Statistical Multipath Channel Models

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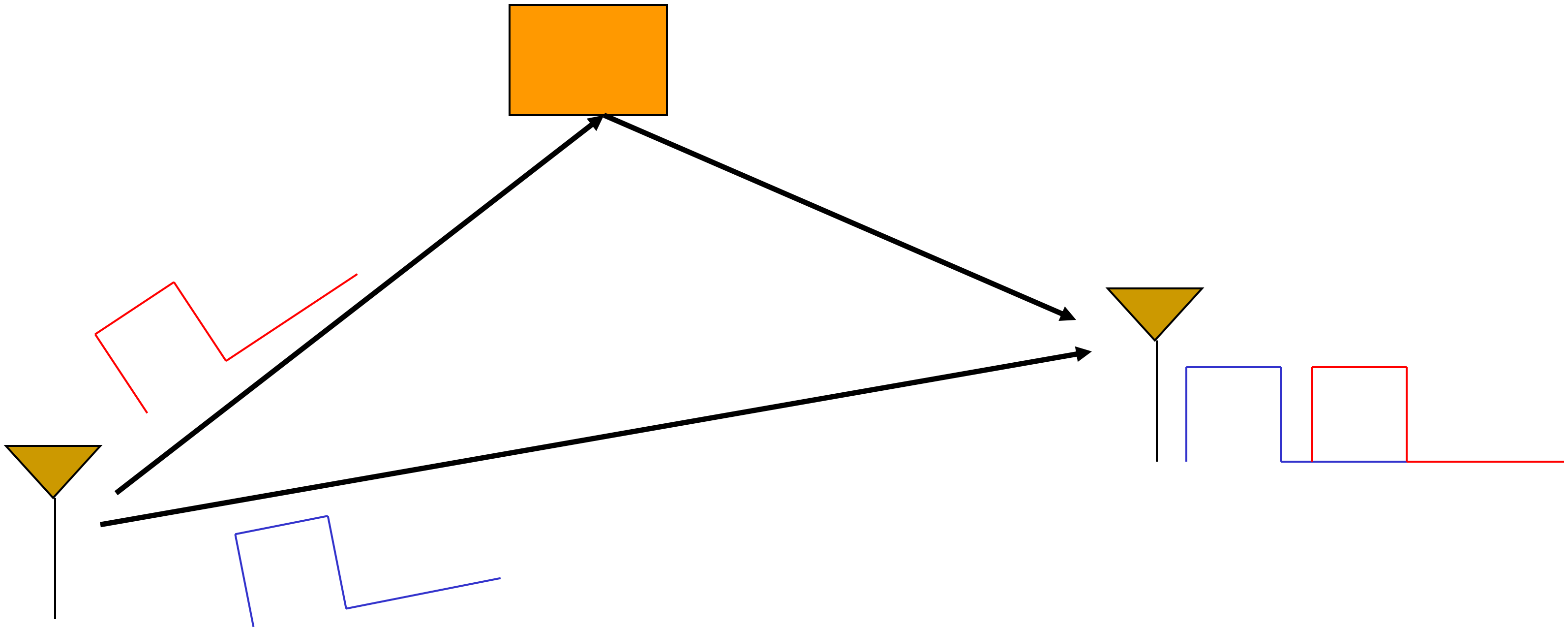
- So far
  - Large scale fading
  - Path loss and shadowing
- We will model the mobile communications channel
  - as a multipath channel
  - with a random time-varying impulse response.



- Different paths
  - Arrive at different times (delay)
  - Have different strengths (gain)
  - May have different carrier frequencies due to mobility and direction (Doppler)

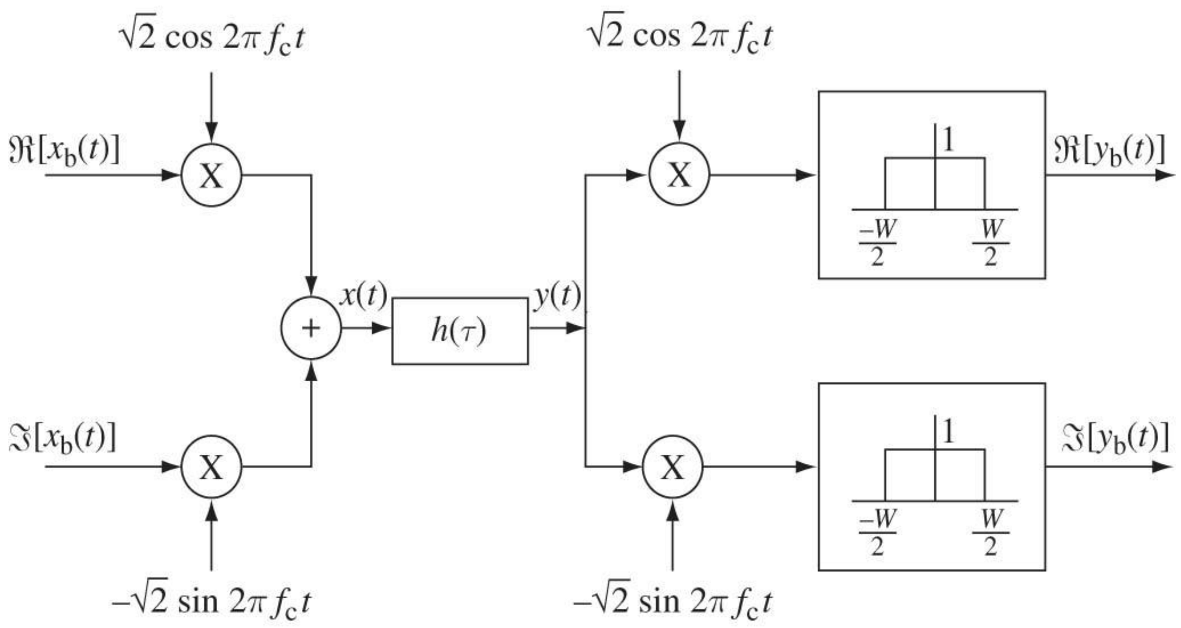
Signal Model

- Transmitted bandpass signal with equivalent lowpass signal

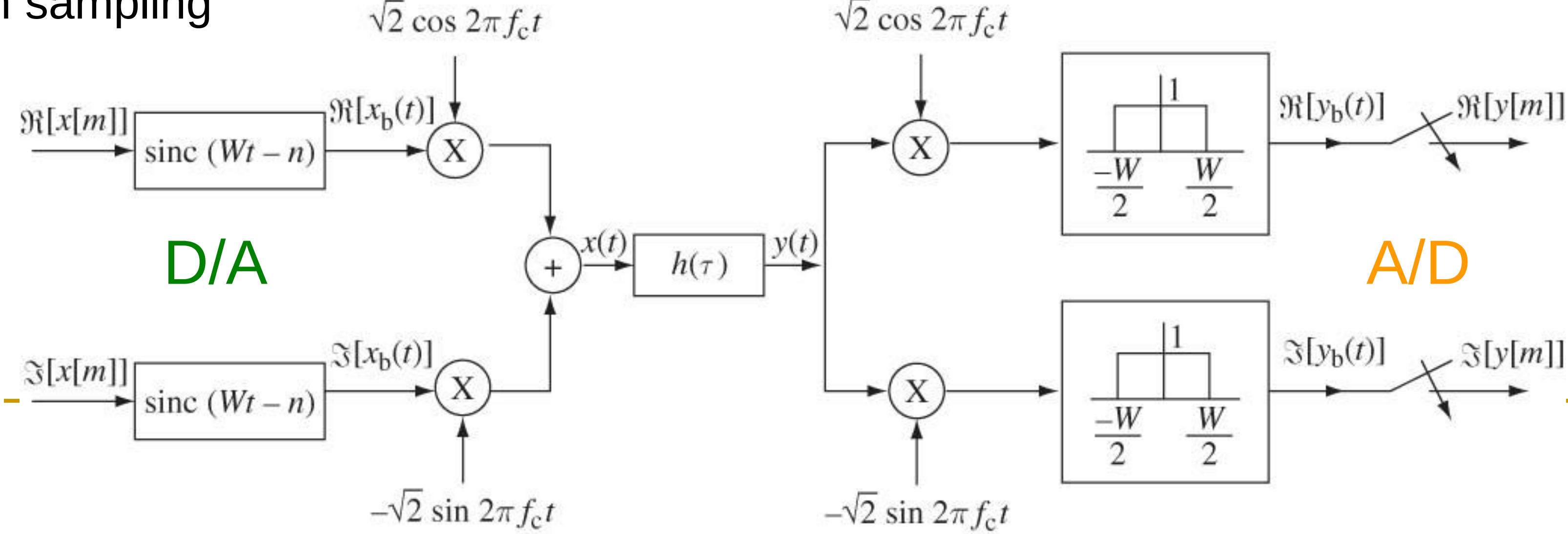
$$\boldsymbol{s}(\boldsymbol{t}) = \Re\left\{\boldsymbol{u}(\boldsymbol{t})\boldsymbol{e}^{j2\pi \boldsymbol{f}_c \boldsymbol{t}}\right\} = \Re\{\boldsymbol{u}(\boldsymbol{t})\}\cos\omega_{\boldsymbol{c}}\boldsymbol{t} - \Im\{\boldsymbol{u}(\boldsymbol{t})\}\sin\omega_{\boldsymbol{c}}\boldsymbol{t}$$

- Communication takes place at $[\boldsymbol{f}_c - \boldsymbol{B}_s/2, \boldsymbol{f}_c + \boldsymbol{B}_s/2]$
- Processing takes place at baseband $[-\boldsymbol{B}_s/2, \boldsymbol{B}_s/2]$
- Bandwidth interchangeably by B_S, B, W

Example: I/Q modulation/demodulation



with sampling



Received signal without noise

Number of resolvable multipath components

Message signal

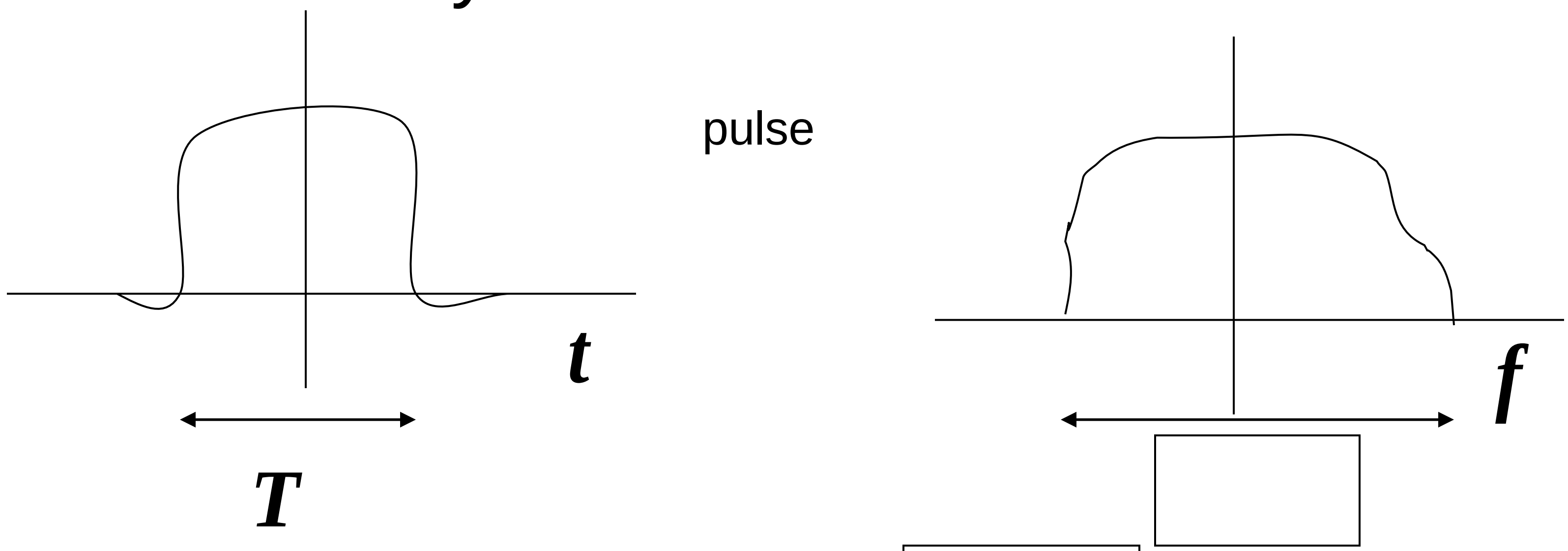
$$r(t) = \Re\left\{ \sum_{n=1}^{N(t)} \alpha_n(t)\, u(t-\tau_n(t))\, e^{j\left\{2\pi f_c (t-\tau_n(t)) + \phi_{D_n}(t)\right\}} \right\}$$

amplitude

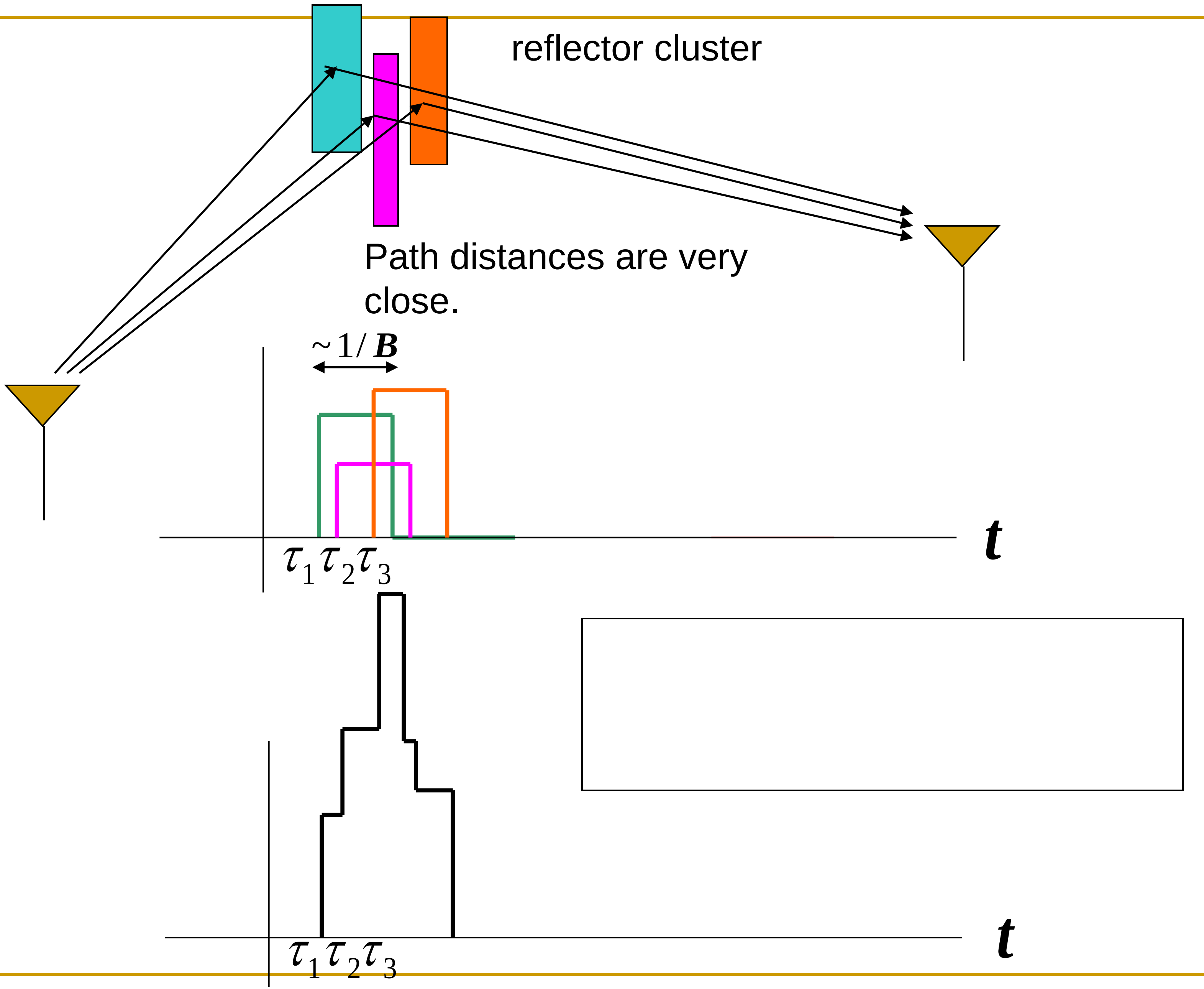
delay

Phase due to Doppler frequency

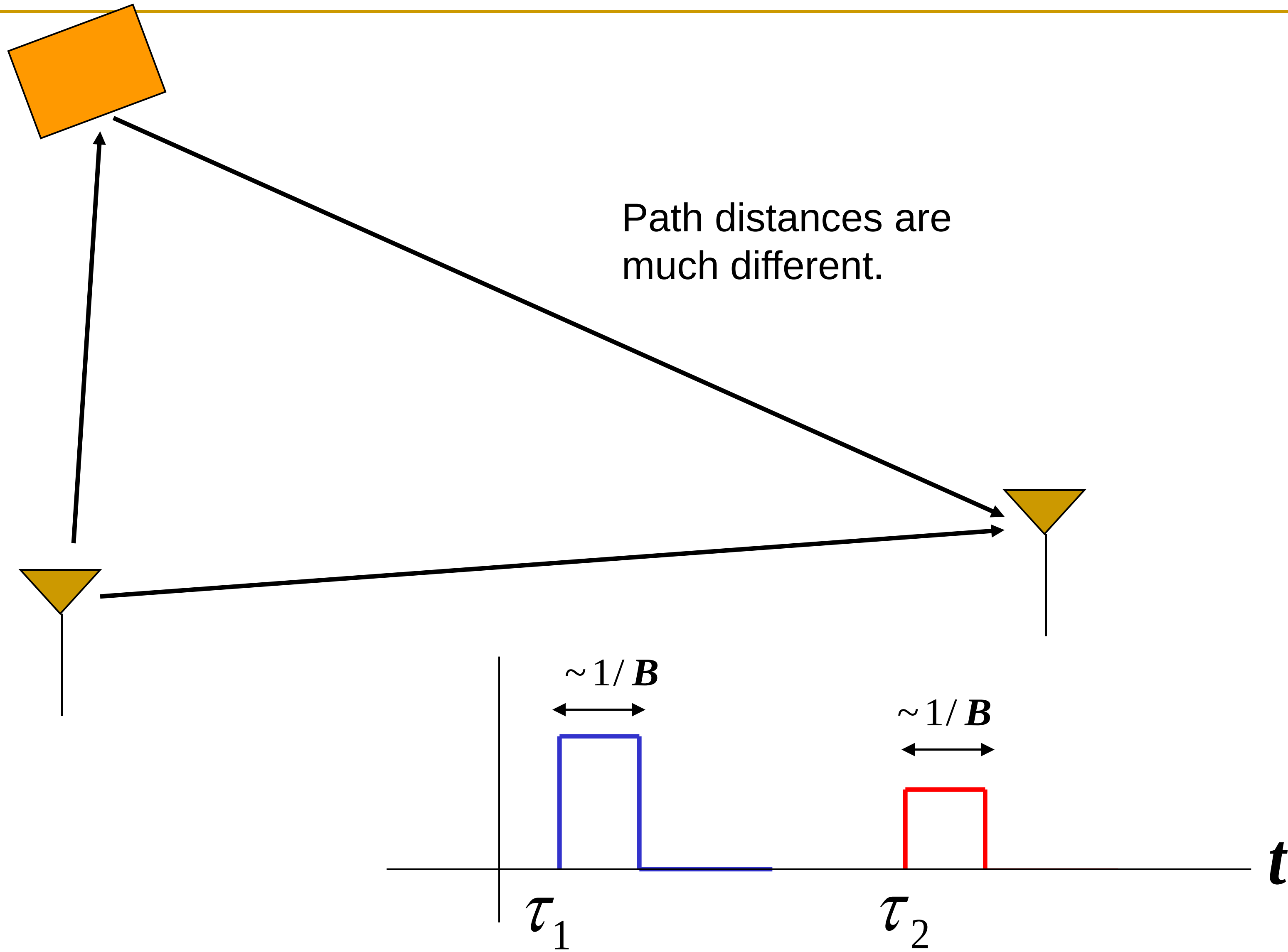
- Resolvability?



- Roughly, pulse duration
- Interested in pulses since linear modulation consists of a train of pulses where each pulse carries information on its amplitude/phase.
- Two paths are resolvable if



reflector cluster
Path distances are very close.
$\sim 1/\boldsymbol{B}$
$\tau_1 \tau_2 \tau_3$
t
$\tau_1 \tau_2 \tau_3$
t



Individual paths are observed => resolvable

$$r(t) = \Re\left\{\left[\sum_{n=1}^{N(t)} \alpha_n(t) e^{-j\left\{2\pi f_c \tau_n(t) - \phi_{D_n}(t)\right\}} u(t - \tau_n(t))\right] e^{j2\pi f_c t}\right\}$$

$$r(t) = \Re\left\{\left[\sum_{n=1}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} u(t - \tau_n(t))\right] e^{j2\pi f_c t}\right\}$$

- $\alpha_n(t)$ a function of path loss and shadowing
- $\phi_n(t)$ depends on delay and Doppler
- Assumption: Two random processes are independent.

$$r(t) = \Re\left\{\left[\sum_{n=1}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} u(t-\tau_n(t))\right] e^{j2\pi f_c t}\right\}$$

$$= \boxed{\phantom{\sum_{n=1}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} u(t-\tau_n(t))}}$$

$$\underbrace{c(\tau, t) = \sum_{n=1}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} \delta(\tau - \tau_n(t))}$$

Equivalent lowpass response of the channel at time *t* to an impulse at time $t-\tau$

Examples

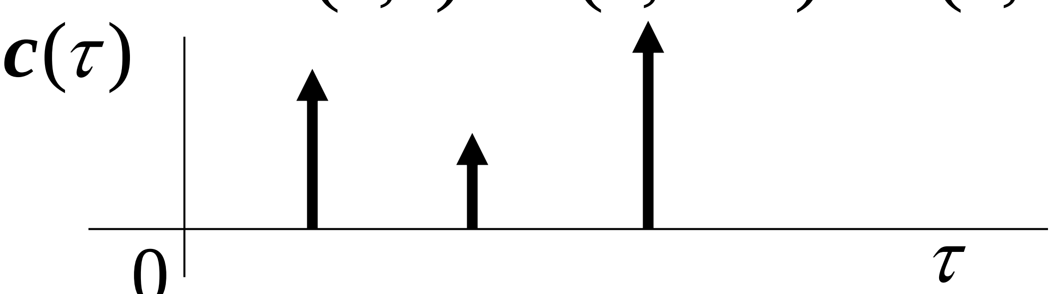
- At time *t* there is no physical reflector with multipath delay $\tau_n(t) = \tau$

$$\Rightarrow$$

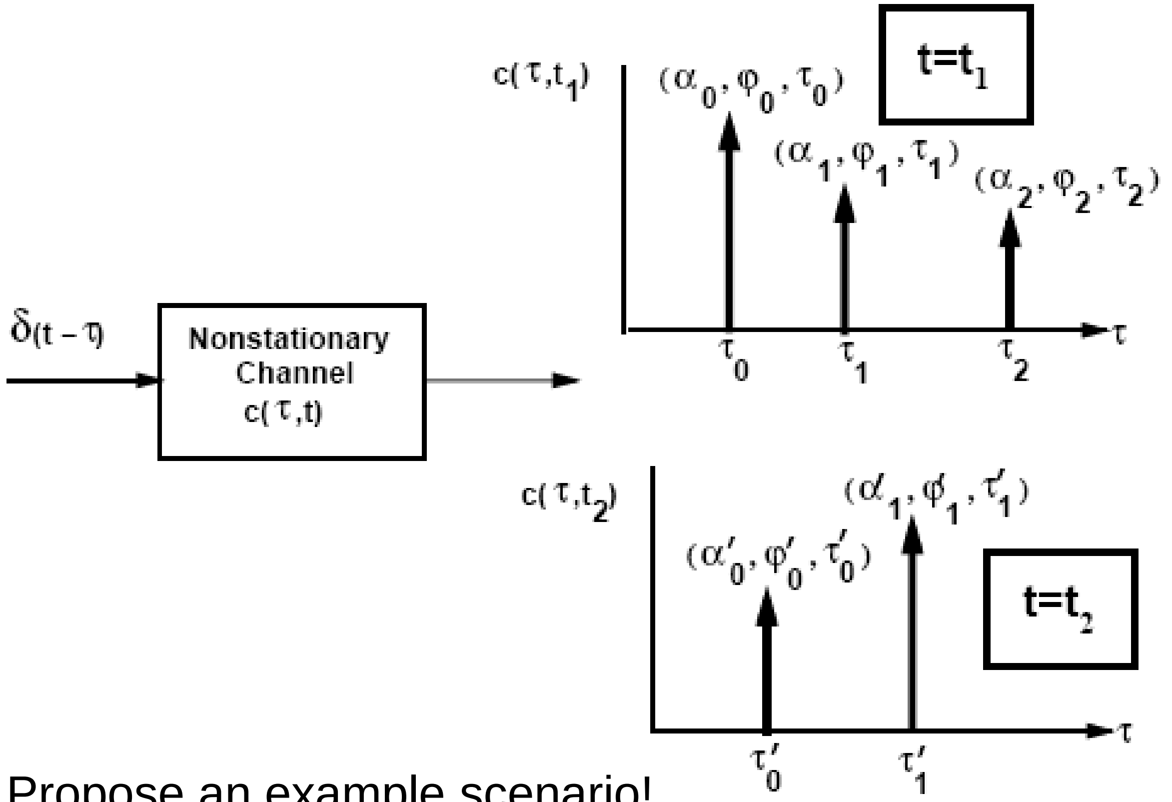
- For time-invariant channels

- In particular

$$c(\tau, t) = c(\tau, t - t) = c(\tau, 0) \doteq c(\tau)$$



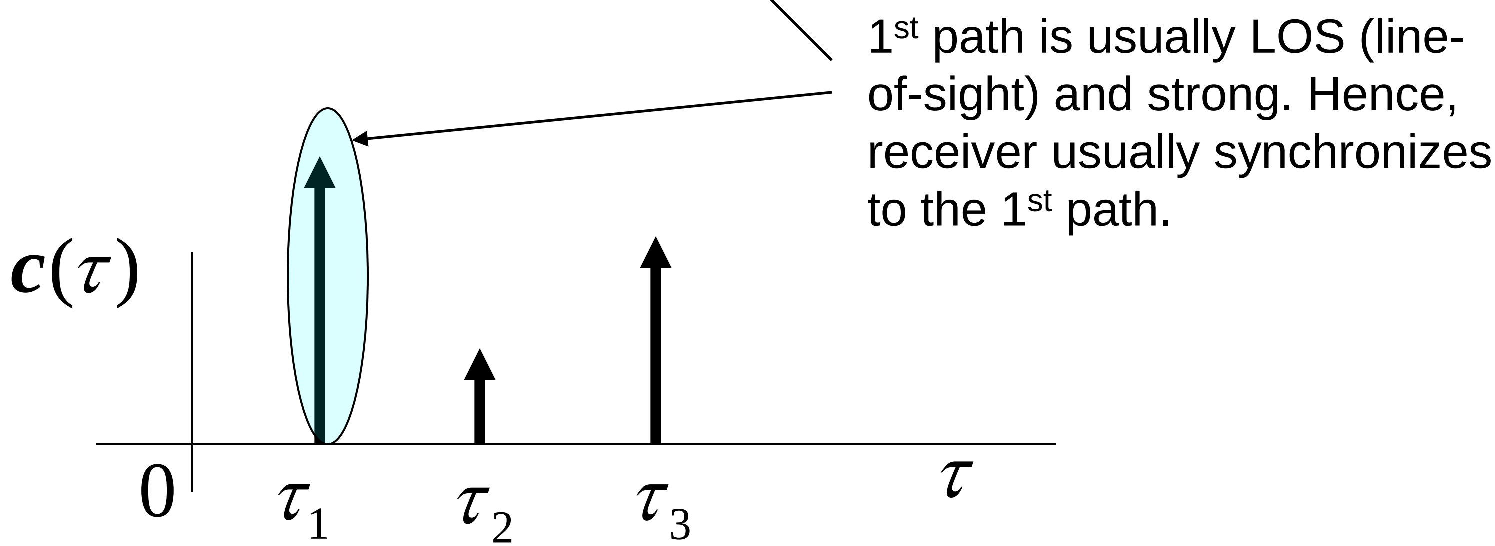
- Nonstationary channel (channel response changes with time, i.e., it is a function of time)



Propose an example scenario!

- Delay spread

$$T_m = \max_n |\tau_n - \tau_1|$$



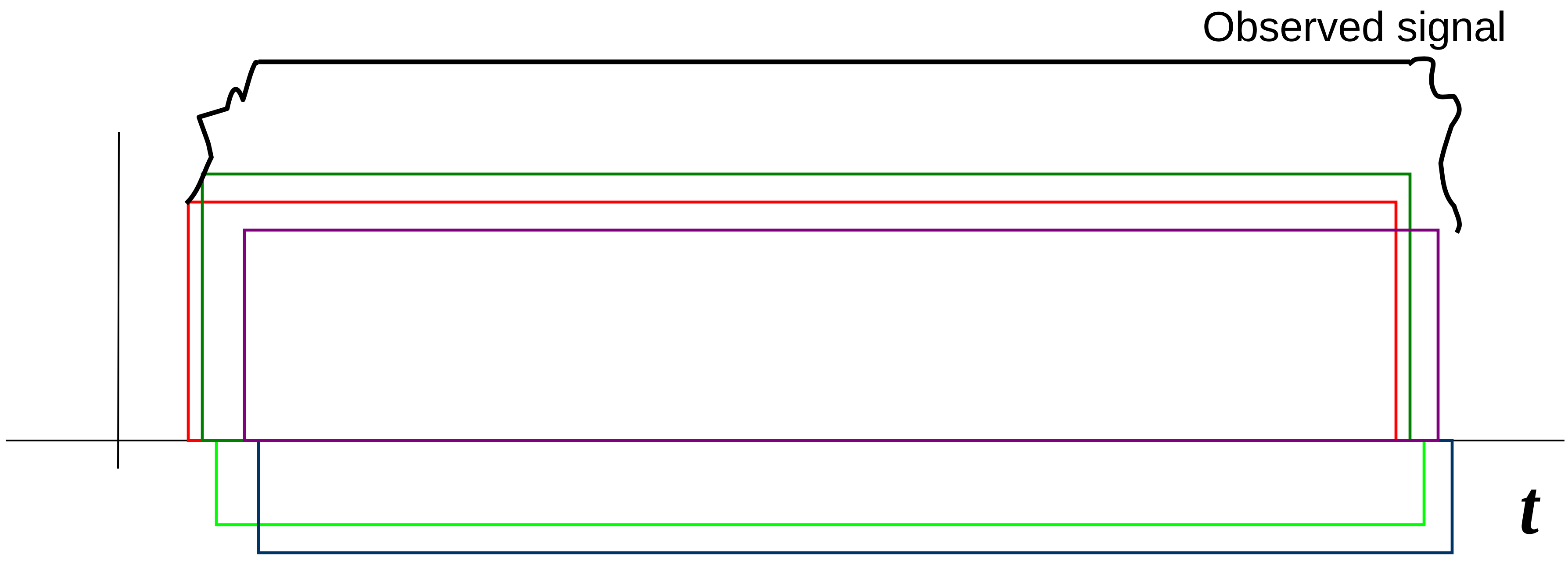
- If delay spread much smaller than pulse duration

Non-resolvable paths

Narrowband fading

Narrowband Fading Models

$$T_m << 1/B$$



- Delay associated with the ith multipath has

$$\tau_i << T_m \Rightarrow u(t-\tau_i) \approx u(t) \text{ for all } i$$

$$r(t) = \Re\left\{\left[\sum_{n=1}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)}\right] u(t) e^{j2\pi f_c t}\right\}$$

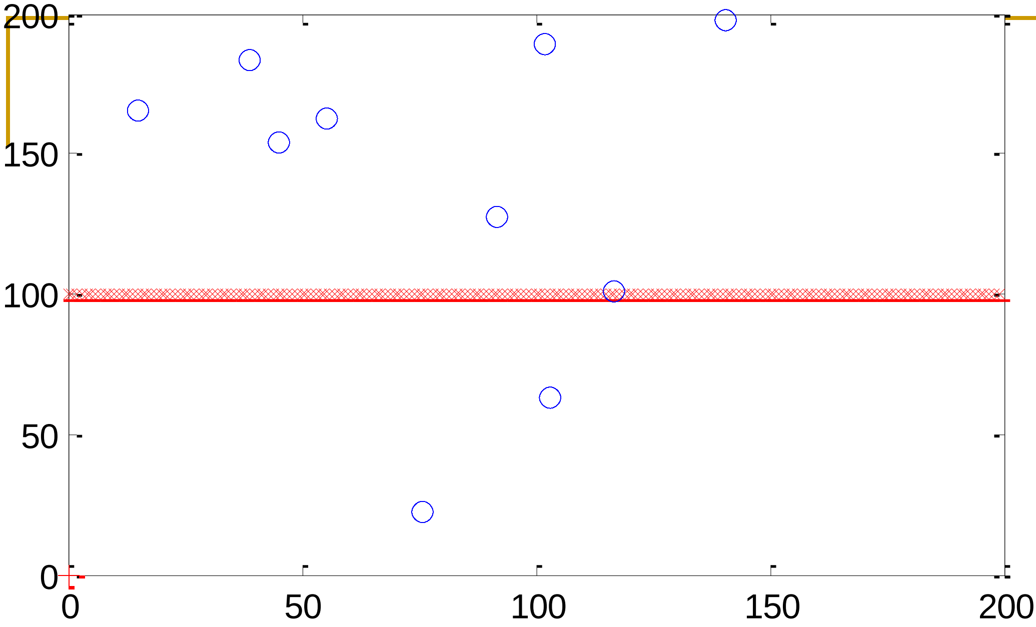
$$c_N(t) = \left[\sum_{n=1}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)}\right]$$

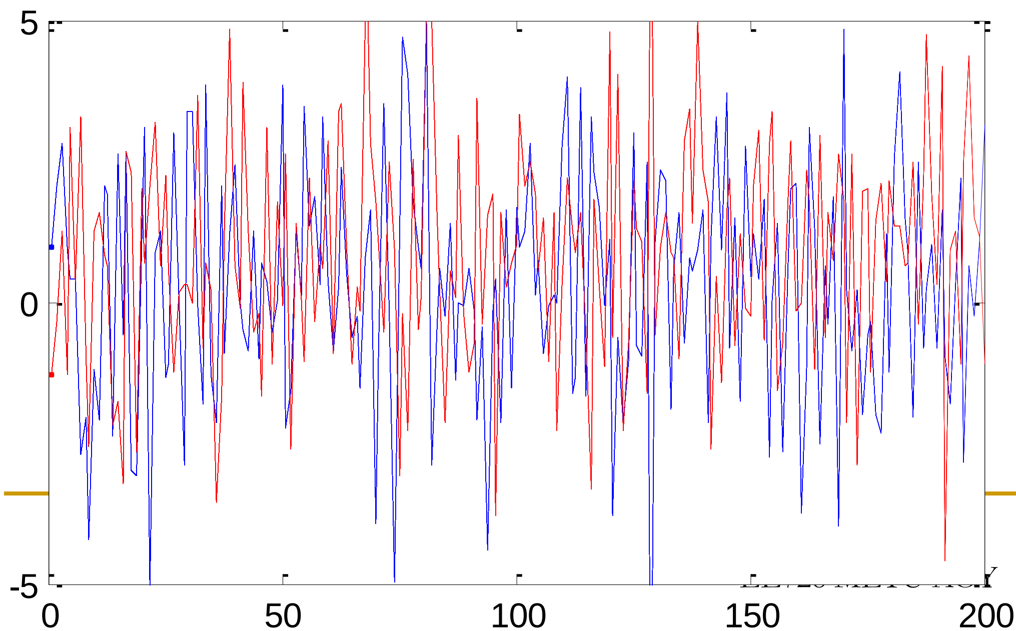
$$c_{N,I}(t) = \sum_{n=1}^{N(t)} \alpha_n(t) \cos\phi_n(t)$$

$$c_{N,Q}(t) = -\sum_{n=1}^{N(t)} \alpha_n(t) \sin\phi_n(t)$$

$$\phi_n(t) = 2\pi f_c \tau_n(t) - \phi_{D_n}(t)$$

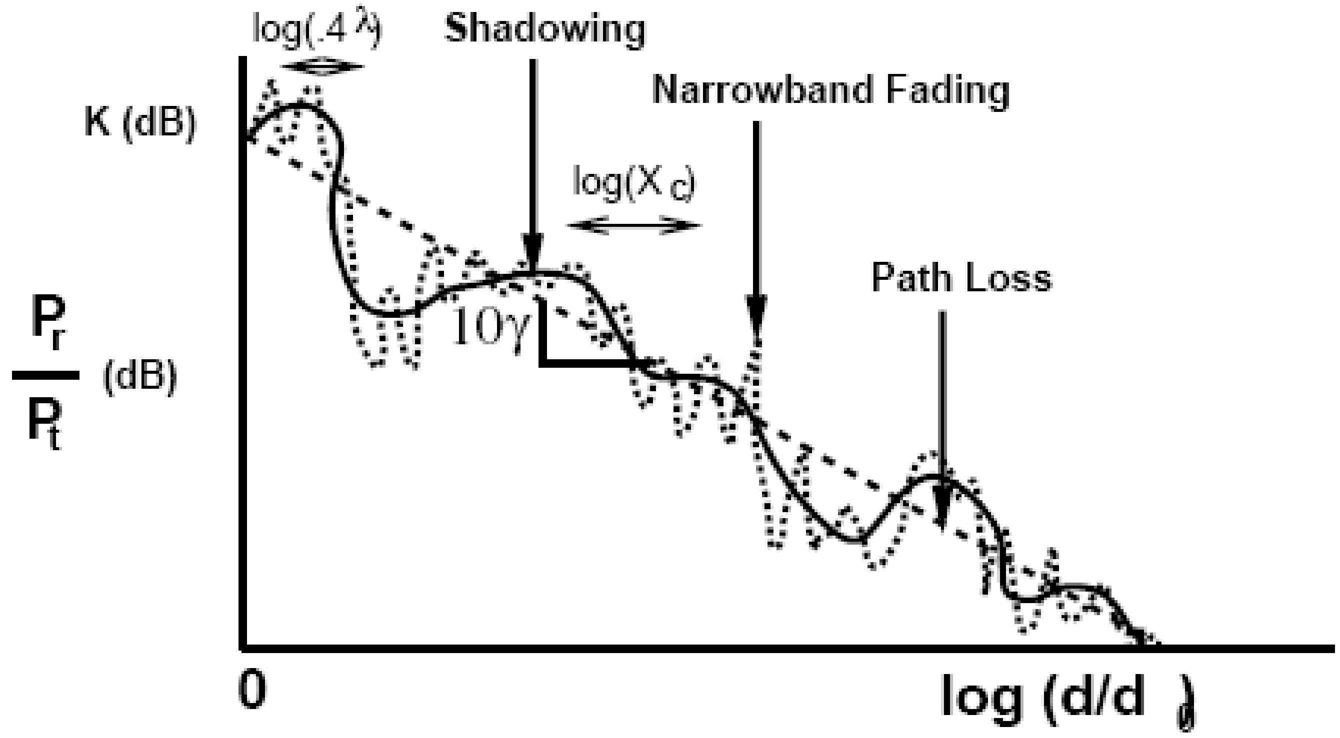
- Recall the central limit theorem
- If number of paths are large
  - channel gain becomes a complex Gaussian r.v.
  - whose amplitude (envelope) is Rayleigh distributed.
- In rich scattering environments, there are many paths and thus channel gains are taken to be complex Gaussian.





- Received power variations due to constructive/destructive addition of paths
- These variations occur over short distances
  - Small-scale
  - on the order of wavelength $\sim 0.4\lambda$
- called FADING

- Large and small scale propagation effects together



- Example
  - 1Ghz carrier, delay of 50ns (a typical value for an indoor system)

$$2\pi f_c \tau_n = 2\pi 50 >> 1$$

  - A small change in delay corresponds to a large phase change
  - Even an additional delay of ns (0.3m) corresponds to full rotation
- Hence fading may change considerably (depending on carrier frequency) with distance.

$$c_{N,I}(t) = \sum_{n=1}^{N(t)} \alpha_n(t)\cos\phi_n(t)$$

$$c_{N,Q}(t) = -\sum_{n=1}^{N(t)} \alpha_n(t)\sin\phi_n(t)$$

- **Assumption**: Phase changes fast $2\pi f_c \tau_n >> 1$ due to large carrier frequency and hence it is uniformly distributed

$$E[c_{N,I}(t)] = E\left[\sum_{n=1}^{N(t)} \alpha_n(t)\cos\phi_n(t)\right]$$

$$= \sum_{n=1}^{N(t)} E[\alpha_n(t)]E[\cos\phi_n(t)] = 0$$

- Similarly, $E[c_{N,Q}(t)] = \sum_{n=1}^{N(t)} E[\alpha_n(t)]E[\sin\phi_n(t)] = 0$

- $c_N(t)$ is a zero-mean Gaussian process.
- Derivations based on key assumptions that generally apply to propagation scenarios without a dominant LOS

- **Assumption**: Amplitude, multipath delay, Doppler frequency change slowly enough to be considered constant over time intervals of interest

$$\alpha_n(t) \approx \alpha_n \quad \tau_n(t) = \tau_n \quad f_{D_n}(t) = f_{D_n}$$

$$\phi_{D_n}(t) = \int_0^t 2\pi f_{D_n}(\lambda) d\lambda \approx 2\pi f_{D_n} t$$

- Autocorrelation for a fixed setting

$$R_{c_{N,I}}(t, t+\tau) = E[c_{N,I}(t) c_{N,I}(t+\tau)]$$

$$= E\left[\sum_n \alpha_n \cos\phi_n(t) \sum_m \alpha_m \cos\phi_m(t+\tau)\right]$$

$$= \sum_n \sum_m E[\alpha_n \alpha_m] E[\cos\phi_n(t) \cos\phi_m(t+\tau)]$$

$$= \sum_m E[\alpha_n^2] E[\cos\phi_n(t) \cos\phi_n(t+\tau)]$$

$$\phi_n(t) = 2\pi f_c \tau_n - 2\pi f_{D_n} t - \phi_0$$

$$E[\cos \phi_n(t) \cos \phi_n(t+\tau)] =$$

$$\frac{1}{2}\left\{E[\cos(2\pi f_{D_n} \tau)] + E[\cos(4\pi f_c \tau_n - 4\pi f_{D_n} t - 2\pi f_{D_n} \tau - 2\phi_0)]\right\}$$

- $4\pi f_c \tau_n$ term changes rapidly w.r.t. other phase terms -> uniform dist.

$$R_{c_{N,I}}(t, t+\tau) = \frac{1}{2}\sum_m E[\alpha_n^2]\cos(2\pi f_{D_n} \tau)$$

$$= \frac{1}{2}\sum_m E[\alpha_n^2]\cos(2\pi v \tau \frac{\cos \theta_n}{\lambda})$$

- By the derived mean and autocorrelation, $c_{N,I}(t)$ is WSS
- $c_{N,Q}(t)$ has the same autocorrelation.
- Cross-correlation

$$R_{c_{N,I},c_{N,Q}}(t,t+\tau) = E[c_{N,I}(t)c_{N,Q}(t+\tau)]$$

$$= \sum_{m} E[\alpha_n^2]\sin(2\pi v\tau\frac{\cos\theta}{\lambda})$$

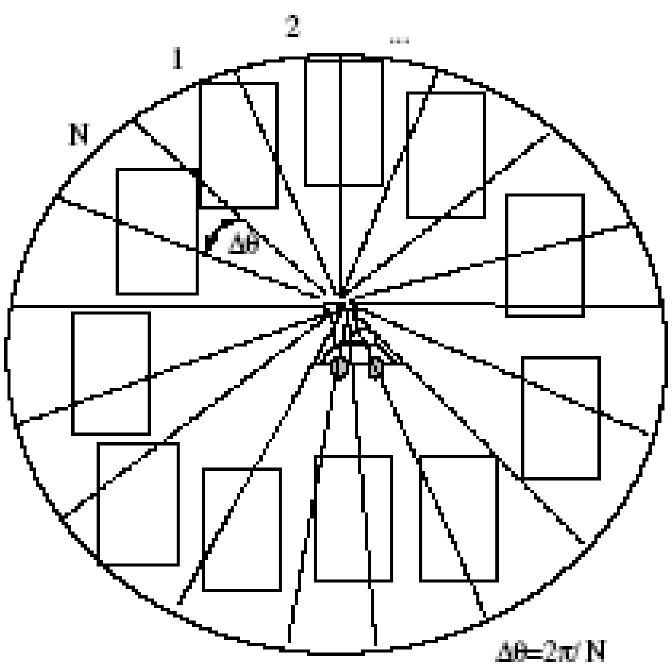
$$= -E[c_{N,Q}(t)c_{N,I}(t+\tau)]$$

- Overall, *c(t)* is also WSS.

$$c(t) = c_{N,I}(t)\cos 2\pi f_c t - c_{N,Q}(t)\sin 2\pi f_c t$$

$$R_c(t, t+\tau) = R_{c_{N,I}}(\tau)\cos(2\pi f_c \tau) + R_{c_{N,I},c_{N,Q}}(\tau)\sin(2\pi f_c \tau)$$

- We now need an assumption to evaluate autocorrelation.
- Dense scattering environment => uniform scattering



- The channel consists of many scatterers densely packed in angle
- It is also assumed that each multipath component has the same power

$$E[\alpha_n^2] = 2P_r / N$$

$$\theta_n = n\Delta\theta, \quad \Delta\theta = \frac{2\pi}{N}$$

$$R_{c_{N,I}}(\tau) = \frac{1}{2}\sum_{n=1}^{N}\cos\left(2\pi v\tau\frac{\cos n\Delta\theta}{\lambda}\right)\Delta\theta$$

$$= \frac{P_r}{2\pi}\sum_{n=1}^{N}\cos\left(2\pi v\tau\frac{\cos n\Delta\theta}{\lambda}\right)\Delta\theta$$

- Let's take the limit as

$$N \to \infty, \Delta\theta \to 0$$

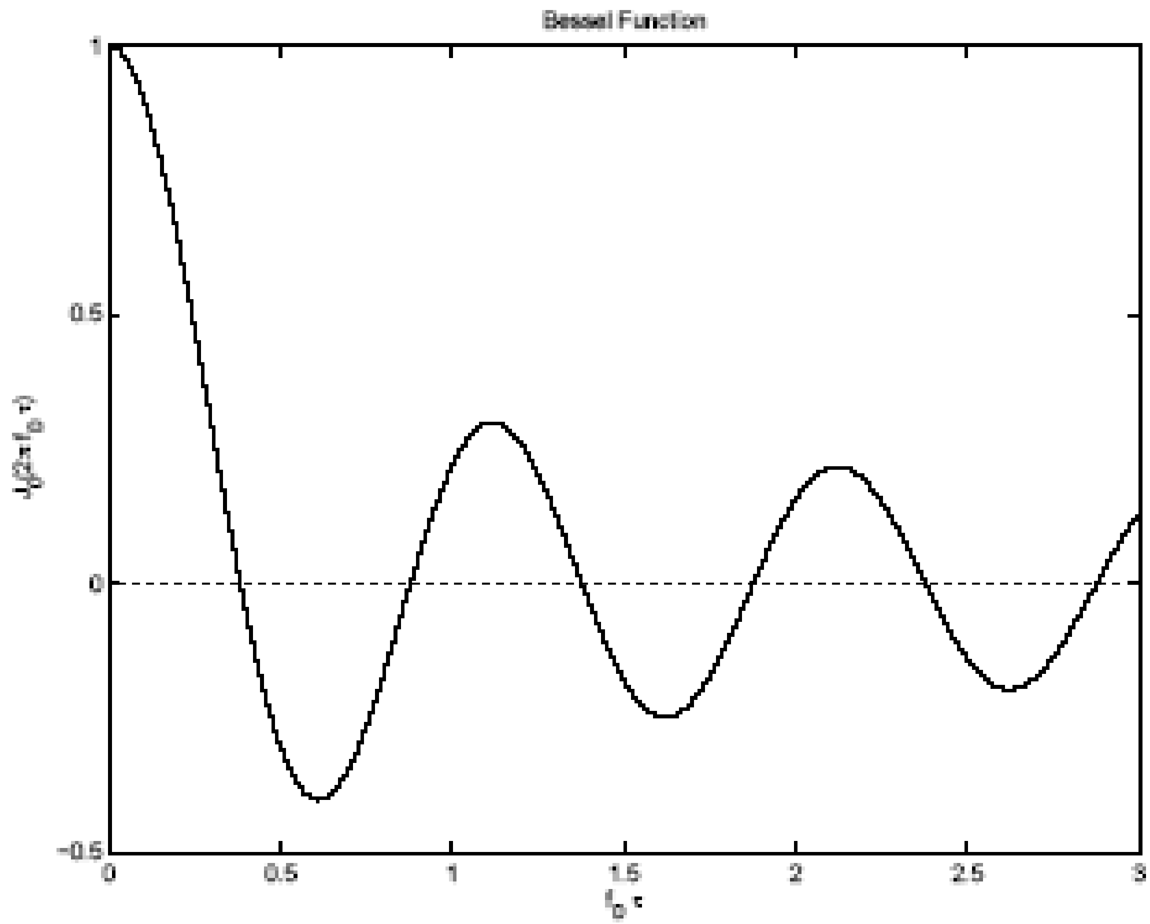
$$R_{c_{N,I}}(\tau) = \frac{P_r}{2\pi}\int_0^{2\pi} \cos(2\pi v\tau \frac{\cos\theta}{\lambda})d\theta$$

$$J_n(z) = \frac{1}{2\pi i^n}\int_0^{2\pi} e^{i z \cos\phi} e^{i n \phi} d\phi$$

Bessel func. of the first kind

$$R_{c_{N,I}}(\tau) = P_r J_0(2\pi f_D \tau)$$

Total received power Maximum Doppler freq.



Signal decorrelates over a distance of ~0.5 wavelength

Note: Recorrelation

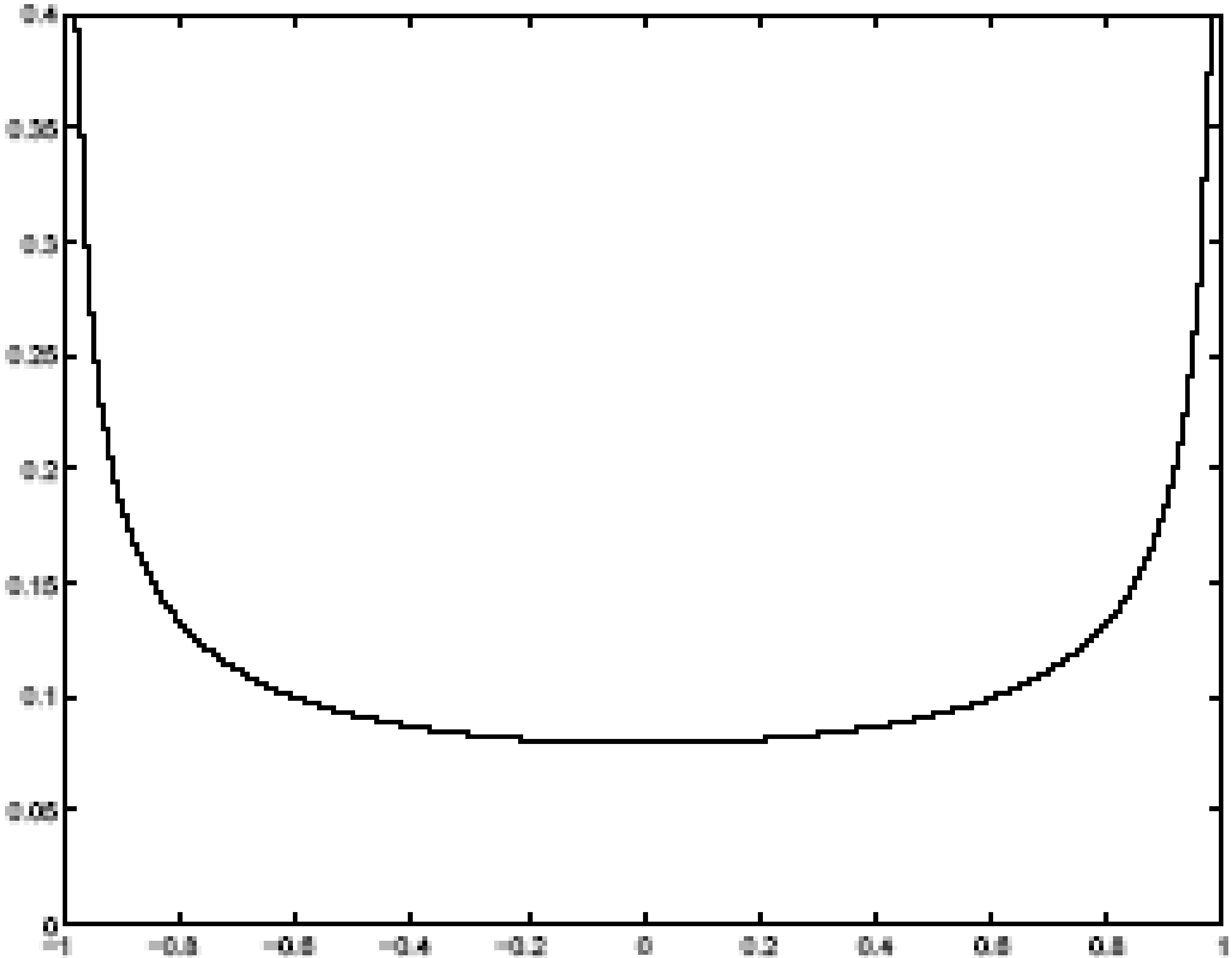
- With uniform scattering

$$R_{c_{N,I},c_{N,Q}}(\tau) = \frac{P_r}{2\pi}\int_0^{2\pi} \sin(2\pi v\tau \frac{\cos\theta}{\lambda})d\theta = 0$$

- Real and imaginary components are uncorrelated and thus independent.
- Power spectral density of fading at baseband

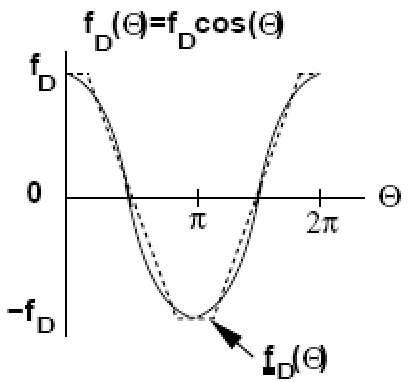
$$S_{c_{N,I}}(f) = S_{c_{N,Q}}(f) = F\{R_{c_{N,I}}(\tau)\} = \begin{cases} \dfrac{2P_r}{\pi f_D}\dfrac{1}{\sqrt{1-(f/f_D)^2}} & |f| \le f_D \\ 0 & o.w. \end{cases}$$

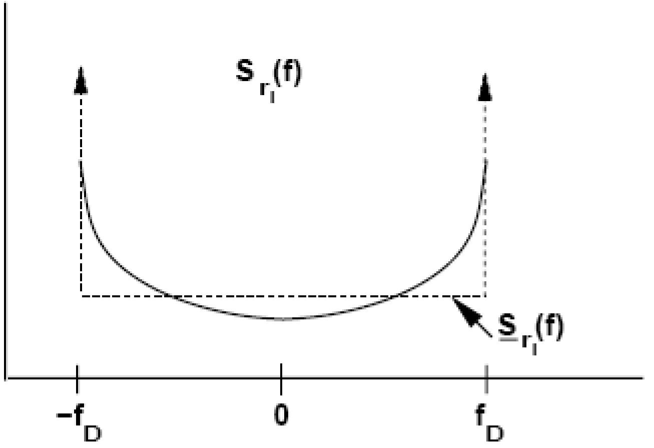
$S_{c_{N,I}}(f)$



f / f_D

- This PSD can be intuitively explained as follows
  - The range of angles for which their cos value around +-1 is larger in comparison to other values.
  - This PSD can actually be directly obtained from the pdf of $\cos\theta$ (Woodward Theorem)





- PSD is useful for generating fading in simulations.
  - Generate white noise
  - Pass the noise through a filter whose square is the desired PSD.

- Envelope and power distributions
  - Real and imaginary components are Gaussian and independent.

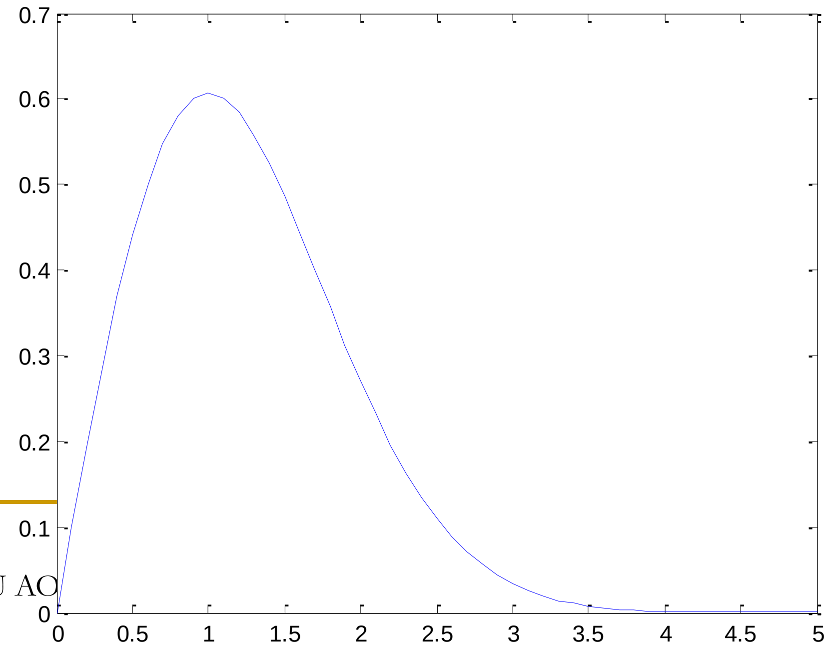
$$z(t)=\sqrt{c_{N,I}^2(t)+c_{N,Q}^2(t)}$$

  - Phase uniformly distributed in $[0,2\pi]$
  - Amplitude has Rayleigh distribution.

$$Z=\sqrt{X^2+Y^2}$$

$$N(0,\sigma^2)$$

$$f_Z(z)=\frac{z}{\sigma^2}\exp\left(-\frac{z^2}{2\sigma^2}\right), z\geq 0$$

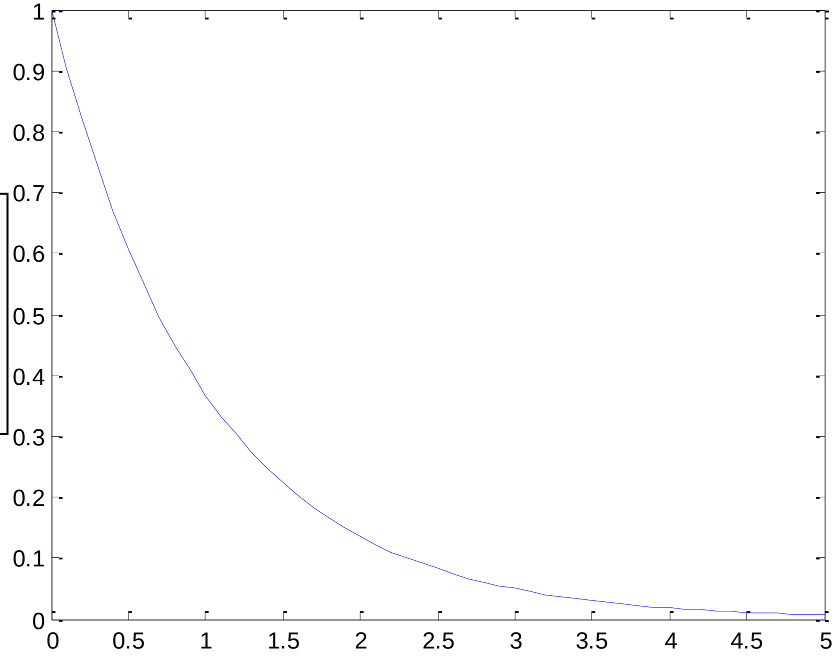


- Power has exponential distribution.

$$Z^2 = X^2 + Y^2$$

$$N(0, \sigma^2)$$

$$f_{Z^2}(x) = \boxed{}$$



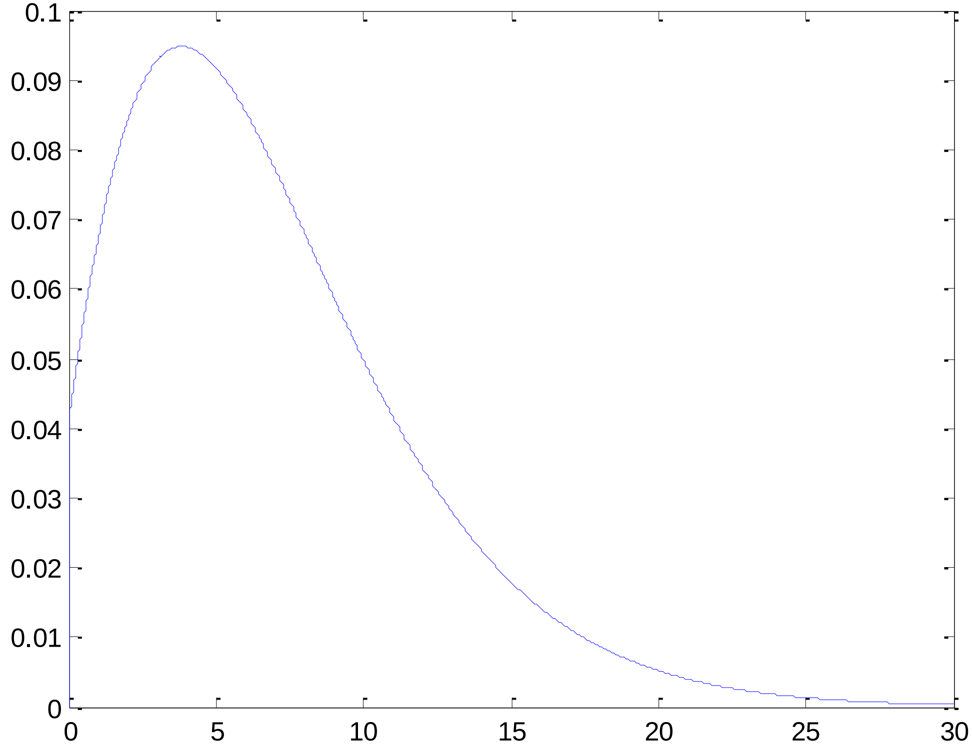
- There is sometimes a LOS component.
- Real and imaginary parts are not zero-mean in that case.
- Chi-square distribution

$$\boldsymbol{Z}^2 = \boldsymbol{X}^2 + \boldsymbol{Y}^2$$

$$N(\mu_{\boldsymbol{X}}, \sigma^2) \quad N(\mu_{\boldsymbol{Y}}, \sigma^2)$$

- Power is not exponential now, it has noncentral chi-square distribution.
- Amplitude then has Rician distribution.

- Power distribution



Rician fading parameter

- Rician distribution

$$f_Z(z) = \frac{z}{\sigma^2} \exp\left(-\frac{z^2 + s^2}{2\sigma^2} \right) I_0\left(\frac{zs}{\sigma^2}\right), z \geq 0$$

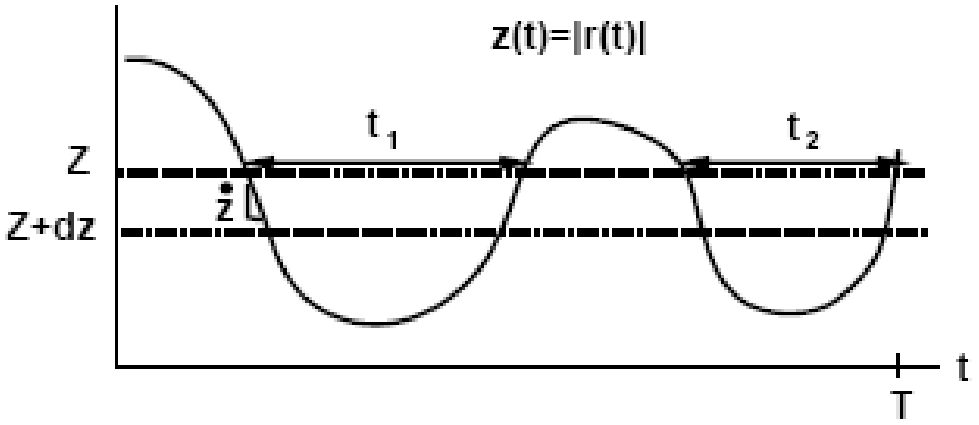
LOS power

modified Bessel func. of zeroth order

- Some experimental data does not fit well into any of the above distributions.
- More general fading distributions were developed whose parameters can be adjusted to fit a variety of empirical measurements.
- Nakagami-m fading distribution

$$p_Z(z) = \frac{2m^m z^{2m-1}}{\Gamma(m) P_r^m} \exp\left[\frac{-mz^2}{P_r}\right], \quad m \geq .5$$

- Level crossing rate and average fade duration

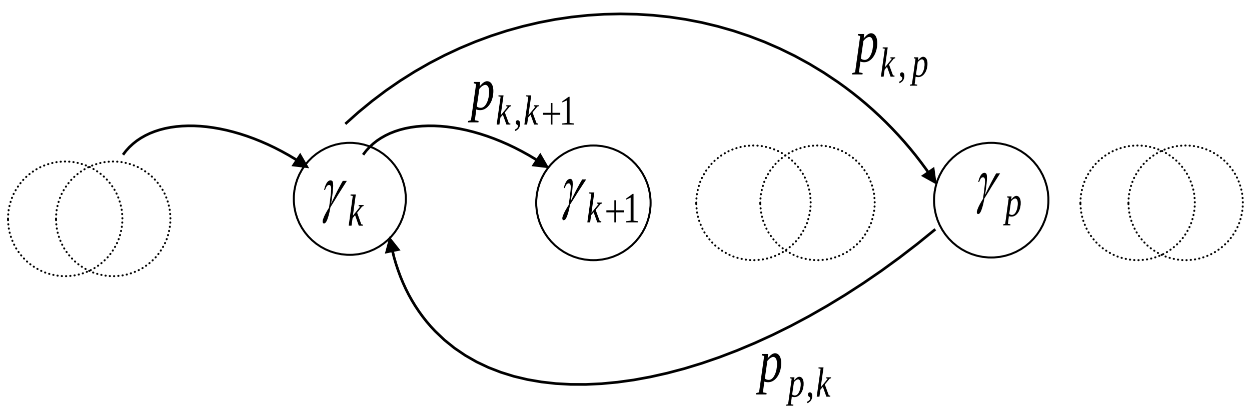


  - HW
- Fading in time are sometimes probabilistically modeled for predicting it.

- Finite-State Markov channels (FSMC)
  - Time-varying SNR $\gamma = z^2 \frac{E_s}{N_0}, \quad 0 \le \gamma < \infty$
  - Fading range discretized into regions

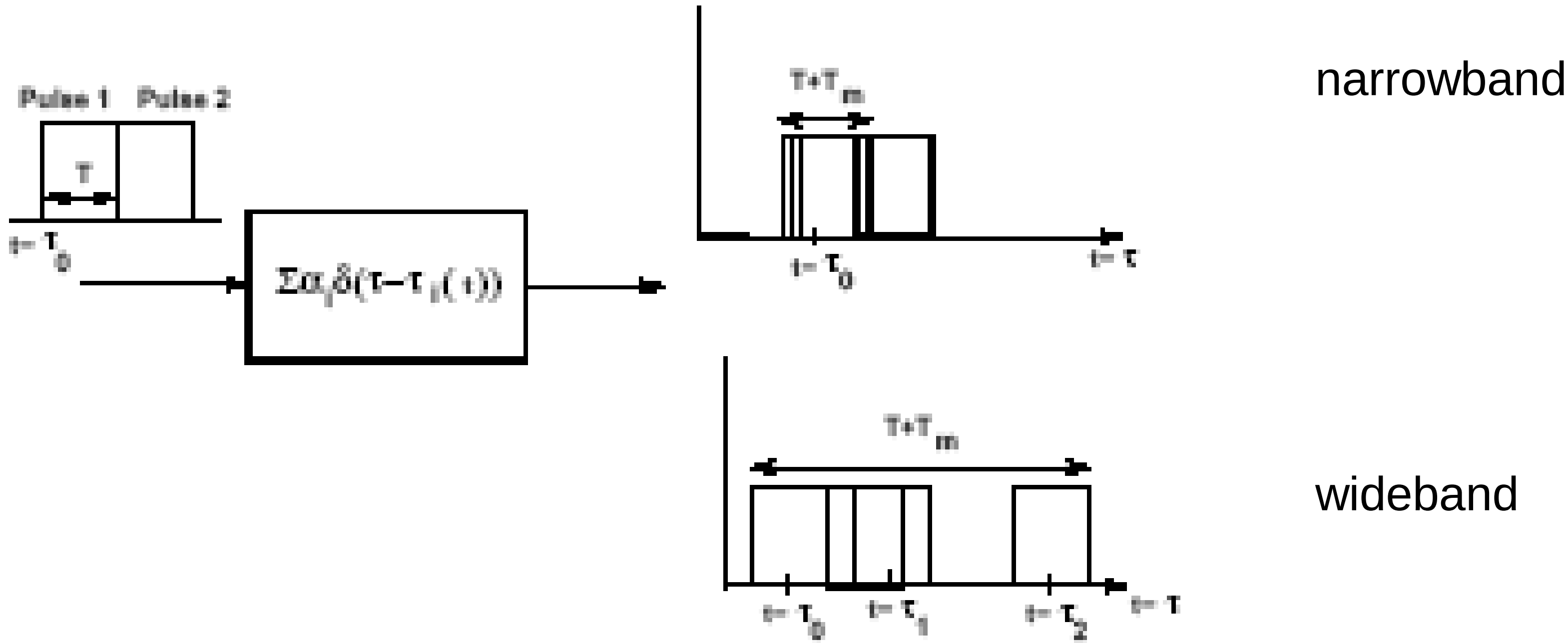
$$R_j = \{\gamma : A_j \le \gamma < A_{j+1}\}$$

  - FSMC assumes that γ stays in the same region for a duration of T and then transitions to another



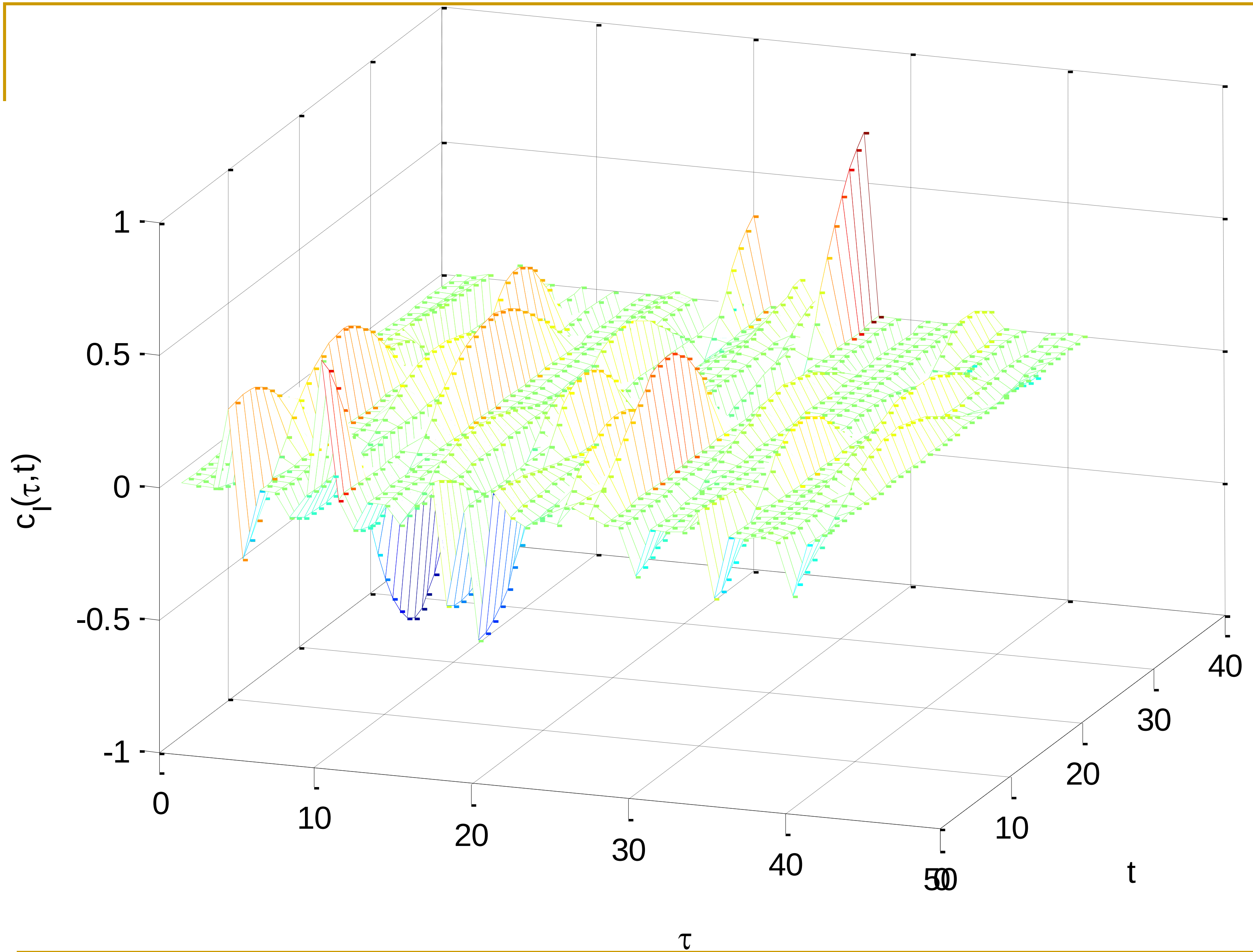
  - First order models are deficient
  - Fade duration statistics can be obtained

Wideband Fading Models

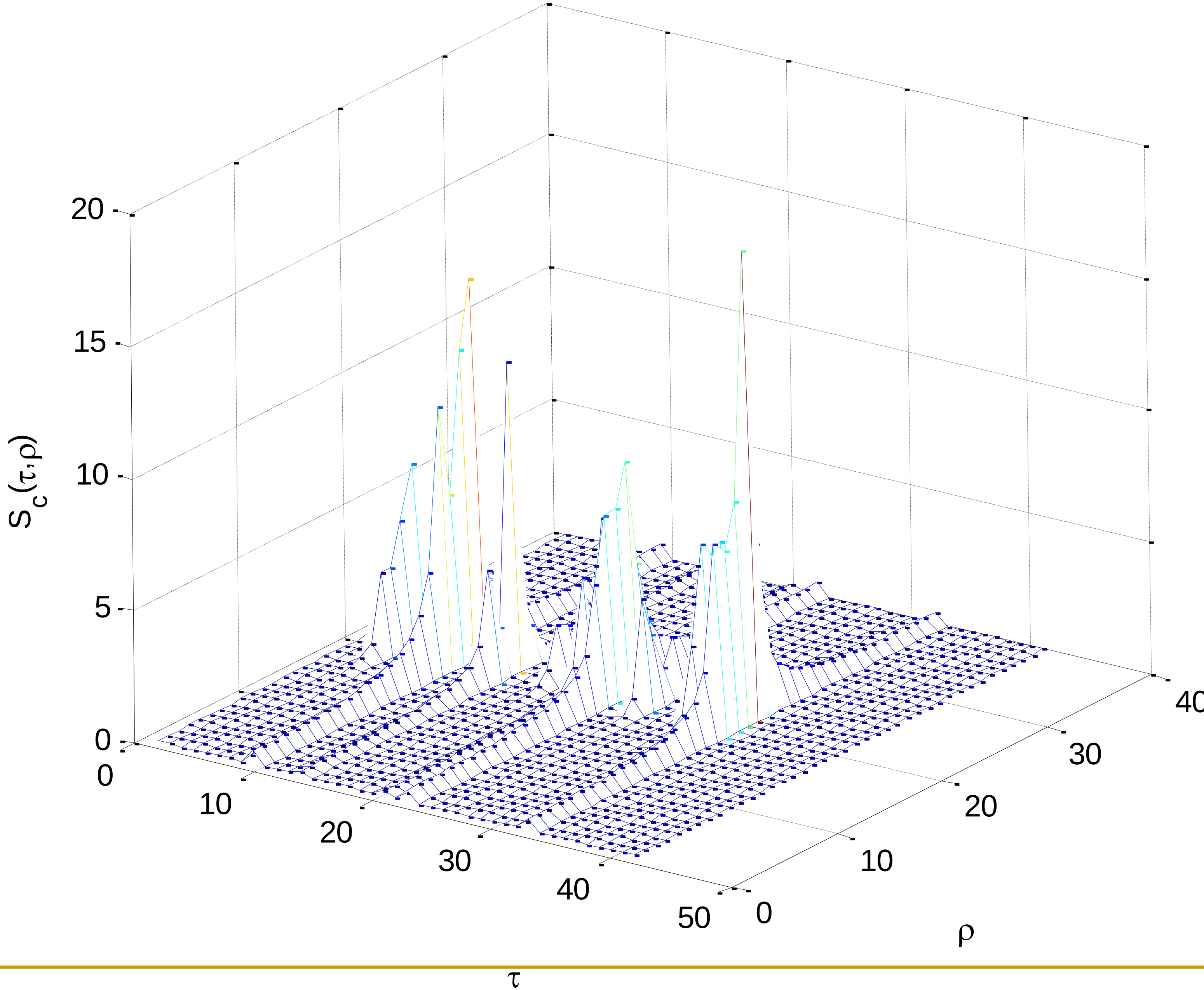


- A large spreading of a pulse
- Multipaths of a pulse interfere with subsequently transmitted pulses (Rayleigh fading demo in matlab)
  - Intersymbol interference (ISI)

- Recall the equivalent lowpass channel impulse response function $c(\tau, t)$
- If deterministic, deterministic scattering func.
- How does a multipath change in time?
- Doppler characteristics of the channel



1
0.5
0
-0.5
-1
$c_l(\tau,t)$
0
10
20
30
40
50
τ
0
10
20
30
40
t



20
15
10
5
0
$S_c(\tau,\rho)$
0
10
20
30
40
50
τ
0
10
20
30
40
ρ

- In general, $c(\tau,t)$ is random due to random amplitudes, phases, and delays.
- Autocorrelation func.

$$R_c(\tau_1,\tau_2;t,t+\Delta t)=E\left[c^*(\tau_1,t)c(\tau_2,t+\Delta t)\right]$$

- We assume that the channel model is WSS.

- In real environments the channel response associated with a given multipath of delay τ_1 is uncorrelated with a multipath at a different delay $\tau_2 \neq \tau_1$ since two multipaths are caused by different scatterers.
- Uncorrelated scattering WSS model (WSSUS)

- With WSSUS

$$\boldsymbol{R_c}(\tau_1,\tau_2;\Delta \boldsymbol{t}) = \boldsymbol{E}\left[\boldsymbol{c}^*(\tau_1,\boldsymbol{t})\boldsymbol{c}(\tau_2,\boldsymbol{t}+\Delta \boldsymbol{t})\right]$$
$$= \boldsymbol{R_c}(\tau_1,\tau_1;\Delta \boldsymbol{t})\delta(\tau_1-\tau_2) \doteq \boldsymbol{R_c}(\tau_1;\Delta \boldsymbol{t})$$

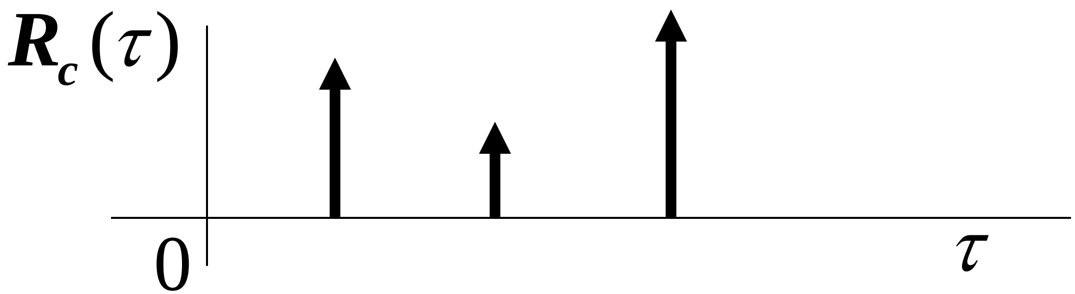
- Scattering function for random channels

- Scattering function characterizes avg output power associated with the channel as a func. of delay τ and Doppler ρ => PSD for τ

- **Power delay profile** (multipath intensity profile)

$$R_c(\tau) \doteq R_c(\tau;0)$$

autocorrelation with 0 time difference corresponds to average power



- Some parameters to quantify delay spread

$$\sigma_{T_m} = \sqrt{\frac{\int_0^\infty (\tau - \mu_{T_m})^2 R_c(\tau)\, d\tau}{\int_0^\infty R_c(\tau)\, d\tau}}$$

Weighting by relative power

Avg delay spread

rms delay spread

Example 3.5:

Consider a wideband channel with multipath intensity profile

$$A_c(\tau) = \begin{cases} e^{-\tau/.00001} & 0 \leq \tau \leq 20 \ \mu\text{sec.} \\ 0 & \text{else} \end{cases} .$$

Find the mean and rms delay spreads of the channel and find the maximum symbol rate such that a linearly-modulated signal transmitted through this channel does not experience ISI.

Solution: The average delay spread is

$$\mu_{T_m} = \frac{\int_0^{20*10^{-6}} \tau e^{-\tau/.00001} d\tau}{\int_0^{20*10^{-6}} e^{-\tau/.00001} d\tau} = \boxed{}$$

The rms delay spread is

$$\sigma_{T_m} = \sqrt{\frac{\int_0^{20*10^{-6}} (\tau - \mu_{T_m})^2 e^{-\tau} d\tau}{\int_0^{20*10^{-6}} e^{-\tau} d\tau}} = \boxed{}$$

We see in this example that the mean delay spread is roughly equal to its rms value. To avoid ISI we require linear modulation to have a symbol period T_s that is large relative to σ_{T_m}. Taking this to mean that $T_s > 10\sigma_{T_m}$ yields a symbol period of $T_s = 52.5$ μsec or a symbol rate of $R_s = 1/T_s = 19.04$ Kilosymbols per second. This is a highly constrained symbol rate for many wireless systems. Specifically, for binary modulations where the symbol rate equals the data rate (bits per second, or bps), high-quality voice requires on the order of 32 Kbps and high-speed data requires on the order of 10-100 Mbps.

- Time-varying multipath channel in freq. domain (channel freq. response at t)

- Under WSSUS, autocorrelation of $C(f;t)$ in freq. depends only on the freq. difference.

- $\boldsymbol{c(\tau;t)}$ WSS Gaussian $\Leftrightarrow$ $\boldsymbol{C(f;t)}$ WSS Gaussian

$$R_C(f_1, f_2;\Delta t) = E\left[C^*(f_1;t)C(f_2;t+\Delta t)\right]$$

$$= E\left[\int_{-\infty}^{\infty} c^*(\tau_1,t)e^{j2\pi f_1\tau_1}d\tau_1 \int_{-\infty}^{\infty} c(\tau_2,t+\Delta t)e^{-j2\pi f_2\tau_2}d\tau_2\right]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E\left[c^*(\tau_1,t)c(\tau_2,t+\Delta t)\right]e^{j2\pi f_1\tau_1}e^{-j2\pi f_2\tau_2}d\tau_1 d\tau_2$$

$=$

$=$

- Example: Two sinusoids transmitted with freq. difference of Δf
- Calculate the cross-correlation of the channel response with time difference of Δt . That would be $R_C(\Delta f;\Delta t)$.
- F.T. of power delay profile
- $R_C(\Delta f) = E\left[C^*(f;t)C(f+\Delta f;t)\right]$ is a regular autocorrelation func.
  - Correlation between frequencies at a given time

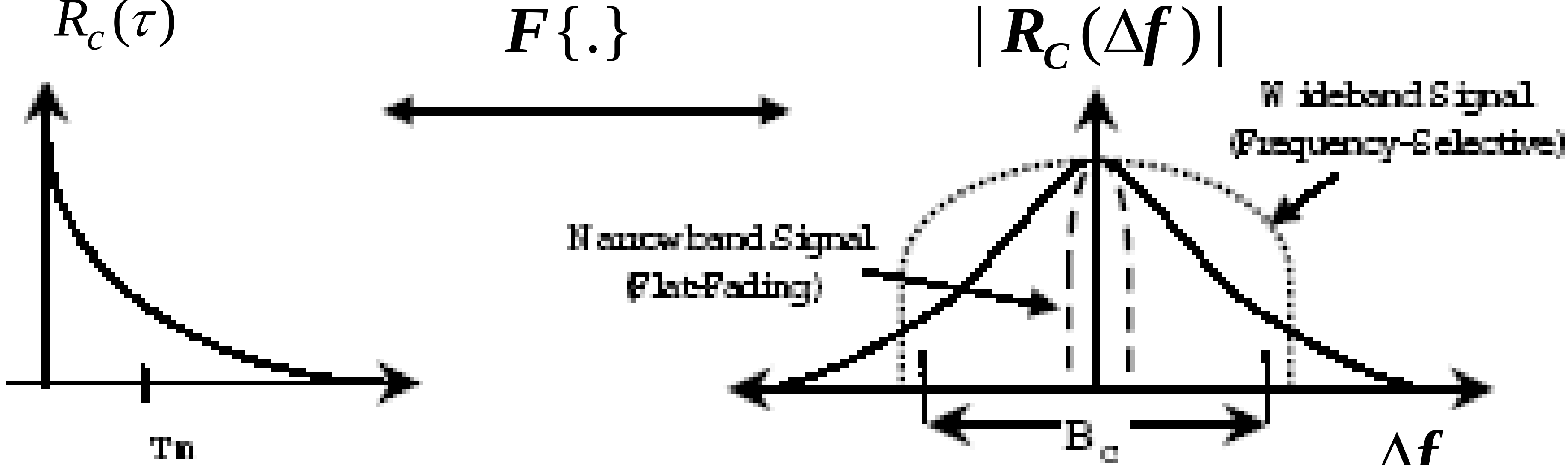
- Channel response approximately <u>independent</u> at freq. separations Δf where

- The freq. B_c where $R_C(\Delta f) \approx 0$ for all $\Delta f > B_c$ is called the **coherence bandwidth**.

$$R_c(\tau) \approx 0 \ \mathbf{for} \ \tau > T \Leftrightarrow R_C(\Delta f) \approx 0 \ \mathbf{for} \ \Delta f > 1/T$$

- T typically taken to be the rms delay spread.

- Other approximations for coherence bandwidth exists.
- The exact form of delay spread is not that important for understanding the general impact of delay spread on multipath channels, as long as the characterization roughly measures multipath distribution.



Channel at a particular band

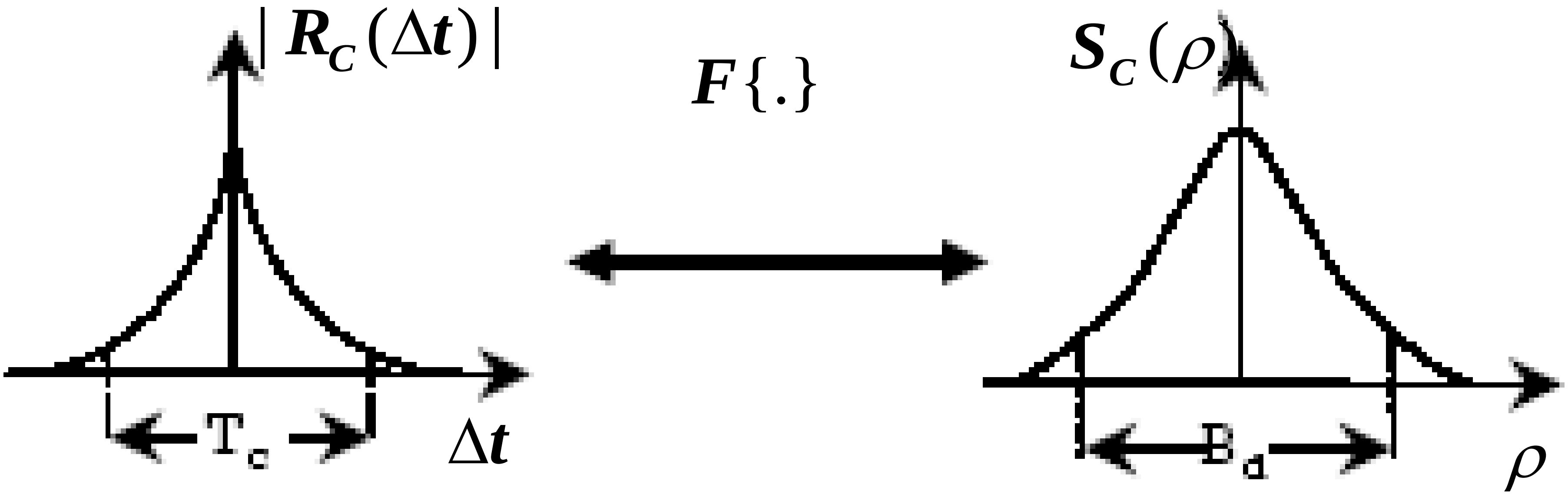
- Flat fading: Fading roughly equal across the entire band if $B << B_c$ (narrowband)
- Frequency selective fading: Fading widely varying across the band if $B >> B_c$ (ISI)
- A signal with symbol duration T_s

- Time variations due to motion

$$S_C(\Delta f, \rho) = \int_{-\infty}^{\infty} R_C(\Delta f, \Delta t) e^{-j2\pi\rho\Delta t} d\Delta t$$

- Doppler at a certain frequency

- $R_C(\Delta t)$ is an autocorrelation func. defining how channel response decorrelates over time.
- Doppler power spectrum $S_C(\rho)$: PSD of the channel response as a func. of Doppler freq.



- **Channel coherence time** T_c
- Time-varying channel decorrelates approximately T_c seconds later.
- **Doppler spread** B_d

$$\boxed{B_d \approx 1/T_c}$$

- Slow fading channel
- Fast fading channel

- Flat fading channel
- Frequency selective channel

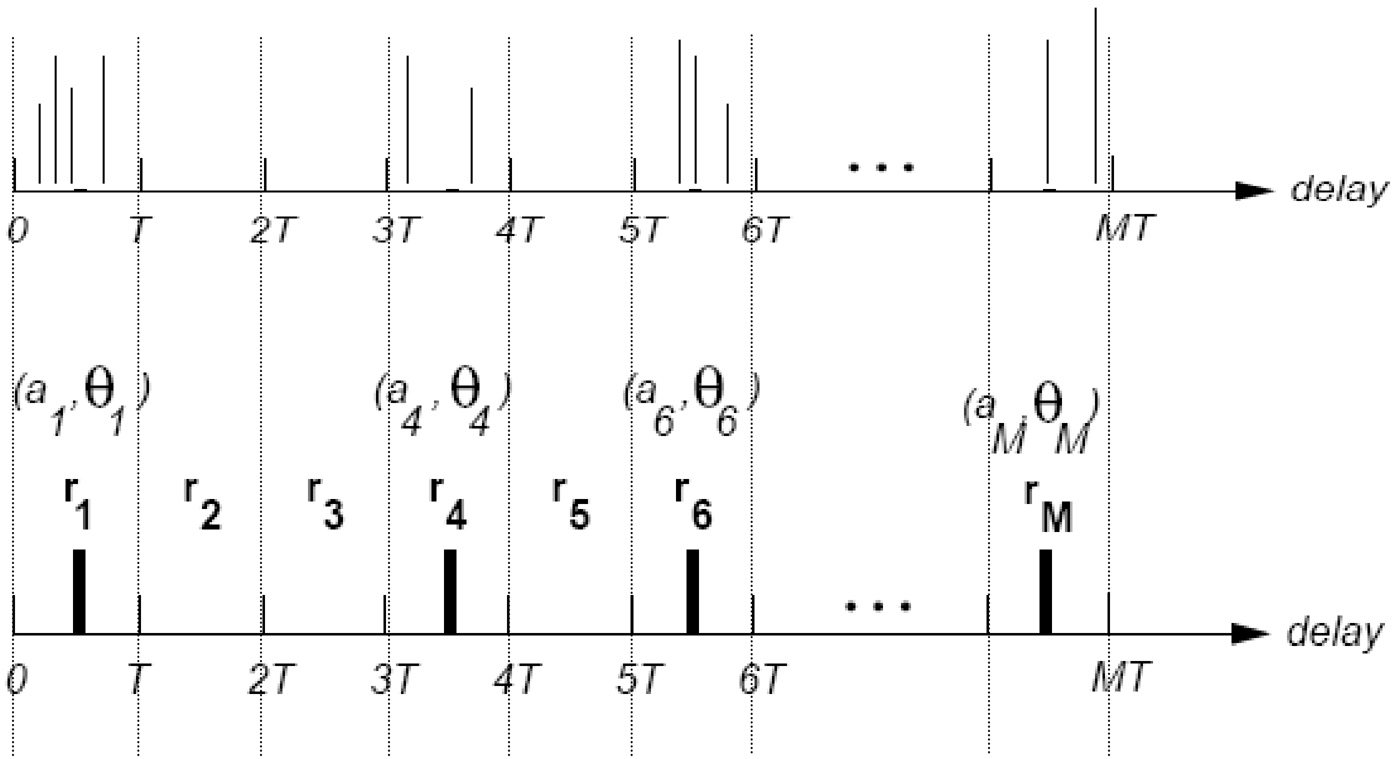
- We almost always employ discrete models to study time-varying channels.

Discrete-Time Model

- Time-varying impulse response model is too complicated in its continuous form

$$c(\tau,t)=\sum_{n=1}^{N(t)}\alpha_n(t)e^{-j\phi_n(t)}\delta(\tau-\tau_n(t))$$

- We know that not all paths are resolvable.
- Combine the paths to clusters whose responses will not be resolvable



- Resolvability $T \approx 1/B$
- Distributions of (a_i, θ_i) may be different
- Model