MIDDLE EAST TECHNICAL UNIVERSITY

Particle Data Analysis in High Energy Physics

Lecture 9

Probability and Random Variables

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Content

In this chapter, wee see the following concepts at introductory level:

Probability

Random Variables

Special Distribution Functions

Probability

Probability

Probability & Statistics plays an essential part in all the sciences as it is the tool which allows the scientist to treat the uncertainties inherent in all measured data and to eventually draw conclusions from the results. For the experimentalist, it is a design and planning tool.

Firstly, We will deal with a central concept known as **Uncertainty**.
In Nuclear and Particle Physics there are various elements of uncertainty:

- Theory is not deterministic (Quantum Mechanics)
- Random measurement errors (present even without quantum effects)
- Things we could know in principle but don’t (e.g. from limitations of cost, time, ...)

We can quantify the uncertainty using **Probability**.

Frequentist Definition of Probability

Suppose we repeat an experiment of tossing a die. Let

s be the number of times a "six" appears

n be the number of tosses

Then the ratio *s*/*n* becomes <u>stable</u> in the long run:

$$f = \frac{s}{n}$$

This stability is the basis of probability theory! So, $f = \frac{s}{n} \to \frac{1}{6} = 0.1666\ldots$

Probability is a long-run frequency.

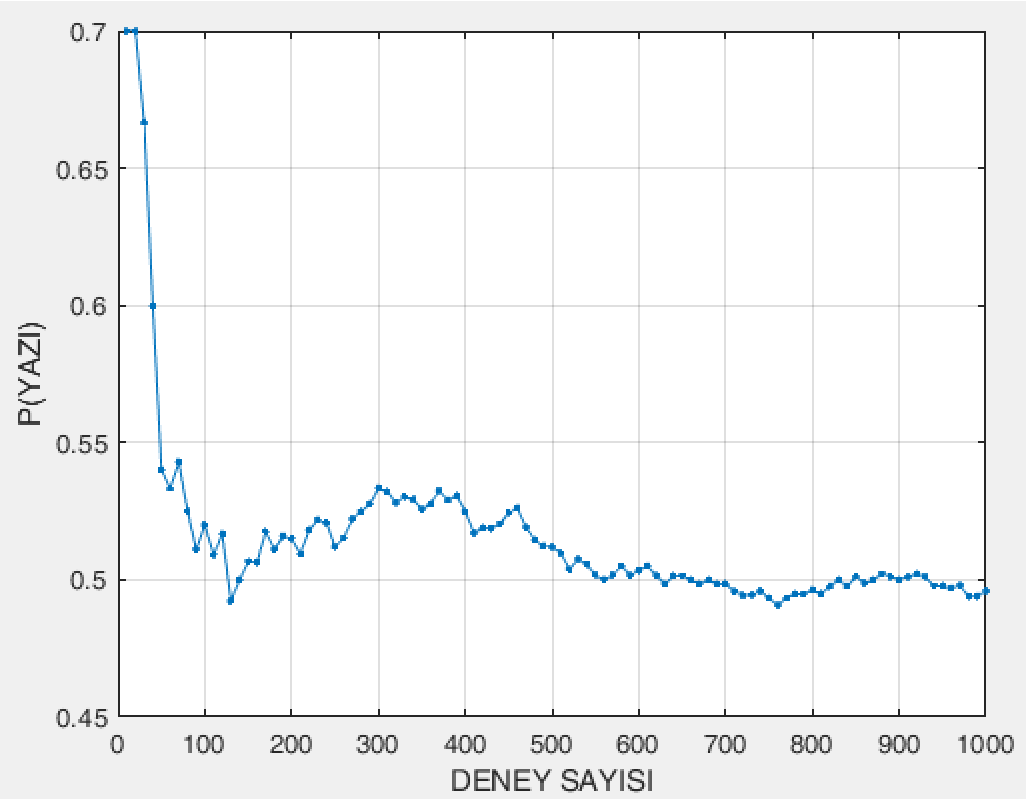
Real Experiment!

Tossing 10 coins 25 times. Results are listed in Table.

P(T) = 128/250 = 0.512

P(H) = 122/250 = 0.488

One of student did this experiment 1000 times.



Experiment#	Tail	Head
1	7	3
2	7	3
3	6	4
4	4	6
5	3	7
6	5	5
7	6	4
8	4	6
9	4	6
10	6	4
11	4	6
12	6	4
13	2	8
14	6	4
15	6	4
16	5	5
17	7	3
18	4	6
19	6	4
20	5	5
21	4	6
22	7	3
23	6	4
24	5	5
25	3	7
Sum	**128**	**122**
Ratio	**0.512**	**0.488**

HEAD

TAIL

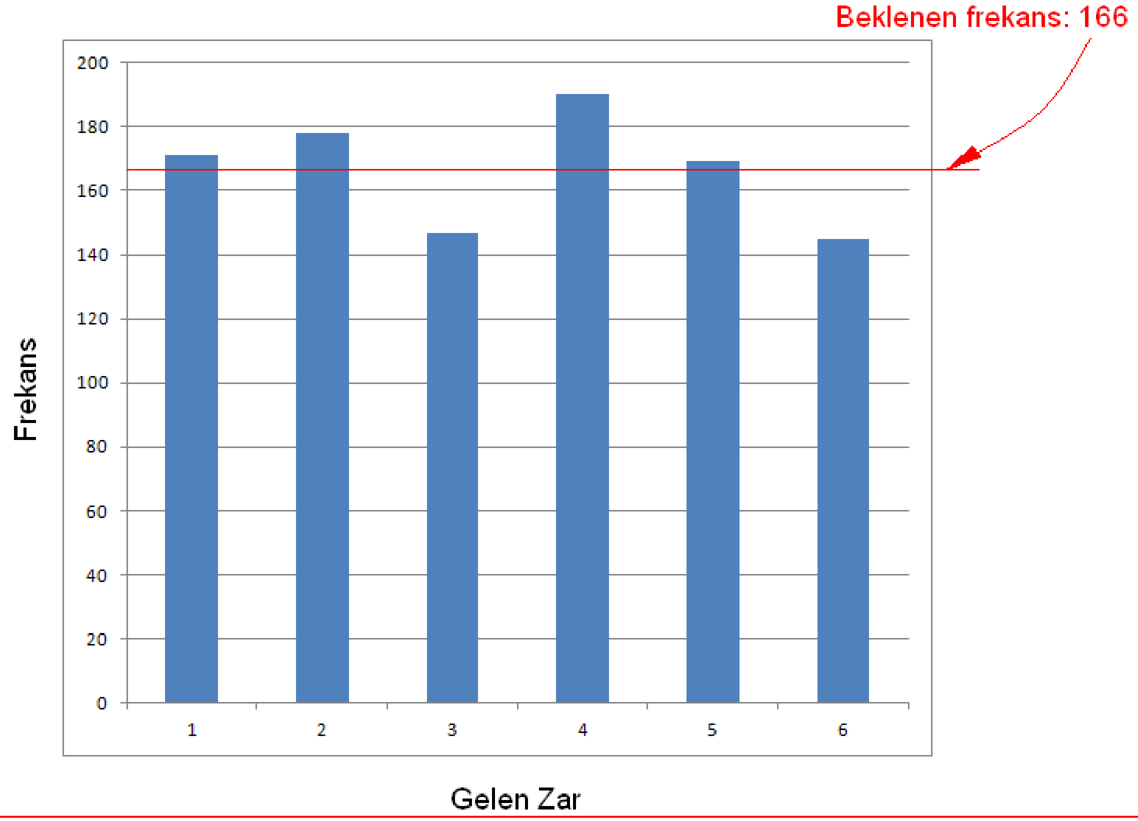
Real Experiment

Another crazy student was thrown a die 1000 times!

P(1) = 171 / 1000 = 0.171

P(2) = 178 / 1000 = 0.178

. . .



Dice's Top Face	s	f = s / n
1	171	0.171
2	178	0.178
3	147	0.147
4	190	0.190
5	169	0.169
6	145	0.145
sum (n)	**1000**	**1.000**

Bayesian Probability

Bayesians see probability as a measure of belief or certainty.

e.g. *Experts say there is a 55% probabilty this candidate will win the election.*

(frequentist definition makes no sense here. One election → no long-run frequency)

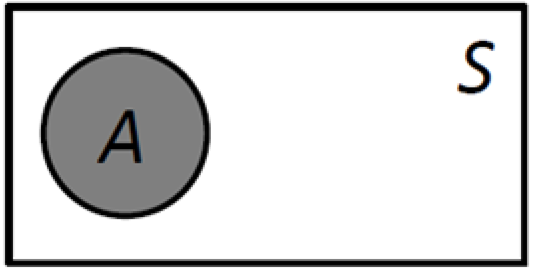
- **Frequentist:**
  Probability is the limit of the relative frequency of an event over many repeated trials.

- **Bayesian:**
  Probability is a representation of an individual's degree of belief in a proposition.

- **Subjective:** (Neither Frequentist nor Bayesian and difficult to determine)
  Example 1: What is the probability of getting a "BA" in this course?
  (Depends on individual confidence and effort).
  Example 2: What is the probability that a ghost is present in this room?
  (Depends on metaphysical beliefs rather than empirical evidence).

Probability of an event

An event describes a set of outcomes of interest to which we can assign a probability.



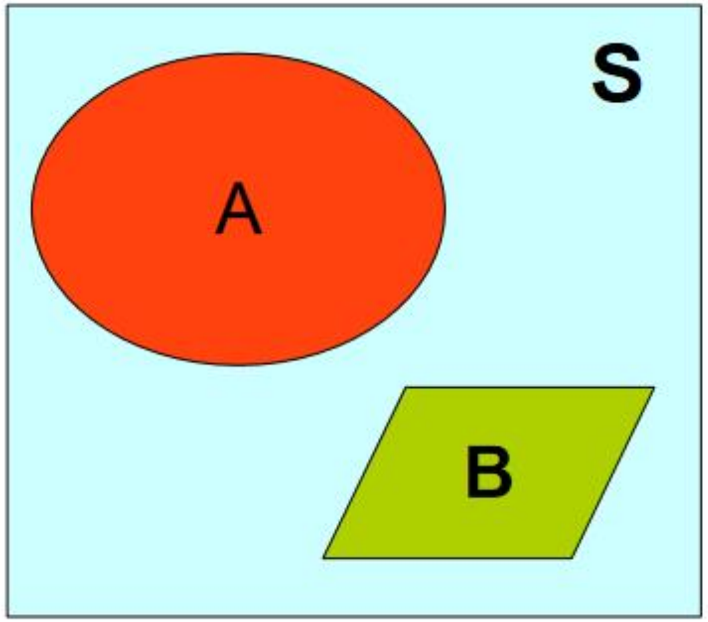
Given a sample space *S* and event *A*, and if every sample point has an equal likelihood of occuring, we can write:

$$P(A) = \frac{n(A)}{n(S)}$$

We can see that the limits of probability are:

$P(S) = 1$, and $P(\phi) = 0$; and so: $0 \leq P(A) \leq 1$

Axioms of Probability



Kolmogorov's axioms (1933)

1. For all $A \subset S$, $P(A) \geq 0$ — Positive definite
2. $P(S) = 1$ — Normalized
3. If $A \cap B = 0$, then $P(A \cup B) = P(A) + P(B)$ — Additive
   (A and B are *disjoint*)

Theorems of Probability

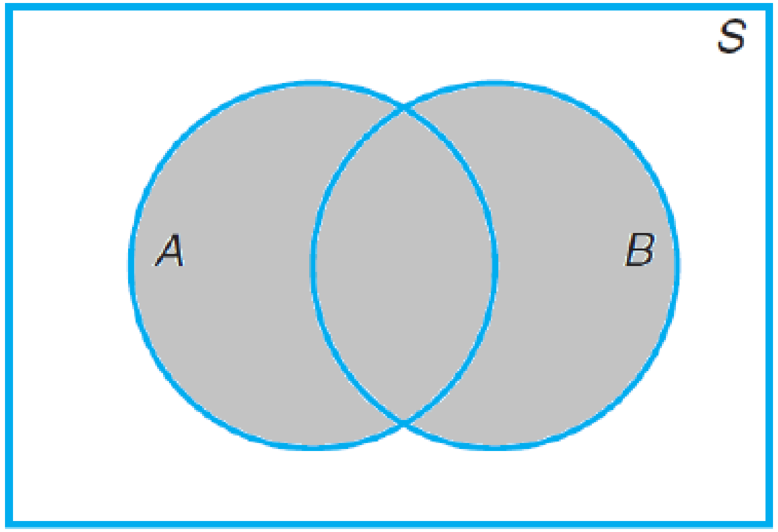
From Kolmogorov's axioms we can derive further properties:

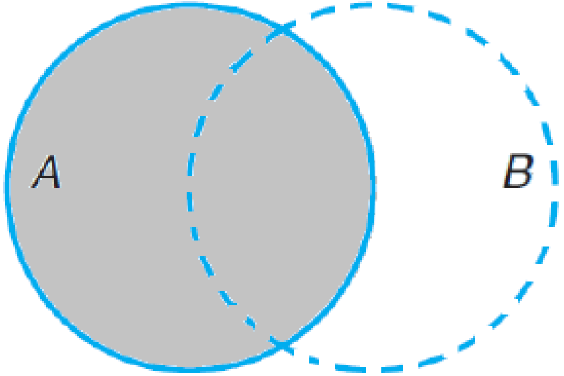
- $P(\bar{A}) = 1 - P(A)$
- $P(A \cup \bar{A}) = 1$
- $P(\emptyset) = 0$
- If $A \subset B$, then $P(A) \leq P(B)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Subsets A and B are said independent if $P(A \cap B) = P(A) P(B)$

Do not confuse with disjoint subsets i.e. $A \cap B = 0$

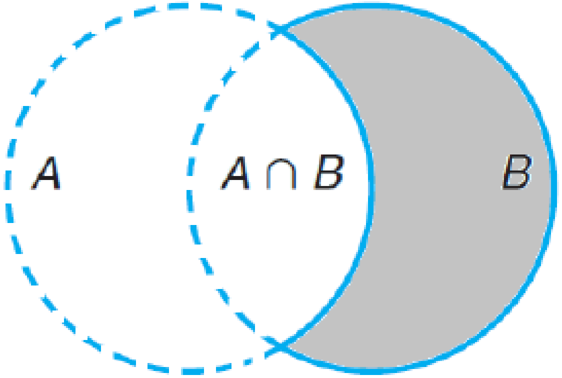
Theorem: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$





$P(A)$

$+$

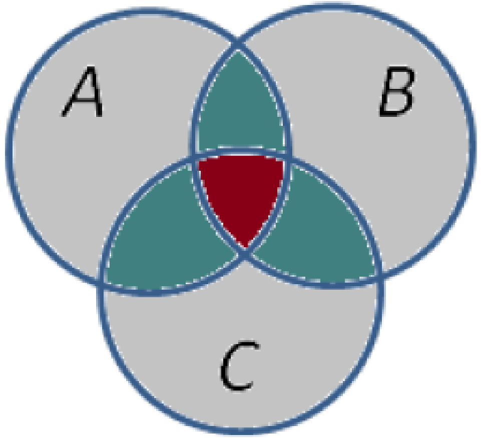


$P(B) - P(A \cap B)$

Theorem

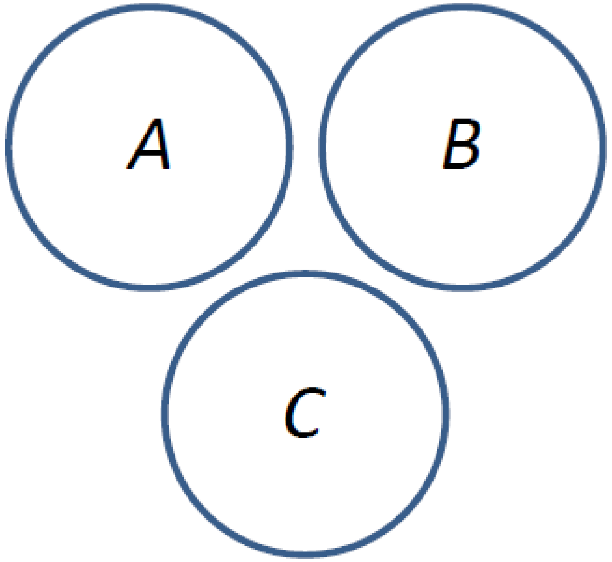
For three events A, B, and C,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) \\ - P(A \cap B) - P(A \cap C) - P(B \cap C) \\ + P(A \cap B \cap C)$$



And for disjoint events we obtain:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) \\ \cancel{- P(A \cap B)} \cancel{- P(A \cap C)} \cancel{- P(B \cap C)} \\ + \cancel{P(A \cap B \cap C)}$$



And if $C = \emptyset$, we obtain:

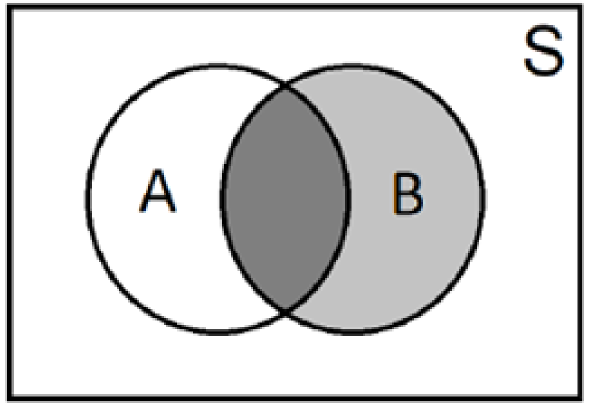
$$P(A \cup B \cancel{\cup C}) = P(A) + P(B) \cancel{+ P(C)} \\ - P(A \cap B) \cancel{- P(A \cap C)} \cancel{- P(B \cap C)} \\ \cancel{+ P(A \cap B \cap C)}$$

Conditional probability

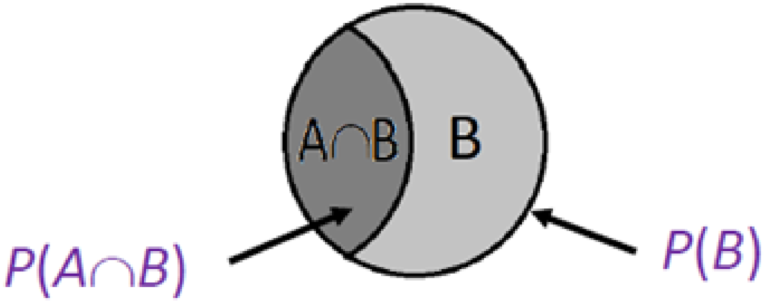
The probability of an event A occurring given that is it known that event B has occurred is called the *conditional probability* and is written as $P(A|B)$; "the probability of A given B".

It can be shown that:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$



Probability is represented by the ***areas*** of events in a *proportional* Venn diagram. Given that event B has occured, the sample space is now reduced to B.



Example

In rolling a dice we have the sample space:

x = {1, 2, 3, 4, 5, 6}

$$P(x < 3|x\ \textbf{even}) = \frac{P((x < 3) \cap (x\ \textbf{even}))}{P(x\ \textbf{even})} = \frac{1/6}{3/6} = \frac{1}{3}$$

Bayes' Theorem

From the definition of conditional probability we have:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{and} \quad P(B|A) = \frac{P(B \cap A)}{P(A)}$$

But $P(A \cap B) = P(B \cap A)$, so:

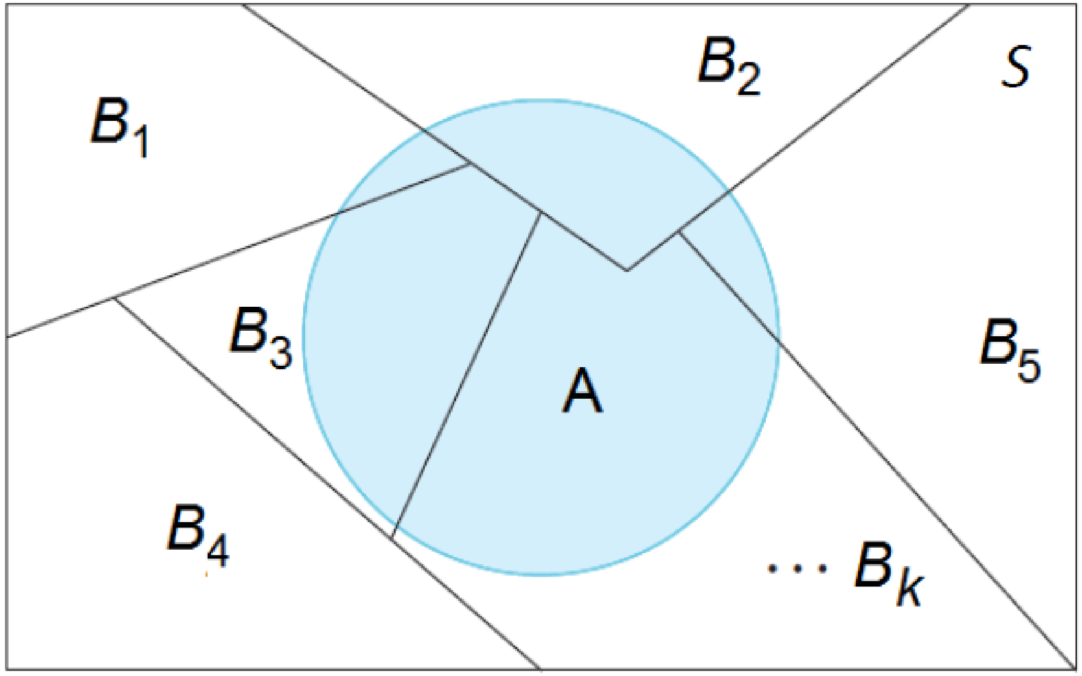
$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Bayes' Theorem

First published (posthumously) by the
Reverend Thomas Bayes (1702−1761)

An essay towards solving a problem in the doctrine of chances,
Philos. Trans. R. Soc. **53c**(1763) 370; reprinted in Biometrika, **45** (1958) 293.

Total Probability and Bayes' theorem



The probability of event A can be constructed from the total probability of all intersecting events:

$$P(A) = \sum_{i=1}^{k} P(A \cap B_i)$$

$$P(A|B_i) = \frac{P(A \cap B_i)}{P(B_i)}$$

$$\Rightarrow \; P(A \cap B_i) = P(A|B_i)P(B_i) \quad \Rightarrow$$

Total probability of event A

$$P(A) = \sum_{i=1}^{k} P(A|B_i)P(B_i)$$

Random Variable

Random Proceses

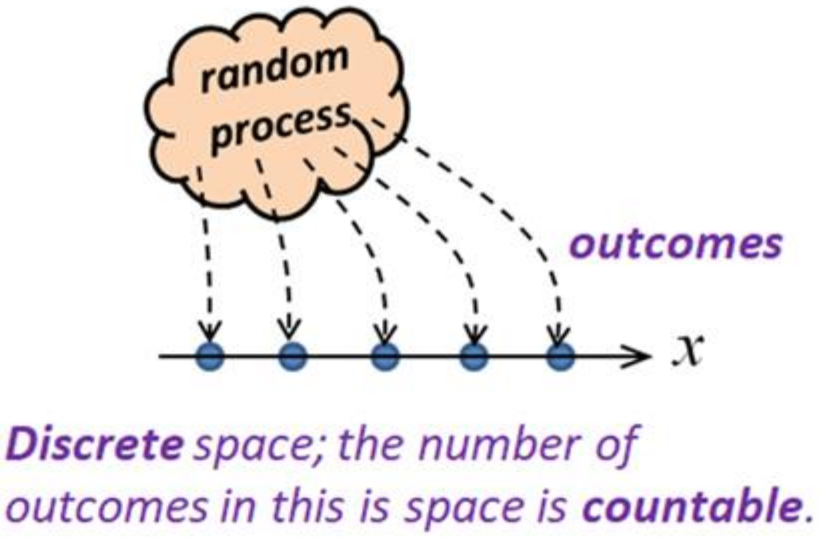
Statistics deals with random processes.

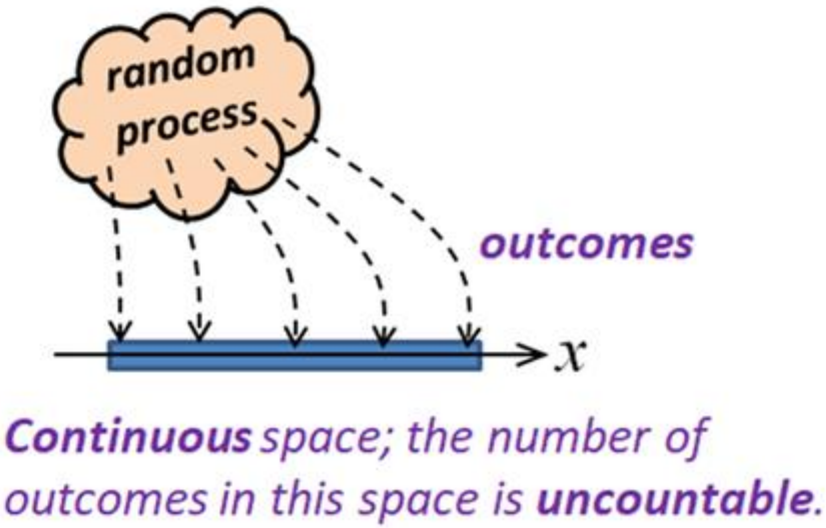
The outcomes of such processes cannot be predictable, for example,

- the throwing of a die
- the number of disintegrations in a particular radioactive source in a period of time T (fluctuate from trial to trial)
- Decay length of a cosmic muon in atmosphere
- Mass of Z Boson

Random Variable

- The outcome of a random process is represented by a **random variable** *x*.
- A random variable can be **discrete** or **continues**:





Probability Density Function (pdf)

Random processes are described by a **probability density** function, $f(x)$.

e.g. If the process is the throwing of a single die, then $x = \{1, 2, 3, 4, 5, 6\}$ *the probability of an outcome x is then given by the density function* $f(x) = 1/6$

The random variable x is then said to be distributed as $f(x)$.

- If x is discrete, $f(x_i)$ then gives the frequency at each point x_i
- If x is continuous, the distribution $f(x)$ is then a continuous density such that the probability of finding x in the interval x to $x + dx$ is $f(x)dx$.

Some Properties

Discrete RV

$$f(x_i) \geq 0$$

$$\sum_i f(x_i) = 1$$

$$\sum_{i=a}^{b} f(x_i) = P(a \leq x \leq b)$$

Continues RV

$$f(x) \geq 0$$

$$\int_{-\infty}^{+\infty} f(x)dx = 1$$

$$\int_a^b f(x)dx = P(a \leq x \leq b)$$

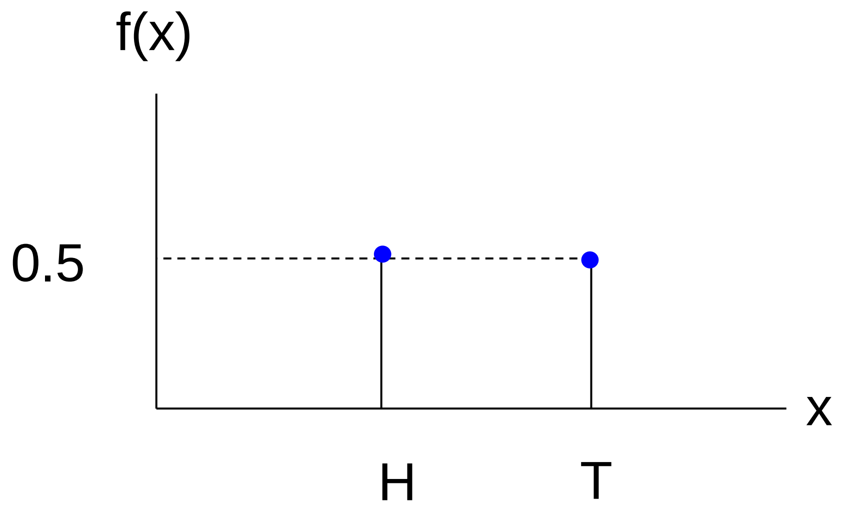
Discrete RV Examples:

pdf

Throwing a coin:

X = { H, T}

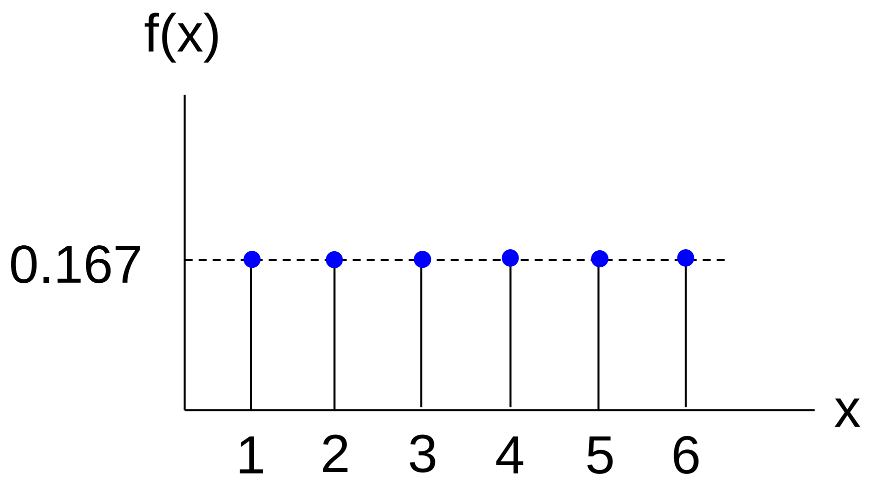
f(x) = 1/2



Throwing a die:

X = {1,2,3,4,5,6}

f(x)= 1/6

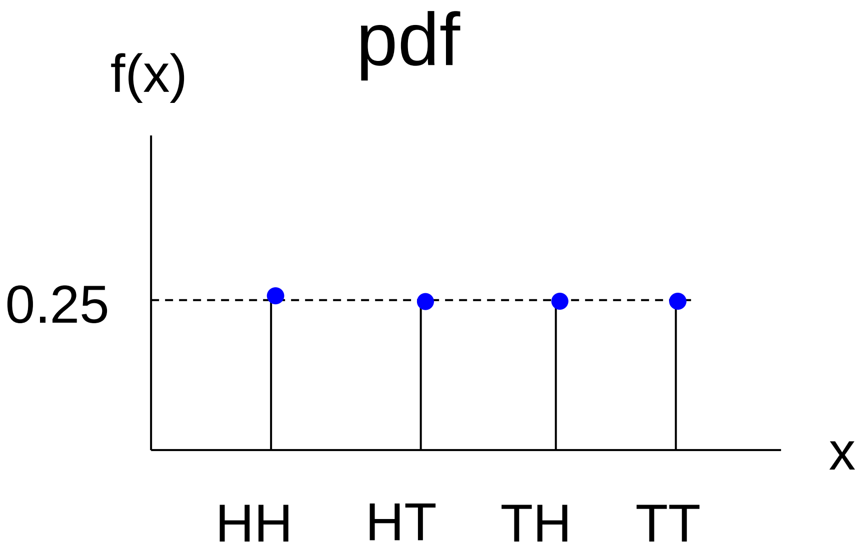


Discrete RV Examples:

Throwing two coins:

```
   X = {HH,   HT,  TH,  TT}
f(x) = 1/4
```

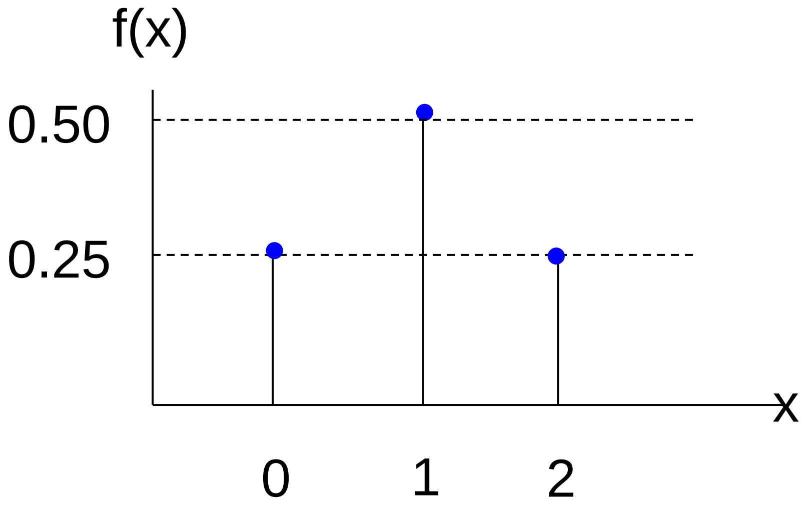
P(HH) = P (HT) = P(TH) = P(TT) = 0.25



Throwing two coins:

Let X = {number of heads}

```
   X = {  0,   1,   2}
f(x) = {1/4, 1/2,  1/4}
```

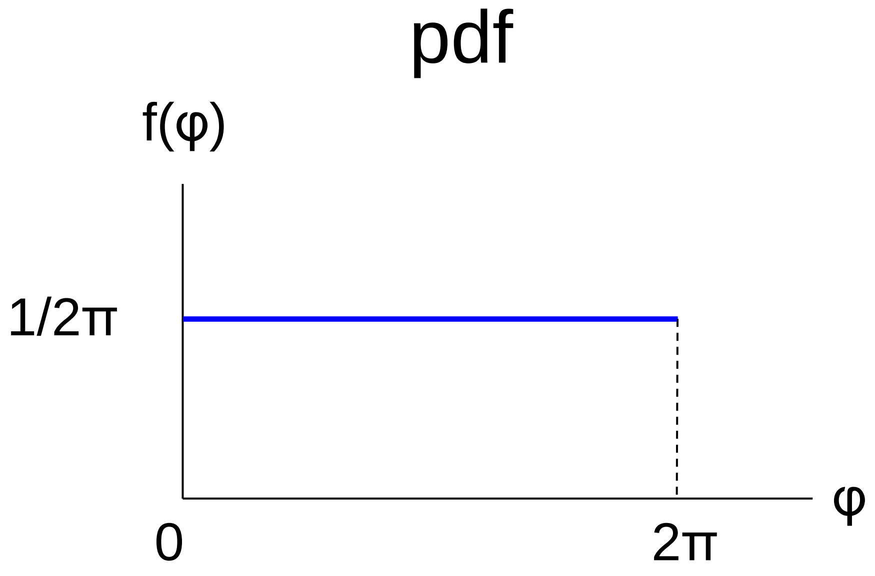


Continues RV Examples:

Random angle:

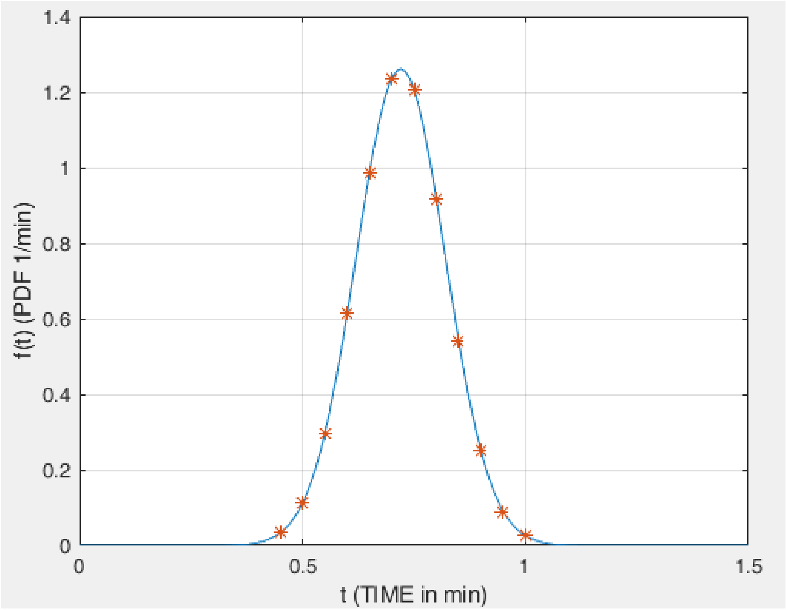
$\Phi = [0, 2\pi]$

$f(\varphi) = 1 / 2\pi$



t = {Run-time of a program in a server}

f(t) = {measured discrete values}



Expectation Values

The expectation (mean) value of a random variable is defined as:

$$\mu = E[x] = \int x f(x) dx \qquad \text{for continues RV}$$

$$\mu = E[x] = \sum x_i f(x_i) \qquad \text{for discrete RV}$$

Expectation value of the function $g(x)$: $E[g(x)] = \int g(x) f(x) dx$

Note1: Notation for mean can be μ , $E[x]$ or $\langle \boldsymbol{x} \rangle$:

Note2: In Quantum Mechanics, expectation value of the position is defined as follows:

$$\langle x \rangle = E[x] = \int_{-\infty}^{+\infty} \Psi^* x \Psi \, dx \qquad \text{where} \qquad \int_{-\infty}^{+\infty} \Psi^* \Psi \, dx = \int_{-\infty}^{+\infty} |\Psi|^2 \, dx = 1$$

Variance

The variance of a random variable is defined as:

$$\sigma^2 = E[(x-\mu)^2] = \int (x-\mu)^2 f(x)dx \quad \text{for continues RV}$$

$$\sigma^2 = E[(x-\mu)^2] = \sum (x_i-\mu)^2 f(x_i) \quad \text{for discrete RV}$$

Note1: Standard deviation of RV : $\sigma = \sqrt{\sigma^2}$

Note2: Root means square of RV : $RMS = \sqrt{E[x^2]}$

Note3: Important relation : $\sigma^2 = E[x^2] - (E[x])^2$ (in QM $(\Delta x)^2 = \langle x^2\rangle - \langle x\rangle^2$)

Example: Tossing a die:

In tossing a die: $x = \{1, 2, 3, 4, 5, 6\}$ and $f(x) = 1/6$

(a) $E[x] = 1 * 1/6 + 2 * 1/6 + \ldots + 6 * 1/6 = 3.50$

(b) $E[x^2] = 1^2 * 1/6 + 2^2 * 1/6 + \ldots + 6^2 * 1/6 = 15.17$

(c) $\text{RMS} = \sqrt{E[x^2]} = 3.89$

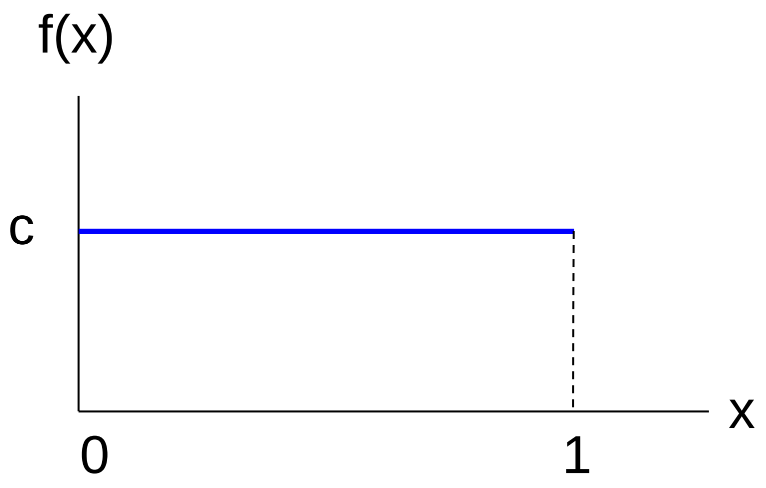
(d) $\sigma^2 = E[x^2] - (E[x])^2 = 2.92$

(e) $\sigma = 1.71$

Example: Uniform Distribution

x = [0, 1]

f(x) = c = constant



(a) Find c

$$\int_{-\infty}^{+\infty} f(x)dx = 1 \longrightarrow \int_{0}^{1} cdx = c(1-0) = 1 \longrightarrow c = 1$$

(b) Find mean and std.dev

$$E[x] = \int_{-\infty}^{+\infty} xf(x)dx = \int_{0}^{1} xdx = [x^2/2]_0^1 = 1^2/2 - 0^2/2 = 1/2 = 0.5$$

$$\sigma^2 = \int_{-\infty}^{+\infty} (x-\bar{x})^2 f(x)dx = \int_{0}^{1} (x-1/2)^2\, dx = 1/12$$

$$\sigma = \sqrt{\sigma^2} = 1/\sqrt{12} = 0.289$$

Covariance

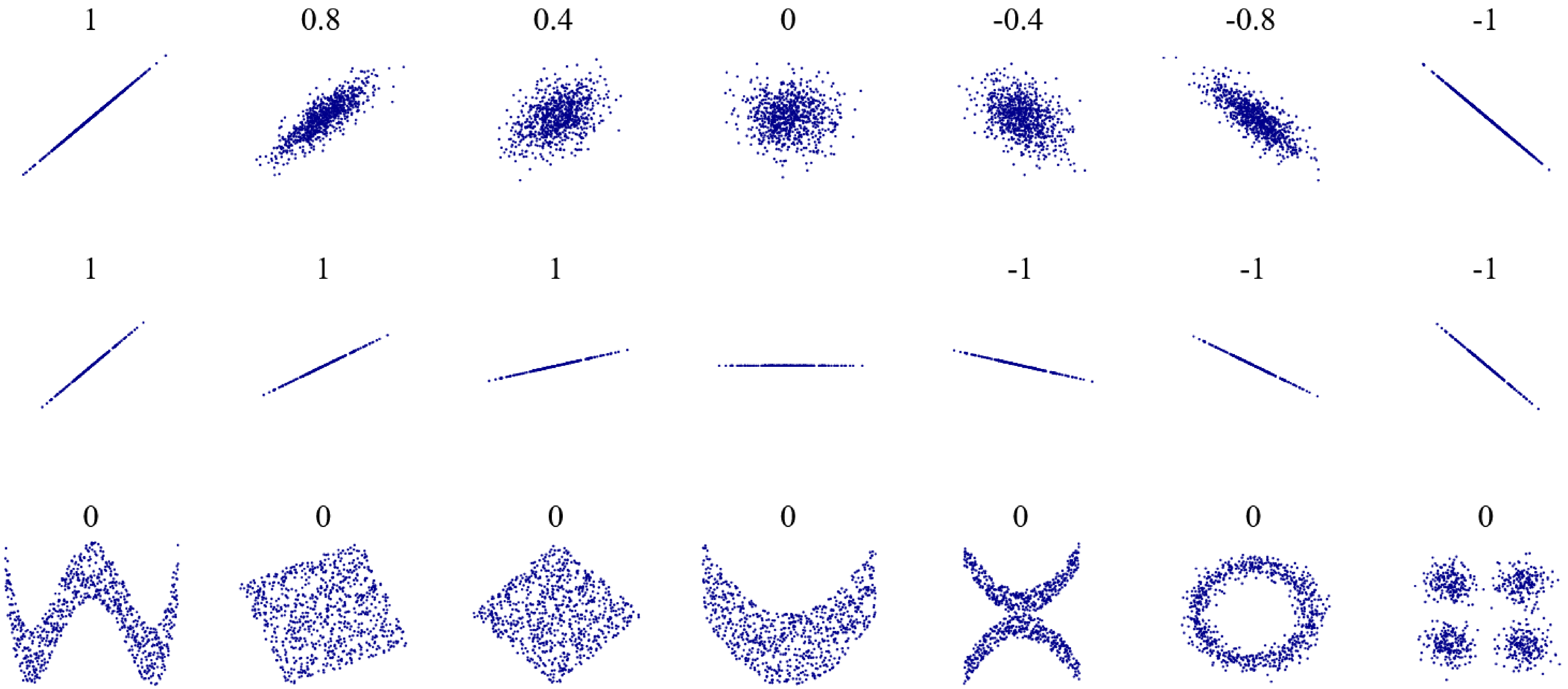
In the more general case, the outcomes of a process may be characterized by several random variables, $x, y, z, \ldots$ The process is then described by a multivariate distribution $f(x, y, z, \ldots)$. **Covariance** of two variables:

$$\operatorname{cov}(x, y) = E\left[(x - \mu_x)(y - \mu_y)\right]$$

The covariance is a measure of the **linear correlation** between the two variables. This is more often expressed as the **correlation coefficient** which is defined as:

$$\rho = \frac{\operatorname{cov}(x, y)}{\sigma_x \, \sigma_y}$$

- ρ varies between -1 and +1 (the sign indicates the sense of the correlation)
- If the variables are perfectly correlated linearly, then $|\rho| = 1$
- If the variables are perfectly uncorrelated, then $|\rho| = 0$ (x and y linearly independent)


1
0.8
0.4
0
-0.4
-0.8
-1
1
1
1
-1
-1
-1
0
0
0
0
0
0
0

Cumulative Distribution Function

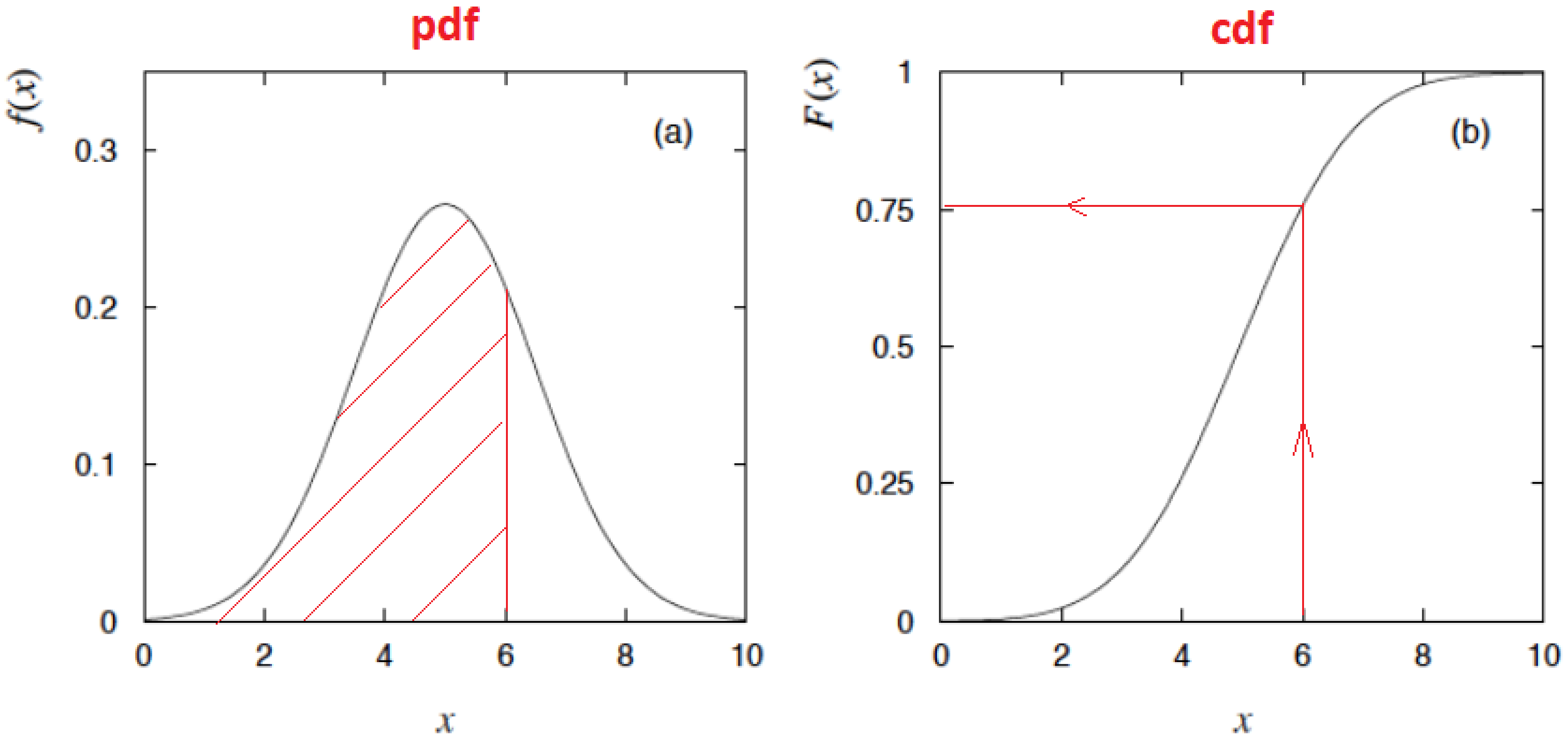
Given a pdf, $f(x')$, probability to have outcome less than or equal to x is:

$\int_{-\infty}^{x} f(x')dx' = F(x)$ for continues RV

$\sum_{i=1}^{x} f(x_i') = F(x)$ for discrete RV

- $F(x)$ is called **cumulative distribution function** (cdf).
- $F(-\infty) = 0$ and $F(+\infty) = 1$
- If cdf is given, then pdf can be calculated from: $f(x) = \frac{dF}{dx}$

$$\int_{-\infty}^{x} f(x')dx' = F(x)$$



Example:

Given $x = \{1.0, 2.0, 3.0, 4.0\}$ and

$$f(x) = \{0.3, 0.5, 0.1, 0.1\}$$

Determine cdf

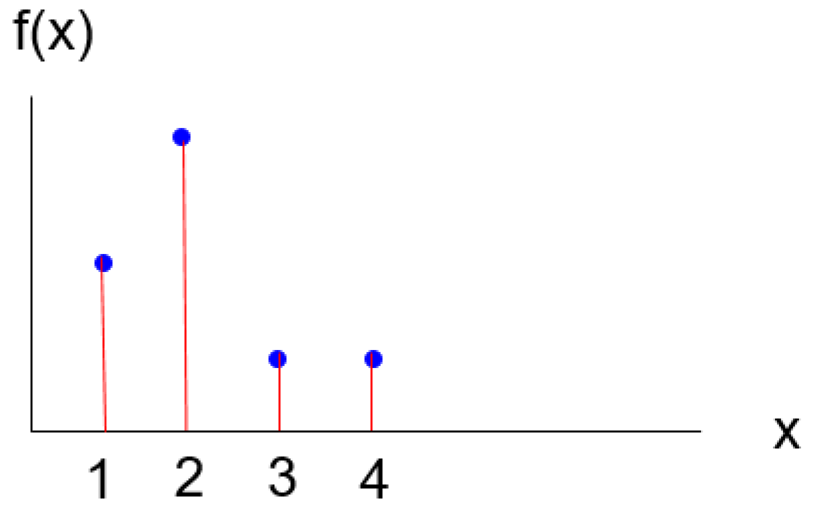
$$\sum_{i=1}^{x} f(x_i') = F(x)$$

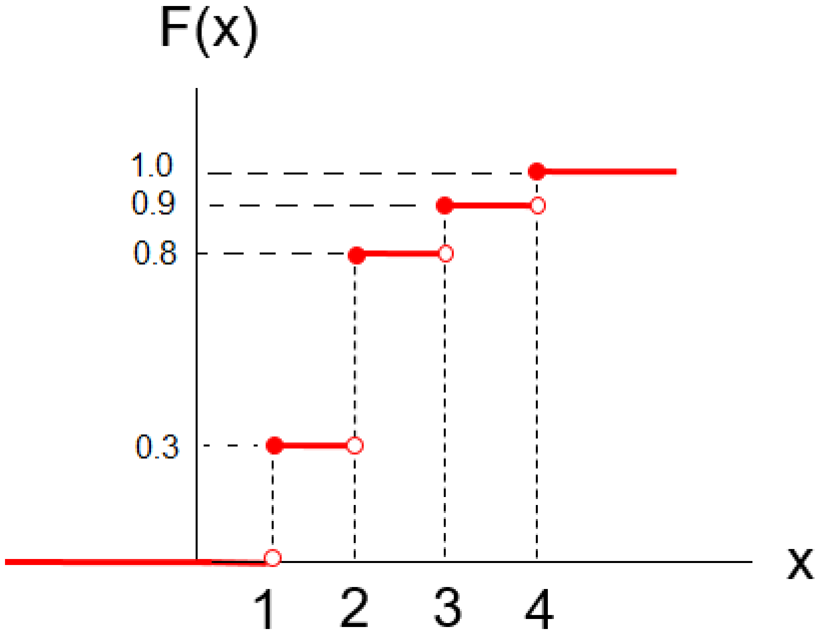
$$F(1) = 0.3$$

$$F(2) = 0.8$$

$$F(3) = 0.9$$

$$F(4) = 1.0$$





Example:

We can describe an unstable particle, that decays with a mean lifetime τ. In this case total probability of finding the particle in space is not a constant and decrease with time (t). Quantum Theory says:

$$f(t) = \int |\Psi(x,t)|^2 \, dx \propto e^{-t/\tau}$$

(a) Determine pdf and cdf. Ans: $f(t) = \frac{1}{\tau} e^{-t/\tau}$ and $F(t,\tau) = 1 - e^{-t/\tau}$

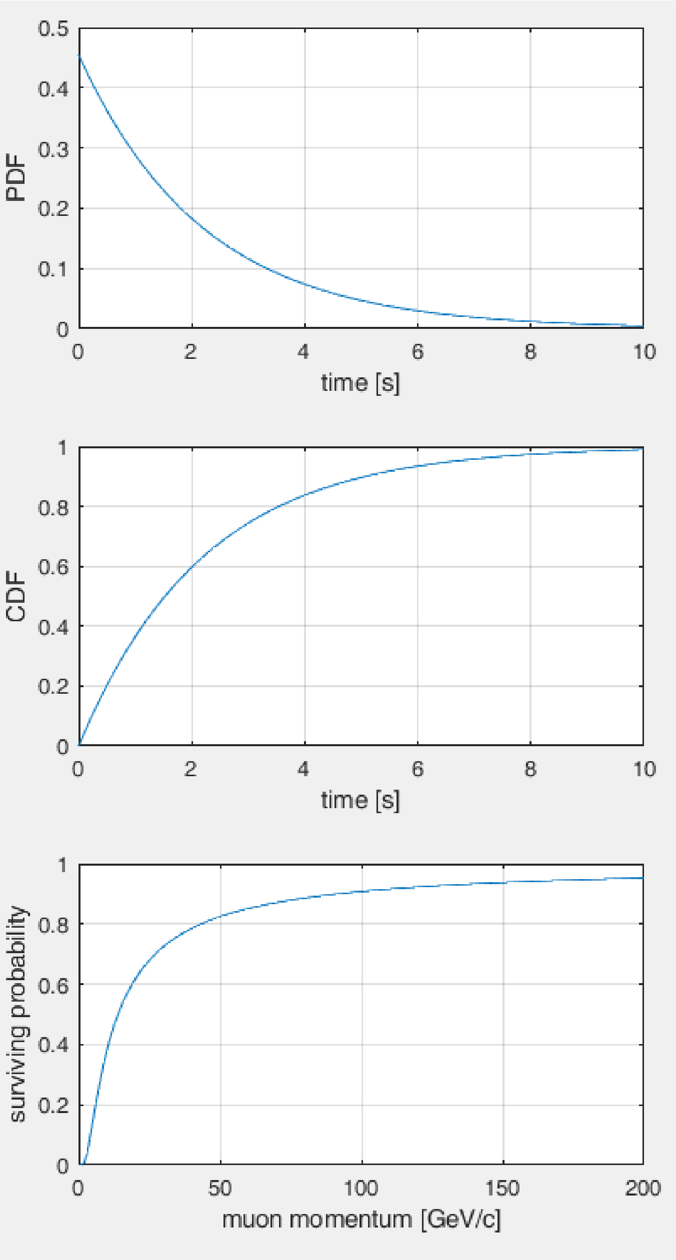
(b) Determine expectation value of t. Ans: $E[t] = \int_0^\infty t f(t) dt = \tau$

(c) For muons $\tau = 2.2\ \mu\text{s}$. What is the probability that a muon can survive at $t = 200\ \mu\text{s}$ if it is at rest? and if it moving at momentum $p = 50\ \text{GeV/c}$?

Solution:

$$\tau' = \frac{\tau}{\sqrt{1-\beta^2}} = \frac{\tau}{\sqrt{1-p/\sqrt{p^2+m^2}}} \text{ (time dilation)}$$

$$P_{survive} = f(t) = \frac{1}{\tau} e^{-t/\tau'}$$



Special Distribution Functions

Root Functions

Distribution functions used in Nuclear and Particle Physics:

Distibution Function	In Root `TRandom` Class
Uniform Distribution	`Double_t Uniform(Double_t x1 = 1)`
Exponential Distribution	`Double_t Exp(Double_t tau)`
Binomial Distribution	`Int_t Binomial(Int_t ntot, Double_t prob)`
Poisson Distribution	`Int_t Poisson(Double_t mean)`
Gaussian Distribution	`Double_t Gaus(Double_t mean=0, Double_t sigma=1)`
Landau Distribution	`Double_t Landau(Double_t mean=0, Double_t sigma=1)`
Breit-Wigner Distribution	`Double_t BreitWigner(Double_t mean=0, Double_t gamma=1)`

See also: Probability Density Functions (PDF)

Binomial Distribution Function

The binomial distribution function specifies the number of times (k) that an event occurs in n independent trials where p is the probability of the event occurring in a single trial.

The Binomial pdf is:

$$f(k) = \frac{n!}{k!\,(n-k)!} p^k (1-p)^{n-k}$$

Mean: $\mu = \sum k f(k) = np$

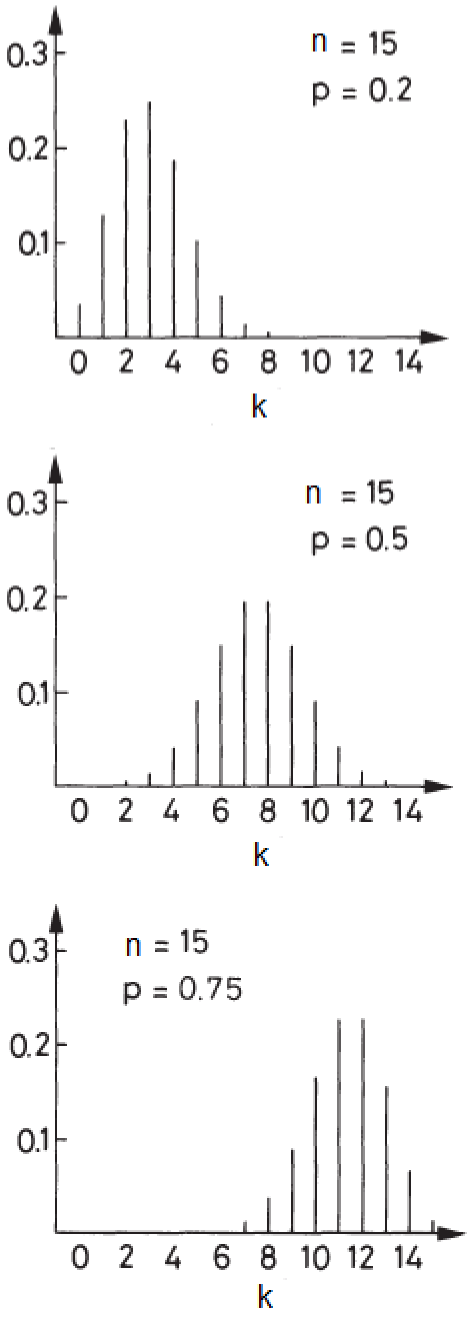
Variance: $\sigma^2 = np(1-p)$

Sum: $\sum f(k) = 1$

Pdf is discrete.

Example:

Detection efficiency (either we detect particle or not).



Example:

In a high-energy physics experiment, a specific sub-detector has an muon detection efficiency of 80% (i.e. it has an 80% probability of successfully detecting a muon passing through it). During a specific beam spill, 10 muons pass through the detector.

What is the probability that exactly 8 muons are successfully detected?

Solution

n = 10 (total number of muons/trials)

k = 8 (number of successful detections/successes)

p = 0.8 (probability of detection)

1−p = 0.2 (probability of a "miss")

$$f(8) = \frac{10!}{8!(10-8)!} 0.8^8 (1-0.8)^{10-8} = 0.302$$

Result:

There is a **30.2%** chance that the detector will record exactly 8 out of the 10 muons.

Example

A coin is thrown 30 times.

(a) Calculate the mean (expected) number heads and standard deviation

$$\mu = np = (30)(0.5) = 15$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{(30)(0.5)(0.5)} = 2.74$$

(b) Imagine you observed 20 heads. Compute how many standard deviations your observation differ from the mean value. Is the coin fair?

$$N = \frac{20-15}{2.74} = 1.83$$

N < 2 sigma
20 heads is consistent with 15 => the coin is fair

(c) Imagine you observed 30 heads. Compute how many standard deviations your observation differ from the mean value. Is the coin fair?

$$N = \frac{30-15}{2.74} = 5.47$$

N > 5 sigma
20 heads is not consistent with 15.
Discovery => the coin is not fair

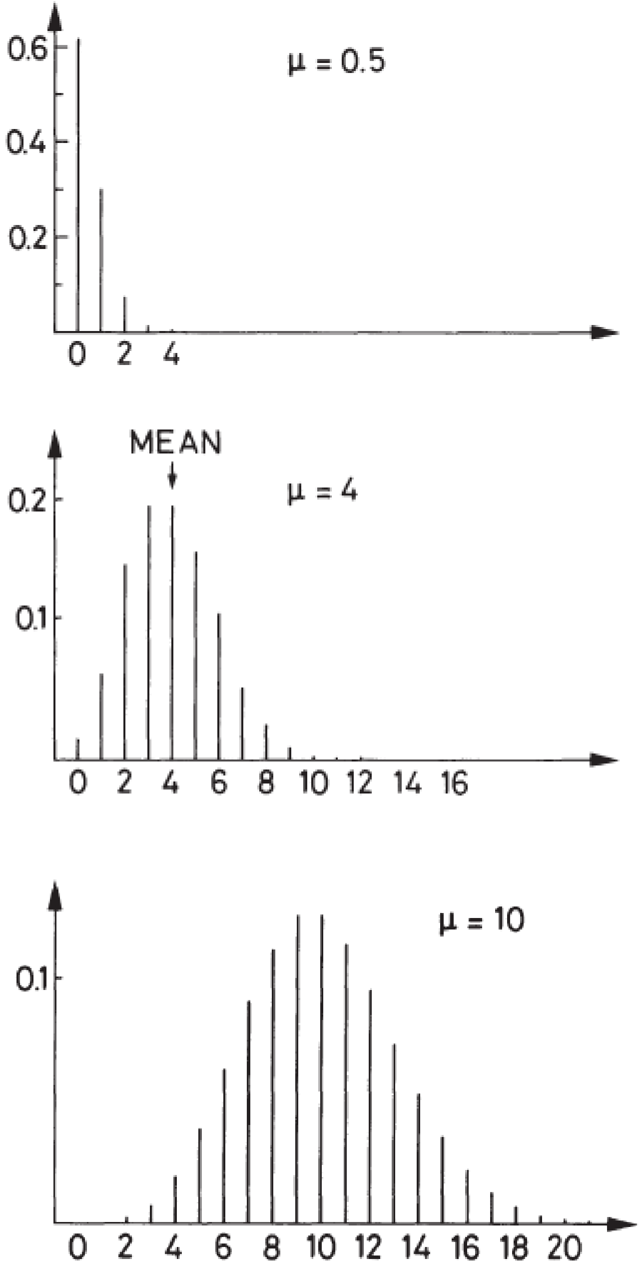
Poisson Distribution Function

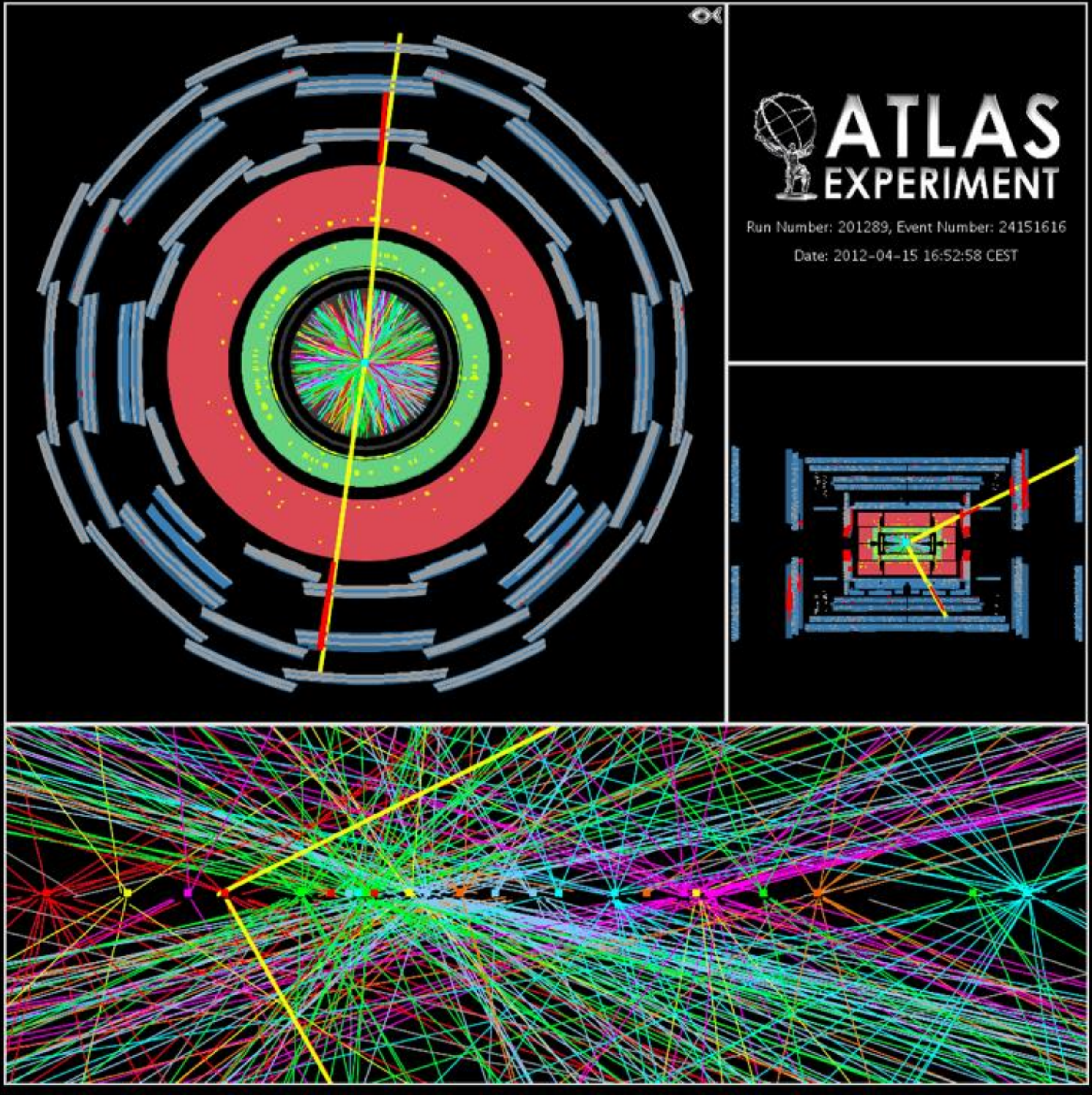
The Poisson distribution occurs as the limiting form of the binomial distribution when the probability $p \to 0$ and the number of trials $n \to \infty$ such that the mean $(\mu = np)$ remains finite.

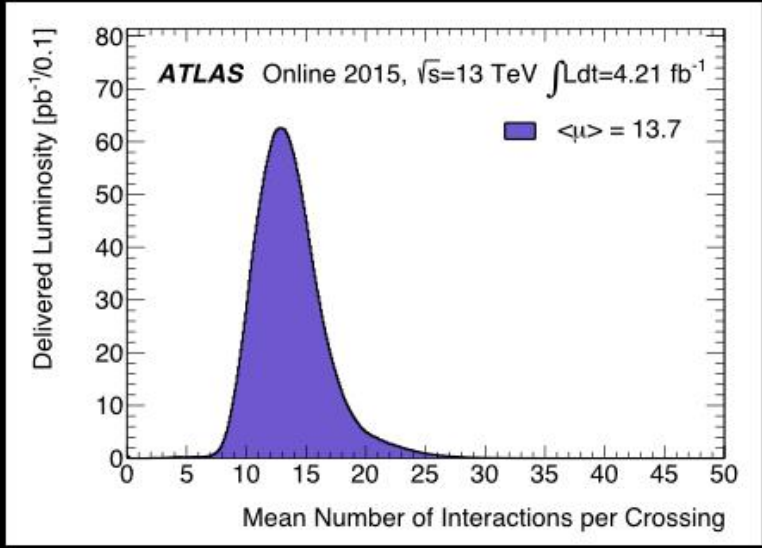
$$f(k) = \frac{\mu^k e^{-\mu}}{k!}$$

Mean: $\mu = \sum k f(k) = np$
Variance: $\sigma^2 = \mu$
Sum: $\sum f(k) = 1$
Pdf is discrete.

Examples:
* Clicks of a Geiger counter in a given time interval
* Mean number of p-p interactions per bunch crossing at LHC (pile-up events)
* Number of atmoshperic muons passing through unit area per unit time
* Number of photons generated in Cherenkov Radiation process







[4] The beat.

Something deep and subtle, with the number of taps per second based on "pileup", the average number of collisions per proton-bunch crossing in 2015. Here it's 16.

Example:

^{137}Cs nucleus has half-life of 27 years.

The probability per unit time for a single nucleus to decay is:

$$\lambda = \frac{\ln 2}{27\ \mathrm{yr}} = 0.026/\mathrm{yr} = 8.2 \times 10^{-10}/\ \mathrm{s}.$$

In 1 pg sample ^{137}Cs contains about 10^9 nuclei.

Mean number of decays from the sample is $10^9 \times 8.2 \times 10^{-10} = 0.82\ \mathrm{decays/s}$

What is the probability to observe 40 decays in 1 min?

Solution

$$\mu = \lambda t = (0.82)(60) = 49.2$$

$$f(40) = \frac{(49.2)^{40} e^{-49.2}}{40!} = 0.025$$

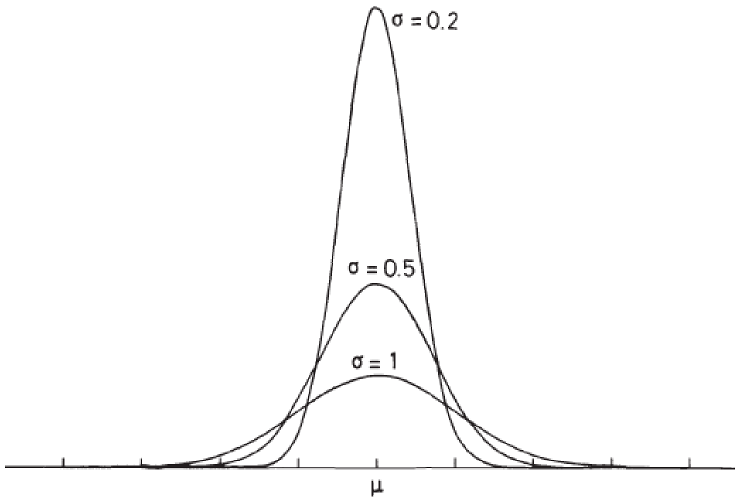
Result: There is a **2.5%** chance that we observe exactly 40 decays in 1 min.

Gaussian or Normal Distribution Function

The Gaussian or normal distribution plays a central role in all of statistics

The Gaussian is a **continuous** pdf:

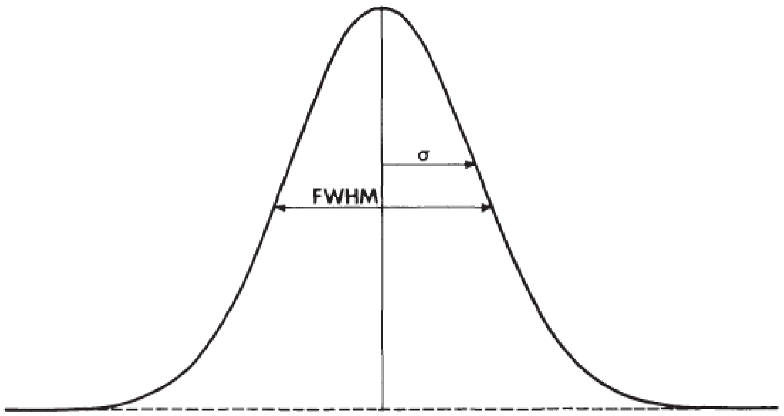
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$



Two parameters μ and σ^2 corresponds mean and variance.

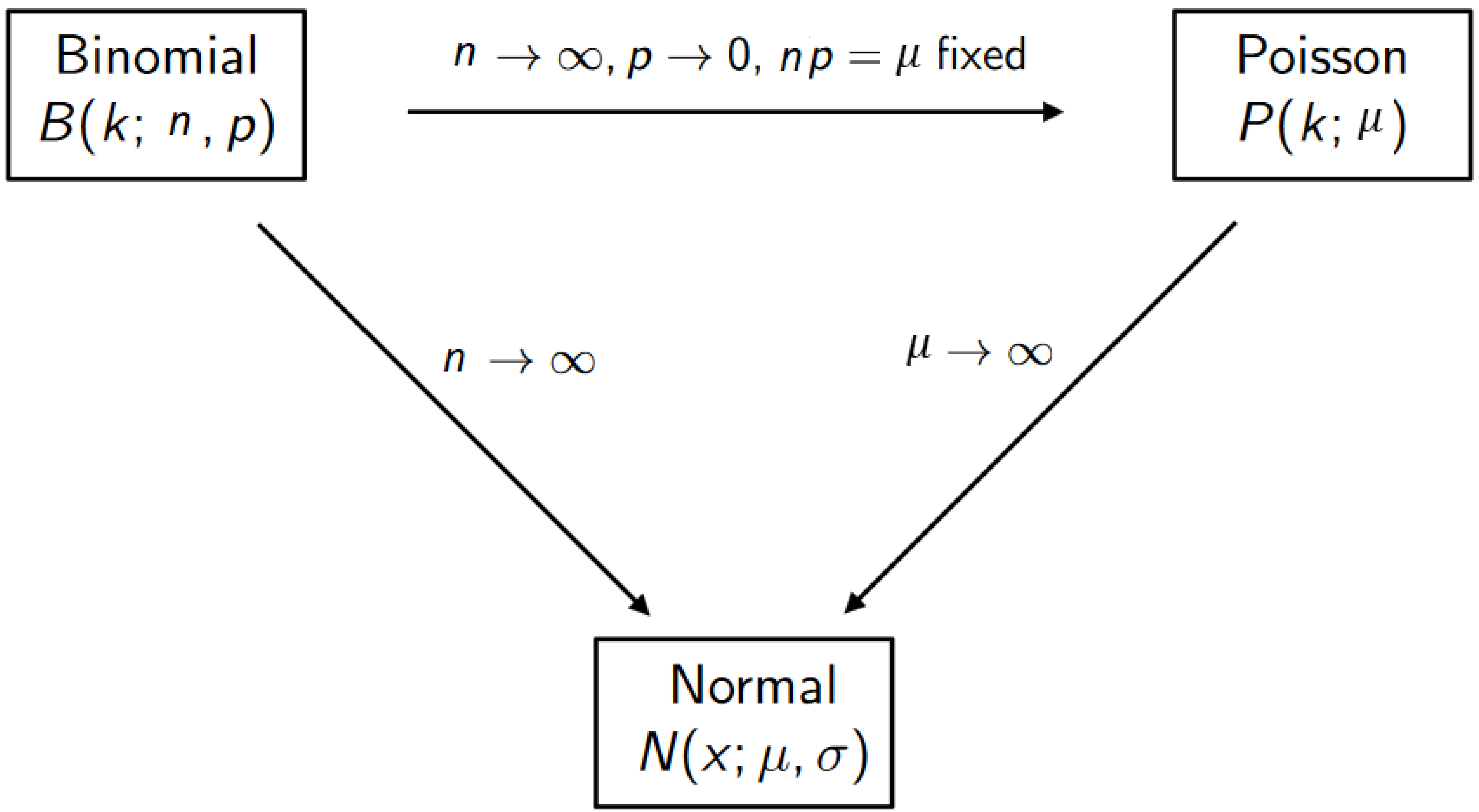
- σ is known as standard deviation of the distribution.
- In some applications, full width at half maximum (FWHM) is used. This is somewhat larger than σ and can easily be shown to be

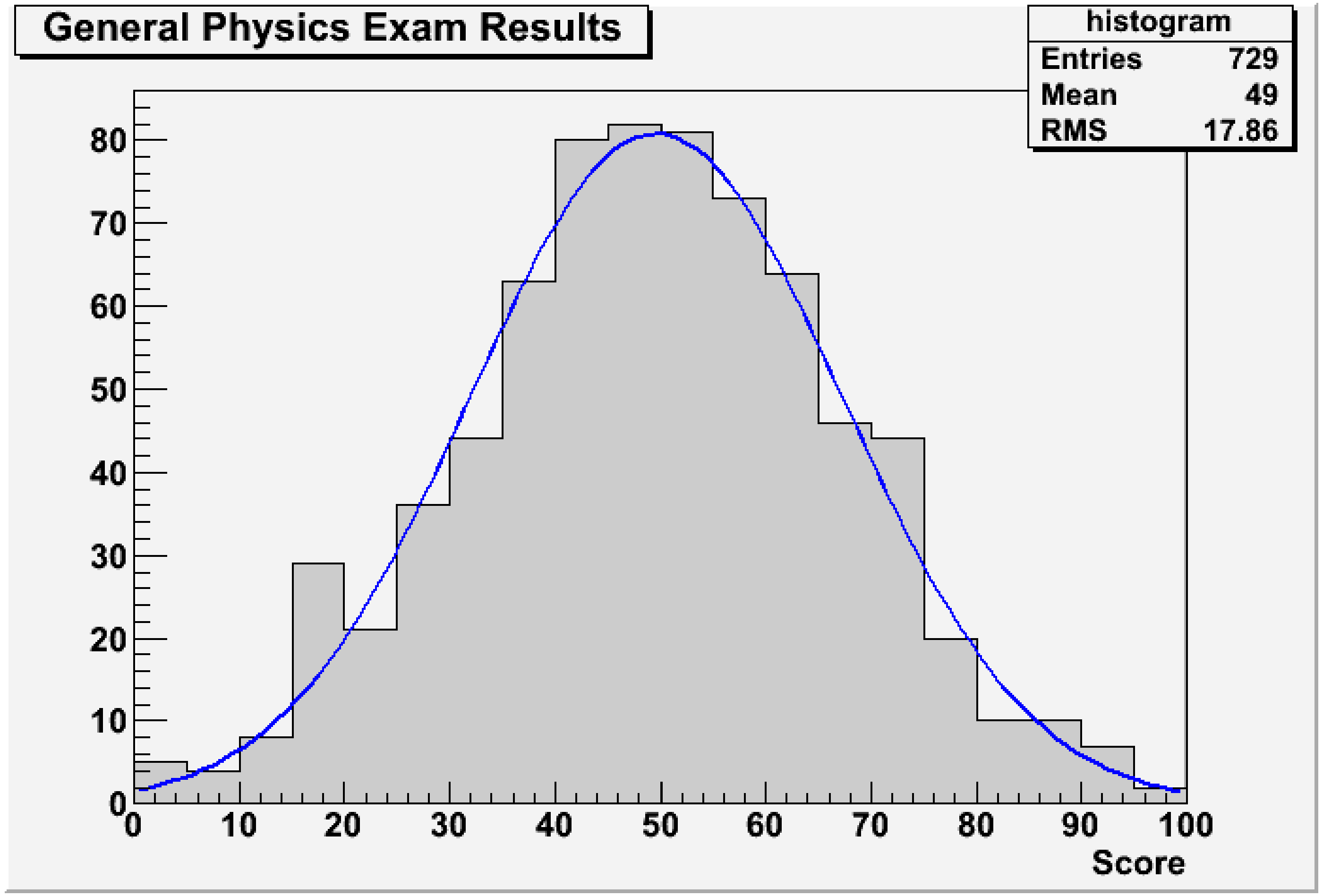
$$FWHM = 2\sigma\sqrt{2\ln(2)} = 2.35\,\sigma$$



Example:

Instrumental errors are generally described by Gaussian distribution.


Binomial
$B(k; n, p)$
$n \to \infty, p \to 0, np = \mu$ fixed
Poisson
$P(k; \mu)$
$n \to \infty$
$\mu \to \infty$
Normal
$N(x; \mu, \sigma)$


General Physics Exam Results
histogram
Entries 729
Mean 49
RMS 17.86
0
10
20
30
40
50
60
70
80
0
10
20
30
40
50
60
70
80
90
100
Score

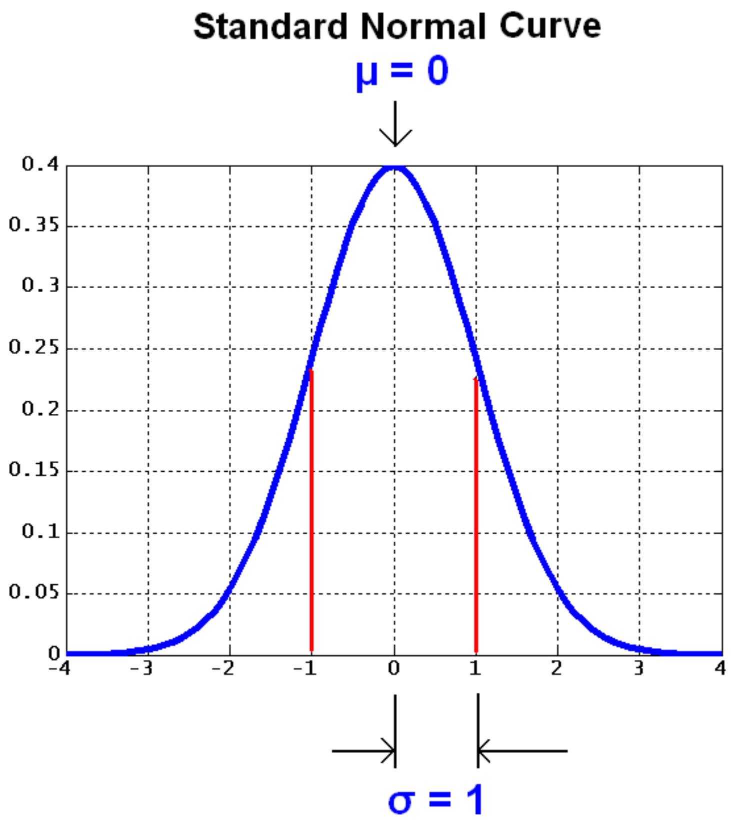
Standard Normal Distribution Function

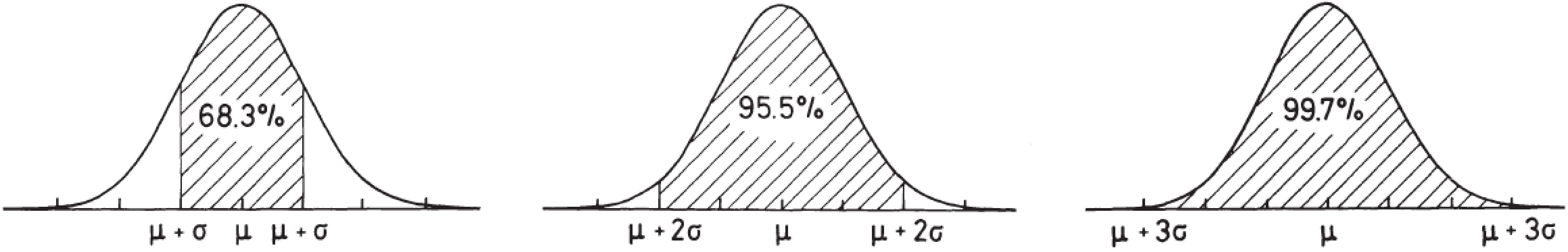
The normal distribution function for $\mu = 0$ and $\sigma = 1$ is called the standard normal distribution function.

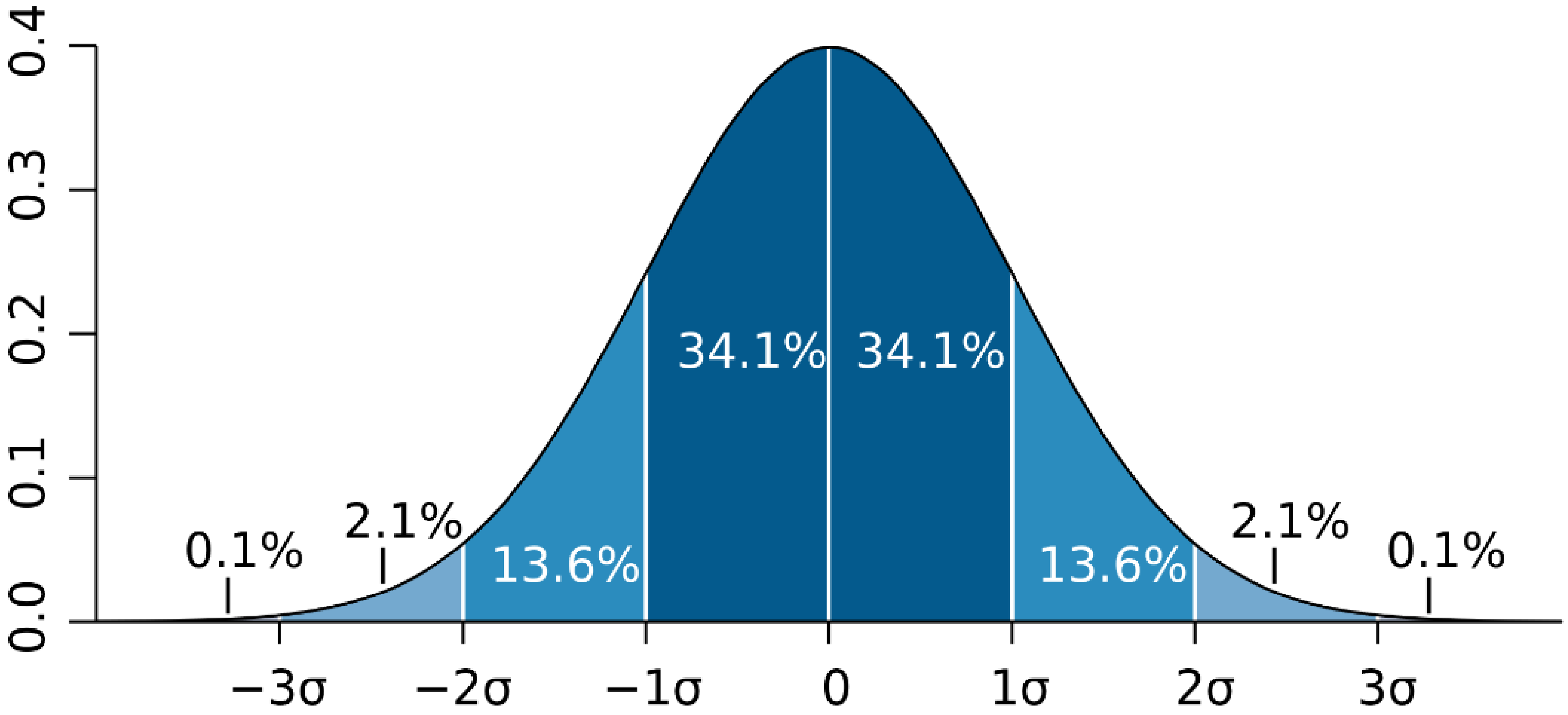
$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$$

All Gaussian distributions may be transformed to this standard form by making the following variable transformation:

$$z = \frac{x - \mu}{\sigma}$$



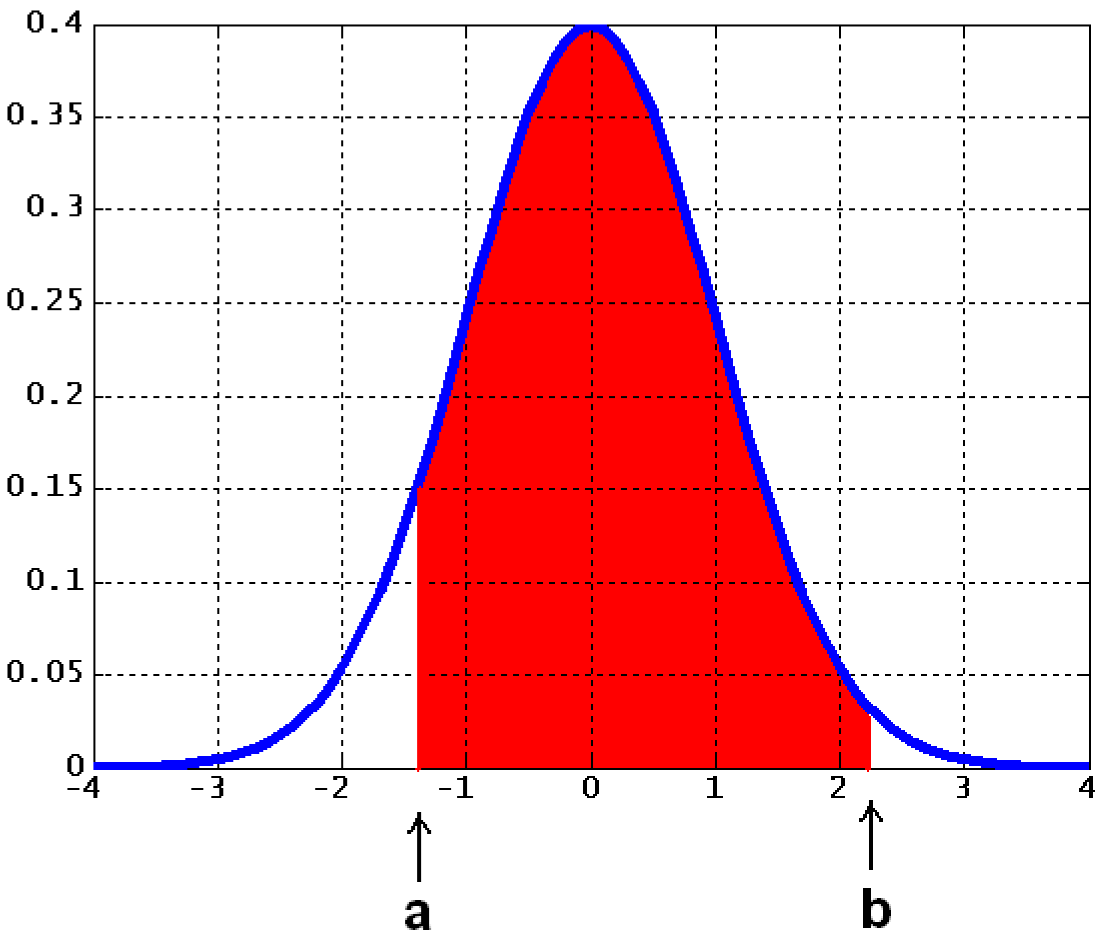

68.3%
μ + σ
μ
μ + σ
95.5%
μ + 2σ
μ
μ + 2σ
99.7%
μ + 3σ
μ
μ + 3σ

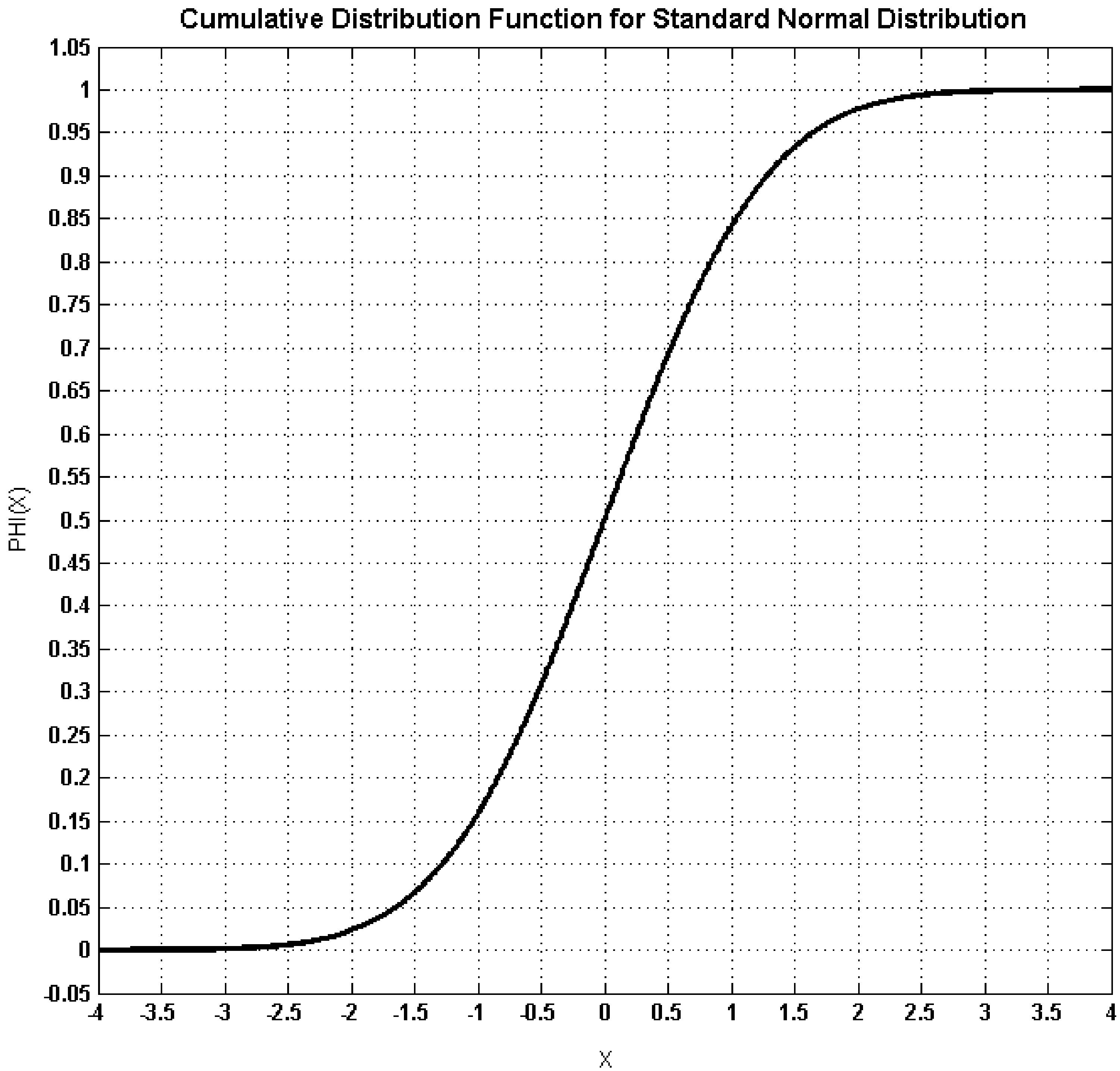

0.0
0.1
0.2
0.3
0.4
0.1%
2.1%
13.6%
34.1%
34.1%
13.6%
2.1%
0.1%
−3σ
−2σ
−1σ
0
1σ
2σ
3σ

Area under the standard normal curve between [a, b] is:

$$\int_a^b \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx \quad = \Phi(b) - \Phi(a)$$

The values of the function $\Phi(x)$ can be taken from a table or from the figure on next page.




Cumulative Distribution Function for Standard Normal Distribution
1.05
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
PHI(X)
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
X

Example:

Suppose the temperature during May is normally distributed with mean 20 °C and standard deviation 3 °C. Find the probability that the temperature is between 21 °C and 27 °C.

Solution

1. Apply standard transformation ($z = (x - \mu)/\sigma$)

$$a = \frac{21-20}{3} = 0.333 \text{ and } b = \frac{26-20}{3} = 2.0$$

2. Compute the integral:

$$\int_a^b \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx = \Phi(b) - \Phi(a)$$

$$= \Phi(2) - \Phi(0.333)$$

$$= 0.9772 - 0.6304$$

$$= \mathbf{0.3468}$$

```
// In root
double a = ROOT::Math::normal_cdf(0.333);
double b = ROOT::Math::normal_cdf(2.0);
double p = b - a;
cout << p << endl;
```

```
# In pyroot
import ROOT
a = ROOT.Math.normal_cdf(0.333)
b = ROOT.Math.normal_cdf(2.0)
p = b - a
print(p)
```

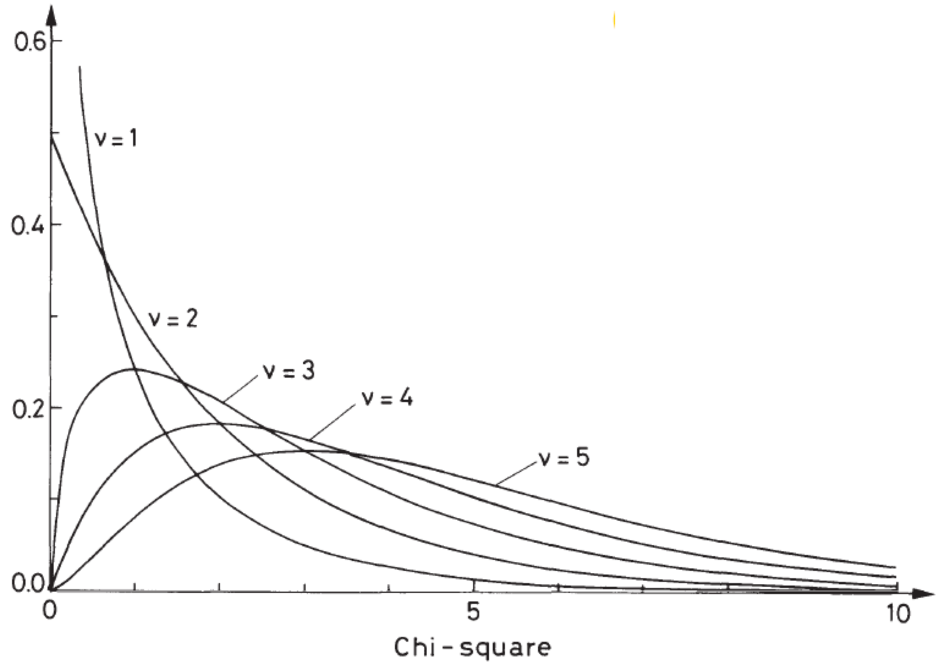
Chi-Square Distribution

Suppose we have a set of *n* independent random variables, x_i, distributed as Gaussian densities with theoretical means μ_i and standard deviations σ_i, respectively.
The sum

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{x_i - \mu_i}{\sigma_i} \right)^2$$

is then known as the chi-square. Let $u = \chi^2$ then chi-square distribution function is

$$P(u)\,du = \frac{(u/2)^{(\nu/2)-1} \exp(-u/2)}{2\Gamma(\nu/2)}\,du$$



where ν is an integer known as degrees of freedom and $\Gamma(\nu/2)$ is the gamma function.
The chi-square distribution is particulary useful for testing the goodness-of-fit of theoretical formulae to experimental data. For a good fit $\chi^2/\nu \approx 1$.

Exercises

Exercises

1. Detector for particle identification

In proton-proton collisions we have: 90% pions, 10% kaons

1. Kaon identification: 95% efficient
2. Pion misidentification: 6%

Question: if the particle identification indicates a kaon, what is the probability that it is a real kaon / a real pion?

Solution: Output can be **particle is identified** (detected) = *I*
or **particle is not identified** (not detected) = *N*

$P(\pi) = 0.90$

$P(K) = 0.10$

$P(I|K) = 0.95$

$P(N|K) = 0.05$

$P(I|\pi) = 0.94$

$P(N|\pi) = 0.06$

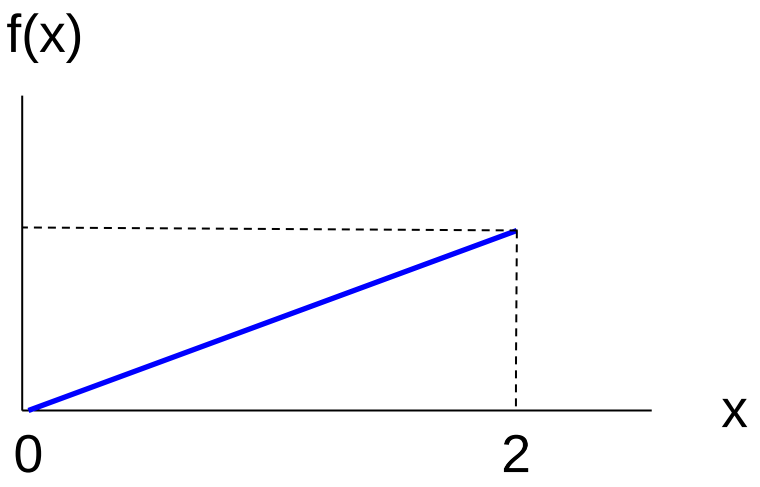
$P(K|I) = ?$

$P(\pi|I) = ?$

2. Show that if x and y are related parabolically, ($y = x^2$), then $\rho = 0$.

3. Consider the triangular pdf.

X = [0,2] and f(x) = kx



Find

(a) the value k

(b) mean and

(c) standard deviation of the distribution.

4. A communication system contains 6 stations. Independent probability of each station being functional is 90%. If the system requires at least 4 stations to be functional what is the probability that the communication system is functional. (Answer: 0.9842)

5. Suppose 2% of the people on the average are left-handed.
   Find the probability of 3 or more left-handed among 100 people. (Ans: 0.325)

6. Inefficency of a muon detector is 1%. Determine the probability of detecting single Higgs Boson from the decay channel: $H \rightarrow ZZ^{*} \rightarrow \mu^{+}\mu^{-}\mu^{+}\mu^{-}$

(a) using Binomial distribution function

(b) using Poisson distribution function

7. Suppose the diameters, D, of screws manufactured by a company are normally distributed with mean 0.25 cm and standard deviation 0.02 cm. A screw is considered defective if its diameter $D < 0.22$ cm. A sample of 250 screws are selected randomly.

Estimate the number of defective screws in this sample (Ans: 17)

8. (a) Calculate average number of photons generated for the visible light (400-700 nm) in water (n = 1.33) of thickness of 2 cm for charged particle of veloctiy $\beta \approx 1$.

(b) What is the probability of observing 450 visible photons?

9. An array of sky-facing cameras operating continuously (24 hours a day) in Gaziantep recorded a total of 540 meteor events over a period of 90 days. Assuming meteor arrivals follow a Poisson process:

(a) What is the probability of observing exactly 2 meteor events tomorrow?

(b) What is the probability of observing at least 2 meteor events tomorrow?