forall i \in N\}$.

- x is **reliably efficient** at θ if $\exists (L_{i,x}^{\theta})_{i \in N}$ s.t. $\forall i \in N$, $x \in L_{i,x}^{\theta} \subset L_i^{\omega}(x)$ for all $\omega \in \mathcal{K}(\theta)$ and $\cup_{i \in N} L_{i,x}^{\theta} = X$. Such alternatives constitute $RE(\theta)$.

  Reliable efficiency parallels the efficiency of de Clippel (2014).

- $RE(\theta) = RPO(\theta) = PO(\pi^*(\theta))$ for all θ, whenever $\mathcal{K}(\theta) = \{\pi^*(\theta)\}$ for all θ.

  I.e., both are **extensions of efficiency** to cases with missing choice data.

- In general, $RPO(\theta) = RE(\theta)$ for all θ. (Proposition 1)

Implementability of RPO in RNE

Proposition (Proposition 2)

Let $\#N \geq 3$. If an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$ induces an **economic environment** *in which $RPO : \Theta \twoheadrightarrow X$* **is nonempty-valued**, *then RPO* **is implementable in RNE**.

Sketch of the proof:

- By Proposition 1, $RPO(\theta) = RE(\theta)$ for all $\theta \in \Theta$.
- RE being nonempty-valued implies the associated profile $\mathbf{L} \equiv (L_{i,x}^{\theta})_{i,\theta,x \in RE(\theta)}$ is reliably-consistent with RE.
- The rest of the proof follows from Theorem 3.

Concluding Remarks

- We *formalize* the implementation problem with missing data,
- *propose* a suitable notion of equilibrium along with resulting concepts of (full) implementation,
- *obtain* necessary conditions that are sufficient in economic environments,
- *establish* that more information enriches implementation opportunities,
- *analyze* implementability of a suitable efficiency notion.

Thank You.

Correlation under Private Information

- We analyze situations where individuals may use both the **incomplete public choice data** and their private information (**payoff type**) when strategizing.

- Under *incomplete information*, the main objects of interest are state contingent allocations, i.e., social choice functions (SCF).

  So, social choice sets composed of SCFs are used instead of SCCs.

  - E.g., Jackson (1991); Bergemann and Morris (2008); Barlo and Dalkıran (2022)

- In our environment, individuals' behavior can be correlated on publicly observable economic states as well.

Correlated Social Choice Sets

- Given desirable alternatives as specified by an SCC $f : \Theta \to \mathcal{X}$, a **correlated social choice set** (CSCS) associated with f is $\Phi_f := (\Phi_{f,\theta})_{\theta \in \Theta}$ with $\Phi_{f,\theta}$ being a non-empty subset of all functions mapping $\mathcal{K}(\theta)$ to $f(\theta)$, for all $\theta \in \Theta$.
- Reliability inherent in RNE parallels the following: Φ_f satisfies the **reliability criterion** if for all $\theta \in \Theta$, $\Phi_{f,\theta}$ equals **constant functions** mapping $\mathcal{K}(\theta)$ to $f(\theta)$ such that for all $x \in f(\theta)$ there is a function in $\Phi_{f,\theta}$ that maps $\mathcal{K}(\theta)$ to $\{x\}$.

▸ An Example

- The CSCS associated with f **satisfying the reliability criterion**, $\bar{\Phi}_f$, is **uniquely determined**.

Correlated Private Strategies

- Given mechanism $\mu = (M, g)$, for each state of the economy $\theta \in \Theta$, individual i's **correlated strategy** at θ is a function $\sigma_{i\theta} : \mathcal{K}_i(\theta) \to M_i$.

- We let $\Sigma_{i\theta}$ be the set of individual i's correlated strategies at $\theta \in \Theta$.

- Given mechanism μ, for each state of the economy $\theta \in \Theta$, individual i's **public correlated strategy** at θ is given by $\varsigma_{i\theta} \in M_i$.

- We let $\Sigma^P_{i\theta}$ be the set of individual i's public correlated strategies at $\theta \in \Theta$.

- **Public correlated strategies** depend **only on** θ and **not on** ω_i.

Ex-Post Correlated Equilibrium

Definition

Given a mechanism $\mu = (M, g)$, and the inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, the correlated strategy profile $\sigma^* \equiv (\sigma^*_{i\theta})_{i \in N, \theta \in \Theta} \in \Sigma$ is an **ex-post correlated equilibrium (ECE)** of μ if for all states of the economy $\theta \in \Theta$, all $i \in N$, and all $\omega_i \in \mathcal{K}_i(\theta)$,

$$g(\sigma^*_{i\theta}(\omega_i), \sigma^*_{-i\theta}(\omega_{-i})) \in C_i^{(\omega_i, \omega_{-i})}(O_i^{\mu}(\sigma^*_{-i\theta}(\omega_{-i}))), \quad \text{for all } \omega_{-i} \in \mathcal{K}_{-i}(\theta).$$

The public correlated strategy profile $\varsigma^* \equiv (\varsigma^*_{i\theta})_{i \in N, \theta \in \Theta} \in \Sigma^P$ is a **public ex-post correlated equilibrium (PECE)** of μ if for all $\theta \in \Theta$, all $i \in N$, and all $\omega_i \in \mathcal{K}_i(\theta)$,

$$g(\varsigma^*_{i\theta}, \varsigma^*_{-i\theta}) \in C_i^{(\omega_i, \omega_{-i})}(O_i^{\mu}(\varsigma^*_{-i\theta})), \quad \text{for all } \omega_{-i} \in \mathcal{K}_{-i}(\theta).$$

▸ Back to RNE

Implementation in Ex-Post Correlated Equilibrium

The implementation of CSCS Φ (not necessarily associated with an SCC f) in ECE requires: Given $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, CSCS Φ is implementable by mechanism μ in ECE if

(*i*) for all $\theta \in \Theta$ and all $\varphi_\theta \in \Phi_\theta$, there is an ECE $\sigma^* \in \Sigma$ with $g(\sigma^*_\theta(\omega)) = \varphi_\theta(\omega)$ for all $\omega \in \mathcal{K}(\theta)$; and

(*ii*) if $\sigma^* \in \Sigma$ is an ECE of μ, then for all $\theta \in \Theta$ there is $\varphi_\theta \in \Phi_\theta$ such that $g(\sigma^*_\theta(\omega)) = \varphi_\theta(\omega)$ for all $\omega \in \mathcal{K}(\theta)$.

We focus on the implementation of $\bar{\Phi}_f$, the unique CSCS associated with SCC f satisfying the **reliability criterion**.

So, for all $\theta \in \Theta$ and all $\varphi_\theta \in \bar{\Phi}_{f,\theta}$, $\varphi_\theta(\omega) = x$ for some $x \in f(\theta)$ for all $\omega \in \mathcal{K}(\theta)$.

Thus, given $\mathcal{K} : \Theta \twoheadrightarrow \Omega$ and SCC f, we obtain the implementation of $\bar{\Phi}_f$ in ECE without the need to revert to CSCSs. Ergo, we obtain the following definition.

Implementation in Ex-Post Correlated Equilibrium

Definition

Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, an SCC $f : \Theta \to \mathcal{X}$ is **implementable in ex-post correlated equilibrium** by a mechanism $\mu = (M, g)$ if

(*i*) for all $\theta \in \Theta$ and all $x \in f(\theta)$, there is an ECE $\sigma^{(x,\theta)} \in \Sigma$ with $g(\sigma_{i\theta}^{(x,\theta)}(\omega_i), \sigma_{-i\theta}^{(x,\theta)}(\omega_{-i})) = x$ for all $\omega \in \mathcal{K}(\theta)$; and

(*ii*) if $\sigma^* \in \Sigma$ is an ECE of μ, then for all $\theta \in \Theta$, there exists $y \in f(\theta)$ such that for all $\omega \in \mathcal{K}(\theta)$, $g(\sigma_{i\theta}^*(\omega_i), \sigma_{-i\theta}^*(\omega_{-i})) = y$.

▸ Back to Implementation in BCE

Implementation in Public Ex-Post Correlated Equilibrium

Definition

Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, an SCC $f : \Theta \to \mathcal{X}$ is **implementable in public ex-post correlated equilibrium** by a mechanism $\mu = (M, g)$ if

(i) for all $\theta \in \Theta$ and all $x \in f(\theta)$, there is a PECE $\varsigma^{(x,\theta)} \in \Sigma^P$ with $g(\varsigma_{i\theta}^{(x,\theta)}, \varsigma_{-i\theta}^{(x,\theta)}) = x$; and

(ii) if $\varsigma^* \in \Sigma^P$ is a PECE of μ, then for all $\theta \in \Theta$, $g(\varsigma_{i\theta}^*, \varsigma_{-i\theta}^*) \in f(\theta)$.

Implementation in Public Ex-Post Correlated Equilibrium

Definition

Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, an SCC $f : \Theta \rightarrow \mathcal{X}$ is **implementable in public ex-post correlated equilibrium** by a mechanism $\mu = (M, g)$ if

(*i*) for all $\theta \in \Theta$ and all $x \in f(\theta)$, there is a PECE $\varsigma^{(x,\theta)} \in \Sigma^P$ with $g(\varsigma_{i\theta}^{(x,\theta)}, \varsigma_{-i\theta}^{(x,\theta)}) = x$; and

(*ii*) if $\varsigma^* \in \Sigma^P$ is a PECE of μ, then for all $\theta \in \Theta$, $g(\varsigma^*_{i\theta}, \varsigma^*_{-i\theta}) \in f(\theta)$.

Remark

Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$*, an SCC* $f : \Theta \rightarrow \mathcal{X}$ *is* **implementable in PECE** *by a mechanism* μ **if and only if** *it is* **implementable in RNE** *via* μ*.*

Reason: A profile of RNE across the states of the economy is equivalent to a PECE.

Implementation in ECE implies Implementation in PECE

Proposition (Proposition 3)

Given $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, if an SCC $f : \Theta \to \mathcal{X}$ is **implementable in ECE** *via a mechanism μ, then it is* **implementable in PECE** *via μ. But the reverse does not hold.*

Arguments in the proof:

- We can transform any ECE strategy to a PECE strategy by fixing any one of the compatible payoff states with the help of the **reliability criterion**.
- Every PECE is an ECE that is **invariant** across individuals' payoff types.
- The example we use to show that the reverse does not hold also shows:

Mechanisms implementing an SCC f in PECE may possess 'bad' ECE.

The Example of Proposition 3

Dismissing bad ECE via Double Implementation

- To dispense with ▸ 'bad' ECE, one may consider **double implementation** in PECE and ECE (as in Saijo et al. (2007)):

  Demand (*i*) of implementation in PECE and (*ii*) of implementation in ECE.

- By replacing (*ii*) **of implementation in ECE** by the following strengthens the above **double implementation**:

(*ii*′) if $\sigma^* \in \Sigma$ is an ECE of μ, then for all $\theta \in \Theta$ and all $\omega \in \mathcal{K}(\theta)$, $g(\sigma^*_\theta(\omega)) \in f(\theta)$.

Dismissing bad ECE via Double Implementation

- Double implementation based on (i) of implementation in PECE and (ii') above requires the planner to consider individuals' **private information**.
- We can handle unwanted ECE outcomes by using only the **public choice data**
  - as any **ECE** of mechanism μ induces an **NE** of μ at every $\omega \in \mathcal{K}(\theta)$ for any $\theta \in \Theta$.
- Hence, dismissing 'bad' NE ensures the elimination of unwanted ECE as well.
- This leads us to (i) of implementation in PECE and the following:

(ii'') if $m^* \in M$ and $\theta \in \Theta$ are s.t. $g(m^*) \in \bigcap_{i \in N} C_i^{\omega}(O_i^{\mu}(m_{-i}^*))$ for some $\omega \in \mathcal{K}(\theta)$, then $g(m^*) \in f(\theta)$.

We attain the **motivation for safe implementation in RNE**: **(i) of implementation in PECE and (ii'')** is equivalent to **safe implementation in RNE**. (Remark 2)

Bayes Correlated Equilibrium

- For each state of the economy θ, and for each payoff state compatible with θ, $\omega \in \mathcal{K}(\theta)$, individual i's preferences admit a **conditional expected utility** representation via the expected utility function $u_{i\theta}(\,\cdot \mid \omega_i) : X \to \mathbb{R}$.
- For each θ, i's **belief** at his payoff type $\omega_i \in \mathcal{K}_i(\theta)$ is $p_{i\theta}(\omega_i) \in \Delta(\mathcal{K}_{-i}(\theta))$, where $\Delta(\mathcal{K}_{-i}(\theta))$ denotes the probability simplex on $\mathcal{K}_{-i}(\theta)$.

Definition

Given mechanism μ, the inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, and the **belief profile p**, the correlated strategy profile $\sigma^* \in \Sigma$ is a **Bayes correlated equilibrium (BCE)** of μ if for all $i \in N$, for all $\theta \in \Theta$, and for all $\omega_i \in \mathcal{K}_i(\theta)$,

$$\sum_{\omega_{-i} \in \mathcal{K}_{-i}(\theta)} p_{i\theta}(\omega_{-i}|\omega_i) \left[u_{i\theta}\left(g(\sigma^*_{i\theta}(\omega_i), \sigma^*_{-i\theta}(\omega_{-i})) \mid \omega_i \right) - u_{i\theta}\left(g(m_i, \sigma^*_{-i\theta}(\omega_{-i})) \mid \omega_i \right) \right] \geq 0,$$

for all $m_i \in M_i$.

Bayes Correlated Equilibrium

A public correlated strategy profile $\varsigma^* \in \Sigma^P$ is a **public Bayes correlated equilibrium** (PBCE) of μ if for all i, all θ, and all $\omega_i \in \mathcal{K}_i(\theta)$,

$$\sum_{\omega_{-i} \in \mathcal{K}_{-i}(\theta)} p_{i\theta}(\omega_{-i}|\omega_i) \ \left[u_{i\theta}\left(g(\varsigma^*_\theta)|\omega_i\right) - u_{i\theta}\left(g(m_i, \varsigma^*_{-i\theta})|\omega_i\right) \right] \geq 0$$

for all $m_i \in M_i$.

The PBCE and the PECE are equivalent as ς^* is a public correlated strategy profile.

Since any RNE profile is equivalent to a PECE, the equivalence of PBCE and PECE delivers further **robustness properties for RNE** as

- **every RNE profile induces a PBCE and a BCE no matter what the beliefs are**.

Back to RNE

Implementation in Bayes Correlated Equilibrium

Given $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, the belief profile $\mathbf{p}$, and an SCC $f : \Theta \to \mathcal{X}$ we say that a CSCS Φ_f associated with f is **implementable in BCE** by a mechanism μ if

(*i*) for all $\theta \in \Theta$ and all $\varphi_\theta \in \Phi_{f,\theta}$, there exists a BCE $\sigma^{(\varphi_\theta)} \in \Sigma$ with $g(\sigma_\theta^{(\varphi_\theta)}(\omega)) = \varphi_\theta(\omega)$ for all $\omega \in \mathcal{K}(\theta)$; and

(*ii*) if $\sigma^* \in \Sigma$ is a BCE of μ, then for all $\theta \in \Theta$, there exists $\varphi \in \Phi_{f,\theta}$ such that $g(\sigma_\theta^*(\omega)) = \varphi(\omega)$ for all $\omega \in \mathcal{K}(\theta)$.

For the unique CSCS associated with f under the reliability criterion, $\bar{\Phi}_f$,

- (*i*) above becomes: for all $\theta \in \Theta$ and all $x \in f(\theta)$, there is a BCE $\sigma^{(x,\theta)} \in \Sigma$ with $g(\sigma_\theta^{(x,\theta)}(\omega)) = x$ for all $\omega \in \mathcal{K}(\theta)$
- (*ii*) above becomes: if $\sigma^* \in \Sigma$ is a BCE of μ, then for all $\theta \in \Theta$, there exists $y \in f(\theta)$ such that $g(\sigma_\theta^*(\omega)) = y$ for all $\omega \in \mathcal{K}(\theta)$.

Implementation in Bayes Correlated Equilibrium

Given $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, the belief profile $\mathbf{p}$, and an SCC $f : \Theta \to \mathcal{X}$ we say that a CSCS Φ_f associated with f is **implementable in BCE** by a mechanism μ if

(*i*) for all $\theta \in \Theta$ and all $\varphi_\theta \in \Phi_{f,\theta}$, there exists a BCE $\sigma^{(\varphi_\theta)} \in \Sigma$ with $g(\sigma_\theta^{(\varphi_\theta)}(\omega)) = \varphi_\theta(\omega)$ for all $\omega \in \mathcal{K}(\theta)$; and

(*ii*) if $\sigma^* \in \Sigma$ is a BCE of μ, then for all $\theta \in \Theta$, there exists $\varphi \in \Phi_{f,\theta}$ such that $g(\sigma_\theta^*(\omega)) = \varphi(\omega)$ for all $\omega \in \mathcal{K}(\theta)$.

For the unique CSCS associated with f under the reliability criterion, $\bar{\Phi}_f$,

- implementation in BCE shares many similarities with ▸ implementation in ECE.

As implementation in RNE is equivalent to implementation in PECE, the equivalence of the PBCE and the PECE implies

- **implementation in RNE** is **equivalent** to **implementation in PBCE**.

Concluding Remarks

- We *formalize* the implementation problem with missing data,
- *propose* a suitable notion of equilibrium along with resulting concepts of (full) implementation,
- *obtain* necessary conditions that are sufficient in economic environments,
- *establish* that more information enriches implementation opportunities,
- *analyze* implementability of a suitable efficiency notion.

Thank You.

An Example: Reliable Pareto Optimality

	(S)trong		*(N)ormal*		*(W)eak*	
	CFO	CMO	CFO	CMO	CFO	CMO
$\{c, e, p\}$	$\{e\}$				$\{p\}$	$\{p\}$
$\{c, e\}$						
$\{c, p\}$	$\{p\}$		$\{p\}$			
$\{e, p\}$		$\{e\}$		$\{p\}$		

- At S, $e \in \bigcap_{\omega \in \mathcal{K}(S)} PO(\omega)$ and $c, p \notin PO(\omega)$ for $\omega = (epc, epc) \in \mathcal{K}(S)$.
- At N, $p \in \bigcap_{\omega \in \mathcal{K}(N)} PO(\omega)$, and $c, e \notin PO(\omega)$ for $\omega = (pec, pec) \in \mathcal{K}(N)$.
- At W, $PO(\omega) = \{p\}$ for all $\omega \in \mathcal{K}(W)$.

▸ Back

Our Example - Safe Implementation in RNE - S

State of the economy: S	State of the economy: N	State of the economy: W
$f(S) = \{e\}$	$f(N) = \{p\}$	$f(W) = \{p\}$
$\mathcal{K}(S) : \begin{array}{c} \{epc\} \\ \times \\ \{cep, ecp, epc\} \end{array}$	$\mathcal{K}(N) : \begin{array}{c} \{epc, pce, pec\} \\ \times \\ \{cpe, pce, pec\} \end{array}$	$\mathcal{K}(W) : \begin{array}{c} \{pce, pec\} \\ \times \\ \{pce, pec\} \end{array}$
$\begin{array}{cccc} & L & M & R \\ \hline U & p & e & c \\ M & \textcircled{e} & p & p \\ D & c & p & c \end{array}$	$\begin{array}{cccc} & L & M & R \\ \hline U & p & e & c \\ M & e & p & \textcircled{p} \\ D & c & p & c \end{array}$	$\begin{array}{cccc} & L & M & R \\ \hline U & \textcircled{p} & e & c \\ M & e & p & p \\ D & c & p & c \end{array}$
RNE: (M, L) Outcomes: $\{e\}$	RNE: (M, R) Outcomes: $\{p\}$	RNE: (U, L) Outcomes: $\{p\}$

S : (M, L) is an RNE because $g(M, L) = e \in C^{\omega}_{CFO}(\{c, e, p\}) \cap C^{\omega}_{CMO}(\{e, p\})$ for all $\omega \in \mathcal{K}(S)$,
(U, L) is not an RNE as $g(U, L) = p \notin C^{\omega}_{CFO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(S)$,
(D, L) is not an RNE as $g(D, L) = c \notin C^{\omega}_{CFO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(S)$,
(M, M) is not an RNE as $g(M, M) = p \notin C^{\omega}_{CFO}(\{e, p\})$ for all $\omega \in \mathcal{K}(S)$,
(D, M) is not an RNE as $g(D, M) = p \notin C^{\omega}_{CFO}(\{e, p\})$ for all $\omega \in \mathcal{K}(S)$,
(U, R) is not an RNE as $g(U, R) = c \notin C^{\omega}_{CFO}(\{c, p\})$ for all $\omega \in \mathcal{K}(S)$,
(M, R) is not an RNE as $g(M, R) = p \notin C^{\omega}_{CMO}(\{e, p\})$ for all $\omega \in \mathcal{K}(S)$,
(D, R) is not an RNE as $g(D, R) = c \notin C^{\omega}_{CFO}(\{c, p\})$ for all $\omega \in \mathcal{K}(S)$.

Our Example - Safe Implementation in RNE - N

State of the economy: S	State of the economy: N	State of the economy: W
$f(S) = \{e\}$	$f(N) = \{p\}$	$f(W) = \{p\}$
$\mathcal{K}(S) : \begin{array}{c} \{epc\} \\ \times \\ \{cep, ecp, epc\} \end{array}$	$\mathcal{K}(N) : \begin{array}{c} \{epc, pce, pec\} \\ \times \\ \{cpe, pce, pec\} \end{array}$	$\mathcal{K}(W) : \begin{array}{c} \{pce, pec\} \\ \times \\ \{pce, pec\} \end{array}$
$\begin{array}{c\|ccc} & L & M & R \\ \hline U & p & e & c \\ M & \textcircled{e} & p & p \\ D & c & p & c \end{array}$	$\begin{array}{c\|ccc} & L & M & R \\ \hline U & p & e & c \\ M & e & p & \textcircled{p} \\ D & c & p & c \end{array}$	$\begin{array}{c\|ccc} & L & M & R \\ \hline U & \textcircled{p} & e & c \\ M & e & p & p \\ D & c & p & c \end{array}$
RNE: (M, L) Outcomes: $\{e\}$	RNE: (M, R) Outcomes: $\{p\}$	RNE: (U, L) Outcomes: $\{p\}$

N : (M, R) is an RNE because $g(M, R) = p \in C^{\omega}_{CFO}(\{c, p\}) \cap C^{\omega}_{CMO}(\{e, p\})$ for all $\omega \in \mathcal{K}(N)$,
(M, L) is not an RNE as $g(M, L) = e \notin C^{\omega}_{CMO}(\{e, p\})$ for all $\omega \in \mathcal{K}(N)$,
(D, L) is not an RNE as $g(D, L) = c \notin C^{\omega}_{CFO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(N)$,
(U, M) is not an RNE as $g(U, M) = e \notin C^{\omega}_{CMO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(N)$,
(U, R) is not an RNE as $g(U, R) = c \notin C^{\omega}_{CFO}(\{c, p\})$ for all $\omega \in \mathcal{K}(N)$,
(D, R) is not an RNE as $g(D, R) = c \notin C^{\omega}_{CFO}(\{c, p\})$ for all $\omega \in \mathcal{K}(N)$.

Our Example - Safe Implementation in RNE - W

State of the economy: S	State of the economy: N	State of the economy: W
$f(S) = \{e\}$	$f(N) = \{p\}$	$f(W) = \{p\}$
$\mathcal{K}(S): \begin{array}{c} \{epc\} \\ \times \\ \{cep, ecp, epc\} \end{array}$	$\mathcal{K}(N): \begin{array}{c} \{epc, pce, pec\} \\ \times \\ \{cpe, pce, pec\} \end{array}$	$\mathcal{K}(W): \begin{array}{c} \{pce, pec\} \\ \times \\ \{pce, pec\} \end{array}$
$\begin{array}{cccc} & L & M & R \\ \hline U & p & e & c \\ M & \textcircled{e} & p & p \\ D & c & p & c \end{array}$	$\begin{array}{cccc} & L & M & R \\ \hline U & p & e & c \\ M & e & p & \textcircled{p} \\ D & c & p & c \end{array}$	$\begin{array}{cccc} & L & M & R \\ \hline U & \textcircled{p} & e & c \\ M & e & p & p \\ D & c & p & c \end{array}$
RNE: (M, L) Outcomes: $\{e\}$	RNE: (M, R) Outcomes: $\{p\}$	RNE: (U, L) Outcomes: $\{p\}$

W : (U, L) is an RNE because $g(M, R) = p \in C^{\omega}_{CFO}(\{c, e, p\}) \cap C^{\omega}_{CMO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(W)$,
(M, L) is not an RNE as $g(M, L) = e \notin C^{\omega}_{CFO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(W)$,
(D, L) is not an RNE as $g(D, L) = c \notin C^{\omega}_{CFO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(W)$,
(U, M) is not an RNE as $g(U, M) = e \notin C^{\omega}_{CMO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(W)$,
(U, R) is not an RNE as $g(U, R) = c \notin C^{\omega}_{CMO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(W)$,
(D, R) is not an RNE as $g(D, R) = c \notin C^{\omega}_{CFO}(\{c, p\})$ for all $\omega \in \mathcal{K}(W)$.

▸ Back

Maskin Monotonicity

For any i, ω, x, let $L_i^{\omega}(x) \equiv \{y \mid x R_i^{\omega} y\}$ be i's **lower contour set** of x at ω.

Definition

Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, an SCC $f : \Theta \to \mathcal{X}$ is

(*i*) **reliably Maskin monotonic** if $x \in f(\theta)$ and $L_i^{\omega}(x) \subseteq L_i^{\tilde{\omega}}(x)$ for all $i \in N$, all $\omega \in \mathcal{K}(\theta)$, and all $\tilde{\omega} \in \mathcal{K}(\tilde{\theta})$ implies $x \in f(\tilde{\theta})$.

(*ii*) **safely Maskin monotonic**, if the following holds: if $x \in f(\theta)$ and for some $\omega \in \mathcal{K}(\theta)$ and some $\tilde{\omega} \in \mathcal{K}(\tilde{\theta})$ we have $L_i^{\omega}(x) \subseteq L_i^{\tilde{\omega}}(x)$ for all $i \in N$, then $x \in f(\tilde{\theta})$.

Equivalence of Consistency and Maskin Monotonicity

Proposition

Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$ *and an SCC* $f : \Theta \rightarrow \mathcal{X}$*, there is a profile of sets* $\mathbf{S} := (S_i(x, \theta))_{i \in N,\ \theta \in \Theta,\ x \in f(\theta)}$ *that is*

(*i*) **reliably-consistent** *with* f *if and only if* f *is* **reliably Maskin monotonic**.

(*ii*) **safely-consistent** *with* f *if and only if* f *is* **safely Maskin monotonic**.

▸ Back

Proof of Theorem 1 - I

Reliable-consistency:

- Suppose $\mu = (M, g)$ implements f in RNE. Hence, for all θ and all $x \in f(\theta)$, there is $m^x \in M$ such that $g(m^x) = x$ and $x \in \cap_{i \in N, \omega \in \mathcal{K}(\theta)} C_i^{\omega}(O_i^{\mu}(m_{-i}^x))$.
- Let **S** be defined by $S_i(x, \theta) \equiv O_i^{\mu}(m_{-i}^x)$ for all i, θ, x in $f(\theta)$.
- (i) of reliable-consistency holds as m^x is an RNE of μ at θ.
- For (ii) of reliable-consistency, suppose $x \in f(\theta)$ and $x \notin f(\tilde{\theta})$, $\theta, \tilde{\theta} \in \Theta$.

  If $x \in \bigcap_{i \in N,\ \tilde{\omega} \in \mathcal{K}(\tilde{\theta})} C_i^{\tilde{\omega}}(S_i(x, \theta)) = \bigcap_{i \in N,\ \tilde{\omega} \in \mathcal{K}(\tilde{\theta})} C_i^{\tilde{\omega}}(O_i^{\mu}(m_{-i}^x))$, then $m^x \in M$ is also an RNE at $\tilde{\theta}$.

  Thus, by (ii) of implementation in RNE, $x \in f(\tilde{\theta})$, a contradiction.

Proof of Theorem 1 - II

Safe-consistency:

- Suppose $\mu = (M, g)$ safely implements f in RNE. So, for all θ and all $x \in f(\theta)$, there is $m^x \in M$ such that $g(m^x) = x$ and $x \in \cap_{i \in N, \omega \in \mathcal{K}(\theta)} C_i^{\omega}(O_i^{\mu}(m_{-i}^x))$.
- Let **S** be defined by $S_i(x, \theta) \equiv O_i^{\mu}(m_{-i}^x)$ for all i, θ, x in $f(\theta)$.
- (i) of safe-consistency holds as m^x is an RNE of μ at θ.
- For (iii) of safe-consistency, suppose $x \in f(\theta)$ and $x \notin f(\tilde{\theta})$, $\theta, \tilde{\theta} \in \Theta$.

  If there is $\tilde{\omega} \in \mathcal{K}(\tilde{\theta})$ such that $x \in \bigcap_{i \in N} C_i^{\tilde{\omega}}(S_i(x, \theta)) = \bigcap_{i \in N} C_i^{\tilde{\omega}}(O_i^{\mu}(m_{-i}^x))$.

  Thus, by (ii) of safe implementation in RNE, $x \in f(\tilde{\theta})$, a contradiction.

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Proof of Theorem 2

Theorem 2 - (i)

- Suppose the planner with knowledge $\mathcal{K}$ infers there is $\mathbf{S} \equiv (S_i(x,\theta))_{i,\theta,x\in f(\theta)}$ reliably-consistent with f and $\mathcal{K}(\tilde{\theta}) \subset \mathcal{K}(\theta)$ with $\theta, \tilde{\theta} \in \Theta$.
- By (i) of reliable-consistency, $x \in f(\theta)$ implies $x \in \bigcap_{i\in N,\ \omega\in\mathcal{K}(\theta)} C_i^{\omega}(S_i(x,\theta))$.
- As $\mathcal{K}(\tilde{\theta}) \subset \mathcal{K}(\theta)$, $x \in \bigcap_{i\in N,\ \tilde{\omega}\in\mathcal{K}(\tilde{\theta})} C_i^{\tilde{\omega}}(S_i(x,\theta))$.
- Thus, $x \notin f(\tilde{\theta})$ produces a contradiction to (ii) of reliable-consistency.
- Therefore, $x \in f(\tilde{\theta})$.

Proof of Theorem 2

Theorem 2 - (ii)

- Suppose the planner with knowledge $\mathcal{K}$ infers there is $\mathbf{S} \equiv (S_i(x,\theta))_{i,\theta,x\in f(\theta)}$ safely-consistent with f and there is $\omega^* \in \mathcal{K}(\theta) \cap \mathcal{K}(\tilde{\theta}) = \emptyset$ with $\theta, \tilde{\theta} \in \Theta$.
- By (i) of safe-consistency, $x \in f(\theta)$ implies $x \in \bigcap_{i\in N,\ \omega\in\mathcal{K}(\theta)} C_i^{\omega}(S_i(x,\theta))$ and hence $x \in \bigcap_{i\in N} C_i^{\omega^*}(S_i(x,\theta))$.
- But, $x \notin f(\tilde{\theta})$ implies that for all $\tilde{\omega} \in \mathcal{K}(\tilde{\theta})$, $x \notin \bigcap_{i\in N} C_i^{\tilde{\omega}}(S_i(x,\theta))$ which implies (on account of $\omega^* \in \mathcal{K}(\tilde{\theta})$) $x \notin \bigcap_{i\in N} C_i^{\omega^*}(S_i(x,\theta))$, a contradiction.
- Hence, $x \in f(\tilde{\theta})$. As θ and $\tilde{\theta}$ can be interchanged, we obtain $f(\theta) = f(\tilde{\theta})$.

▸ Back

Proof of Theorem 3

Suppose that given $\mathcal{K}$ and f, the planner infers that

- the environment is economic (*strictly economic*) and that
- there is $\mathbf{S} := (S_i(x,\theta))_{i,\theta,x\in f(\theta)}$ reliably-consistent (*safely-consistent*) with f.

We use the *canonical mechanism* $\mu = (M, g)$:

- $M_i := \Theta \times X \times \mathbb{N}$, where $m_i = (\theta^{(i)}, x^{(i)}, k^{(i)}) \in M_i$.
- The *outcome function* $g : M \to X$ is given by

Rule 1 : $g(m) = x$ if $m_i = (\theta, x, \cdot)$ for all $i \in N$ with $x \in f(\theta)$,

Rule 2 : $g(m) = \begin{cases} x' & \text{if } x' \in S_j(x,\theta) \\ x & \text{otherwise.} \end{cases}$ if $m_i = (\theta, x, \cdot)$ for all $i \in N \setminus \{j\}$ with $x \in f(\theta)$, and $m_j = (\theta', x', \cdot) \neq (\theta, x, \cdot)$,

Rule 3 : $g(m) = x^{(i^*)}$ where $i^* = \min\{j \in N : k^{(j)} \geq \max_{i' \in N} k^{(i')}\}$ otherwise.

▸ Back

An Example for the Reliability Criterion

Let $N = \{1, 2\}$, $X = \{x, y, z\}$, $\Theta = \{\theta_1, \theta_2\}$ and $\Omega_i = \{\omega_{i1}, \omega_{i2}, \omega_{i3}\}$ for $i = 1, 2$ with

- $\mathcal{K}(\theta_1) = \{(\omega_{11}, \omega_{21}), (\omega_{11}, \omega_{22}), (\omega_{12}, \omega_{21}), (\omega_{12}, \omega_{22})\}$ and
- $\mathcal{K}(\theta_2) = \{(\omega_{12}, \omega_{22}), (\omega_{12}, \omega_{23}), (\omega_{13}, \omega_{22}), (\omega_{13}, \omega_{23})\}$.
- The given SCC f is s.t. $f(\theta_1) = \{x, y\}$ and $f(\theta_2) = \{z\}$.
- A CSCS associated with f, Φ_f, could be $\Phi_{f,\theta_1} = \{\langle x, x, x, x\rangle, \langle y, y, y, x\rangle\}$ and $\Phi_{f,\theta_2} = \{\langle z, z, z, z\rangle\}$.

  (e.g., $\langle y, y, y, x\rangle$ denotes the function on $\mathcal{K}(\theta_1)$ which maps the payoff state $(\omega_{12}, \omega_{22})$ to x and all the other payoff states in $\mathcal{K}(\theta_1)$ to y).

An Example for the Reliability Criterion

Let $N = \{1, 2\}$, $X = \{x, y, z\}$, $\Theta = \{\theta_1, \theta_2\}$ and $\Omega_i = \{\omega_{i1}, \omega_{i2}, \omega_{i3}\}$ for $i = 1, 2$ with

- $\mathcal{K}(\theta_1) = \{(\omega_{11}, \omega_{21}), (\omega_{11}, \omega_{22}), (\omega_{12}, \omega_{21}), (\omega_{12}, \omega_{22})\}$ and
- $\mathcal{K}(\theta_2) = \{(\omega_{12}, \omega_{22}), (\omega_{12}, \omega_{23}), (\omega_{13}, \omega_{22}), (\omega_{13}, \omega_{23})\}$.
- The given SCC f is s.t. $f(\theta_1) = \{x, y\}$ and $f(\theta_2) = \{z\}$.
- The **CSCS associated with f that satisfies the reliability criterion**, $\bar{\Phi}_f$, is uniquely determined.
- In this example, $\bar{\Phi}_{f,\theta_1} = \{\langle x, x, x, x\rangle, \langle y, y, y, y\rangle\}$ and $\bar{\Phi}_{f,\theta_2} = \{\langle z, z, z, z\rangle\}$.

▸ Back

Implementation in ECE implies Implementation in PECE

- $N = \{1, 2\}$, $X = \{x, y\}$, $\Theta = \{\theta_1, \theta_2\}$, Ω_i equals all strict rankings of $\{x, y\}$ (where xy means that i strictly prefers x to y).
- $\mathcal{K}_i(\theta_1) = \{xy, yx\}$, and $\mathcal{K}_i(\theta_2) = \{xy\}$ for all $i = 1, 2$.
- The SCC f is such that $f(\theta_1) = \{y\}$ and $f(\theta_2) = \{x, y\}$.
- The following mechanism implements f **in PECE** but **not in ECE**:

Individual 2

Individual 1	a_1	a_2
a_1	x	y
a_2	y	y

- This mechanism has a ▸ **'bad' ECE** that varies with an individuals' payoff type.

A 'bad' ECE

The following mechanism implements f **in PECE** (**in RNE**) but has a **'bad' ECE**:

		Individual 2	
		a_1	a_2
Individual 1	a_1	x	y
	a_2	y	y

- $N = \{1, 2\}$, $X = \{x, y\}$, $\Theta = \{\theta_1, \theta_2\}$, Ω_i equals all strict rankings of $\{x, y\}$. $\mathcal{K}_i(\theta_1) = \{xy, yx\}$, and $\mathcal{K}_i(\theta_2) = \{xy\}$ for all $i = 1, 2$.
- The SCC f is such that $f(\theta_1) = \{y\}$ and $f(\theta_2) = \{x, y\}$.
- Let σ^* be s.t. $\sigma^*_{i\theta_1}(xy) = a_1$, $\sigma^*_{i\theta_1}(yx) = a_2$, and $\sigma^*_{i\theta_2}(xy) = a_1$, $i = 1, 2$.
- σ^* is **an ECE** s.t. $g(\sigma^*_{\theta_1}(xy, xy)) = x \notin f(\theta_1) = \{y\}$ with $(xy, xy) \in \mathcal{K}(\theta_1)$.

▸ Back to Safe Implemention in RNE   ▸ Back to Proposition 3

Concluding Remarks

- We *formalize* the implementation problem with missing data,
- *propose* a suitable notion of equilibrium along with resulting concepts of (full) implementation,
- *obtain* necessary conditions that are sufficient in economic environments,
- *establish* that more information enriches implementation opportunities,
- *analyze* implementability of a suitable efficiency notion.