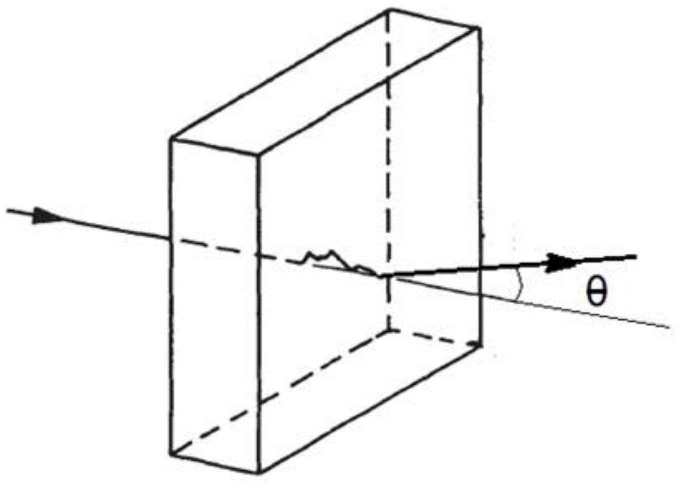
Particle Data Analysis in High Energy Physics

Lecture 5

Particle Interaction with Matter



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Introduction

We will briefly exemine the following topics:

- Interaction of Charged Particles with Matter
- Interaction of Photons with Matter
- Electromagenetic and Hadronic Showers

Constants

$c = 3.00 \times 10^{8}\ \mathrm{m/s}$	Speed of Light
$e = 1.60 \times 10^{-19}\ \mathrm{C}$	Electronic Ch.
$h = 6.63 \times 10^{-34}\ \mathrm{J}.s$	Planck Const.
$N_A = 6.02 \times 10^{23}\ \mathrm{mol}^{-1}$	Avagadro's #

Symbols

ρ	Density [g/cm^3]
λ	Mean Free Path [cm]
λ	Wavelength [nm]
M_u	Molar mass [g/mol]
σ	Cross section [m^2, barn]
X_0	Radiation Length [cm]
Z	Atomic Number
A	Mass Number
$\bar{I}$	Mean Ionization Energy
I	Intensity

Rest mass of some particles

$m_e = 9.11 \times 10^{-31}\ \mathrm{kg} = 0.511\ \mathrm{MeV/c^2}$	Electron
$m_p = 1.67 \times 10^{-27}\ \mathrm{kg} = 938\ \mathrm{MeV/c^2}$	Proton
$m_\pi = 2.50 \times 10^{-28}\ \mathrm{kg} = 140\ \mathrm{MeV/c^2}$	Pion
$m_\mu = 1.88 \times 10^{-28}\ \mathrm{kg} = 106\ \mathrm{MeV/c^2}$	Muon
$m_\alpha = 6.64 \times 10^{-24}\ \mathrm{kg} = 3727\ \mathrm{MeV/c^2}$	Alpha par.

Conversion between kg and eV.

$$1\ \mathrm{eV} = 1.6 \times 10^{-19}\ \mathrm{J} \ \Rightarrow \ 1\ \mathrm{MeV} = 1.6 \times 10^{-13}\ \mathrm{J}$$

$$E = mc^2$$

$$\begin{aligned} E = m_e c^2 &= (9.11 \times 10^{-31}\ \mathrm{kg})(3 \times 10^{8}\ \mathrm{m/s}) \\ &= 2.733 \times 10^{-22}\ \mathrm{J} \\ &= 0.511\ \mathrm{MeV} \end{aligned}$$

$$m_e = \frac{E}{c^2} = 0.511 \frac{\mathrm{MeV}}{c^2}$$

Some Material Data

Material	Density ρ (g / cm^3)	Atomic Number Z	Mass Number A	Molar Mass M_u (g / mol)	Electron Density n (10^{24} / cm^3)	Radiation Length X_0 (g / cm^2)
Aluminum (Al)	2.70	13	27	26.98	0.78	24.28
Copper (Cu)	8.94	29	64	63.55	2.46	13.05
Iron (Fe)	7.87	26	56	55.85	2.21	14.18
Lead (Pb)	11.40	82	207	207.20	2.72	6.30
Carbon (C)	2.27	6	12	12.01	0.68	42.70
Silcon (Si)	2.33	14	28	28.085	0.70	21.82
Water (H_2O)	1.00	7.42 (eff)	18	18.015	0.334	36.08
Air	0.0012	7.3 (eff)	29	28.97	0.0004	36.62

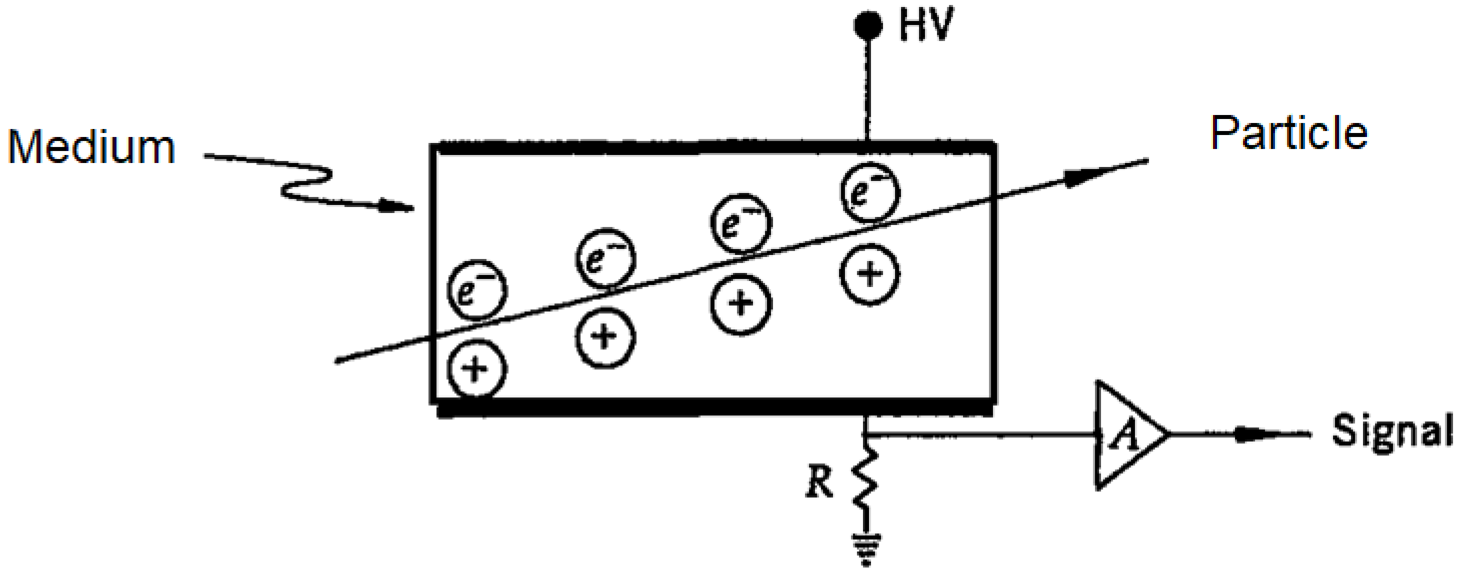
Interaction of Charged Particles with Matter

Interaction of Charged Particles

When an electrically charged particle travels in a medium, it:

- interacts with atomic electrons (inelastic collision).
- interacts with atomic nuclei (elastic scattering).

If the particle has sufficient kinetic energy, it can deposit this energy in the medium by ionizing the atoms in its path or by exciting the atoms (or molecules) to higher energy states.



How does a charged particle ionize particles in a medium while traveling through it?

The most suitable variable describing the ionization properties of a medium is **stopping power** or **ionization energy loss**.

Average energy loss in unit length is:

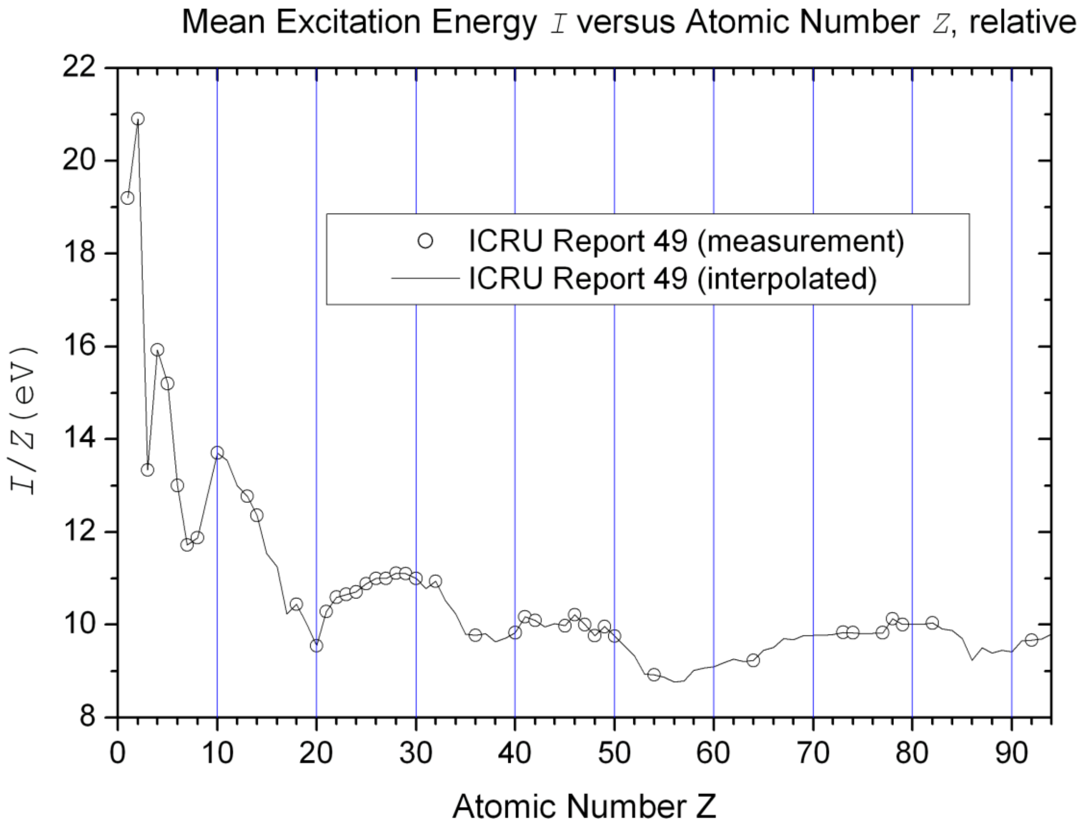
$$-\frac{dE}{dx} = n_i \bar{I}$$

E = Particle energy

n_i = number of electron-ion pair per unit length

$\bar{I}$ = mean ionization/exaction energy

if $Z > 20$: $\bar{I} \approx 10(\mathrm{eV})\, Z$



Bethe-Bloch Formula

For relativistic particles, the simplified version for dE/dx is given by:

$$-\frac{dE}{dx} = \frac{4\pi}{m_e c^2} \cdot \frac{nz^2}{\beta^2} \cdot \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \cdot \left[\ln\left(\frac{2m_e c^2 \beta^2}{I \cdot (1-\beta^2)}\right) - \beta^2\right]$$

This formula developed by Hans Bethe and Felix.

Here:

$n = \frac{N_A Z \rho}{M_u}$	electron density of material
ρ	mass density
Z, A	atomic and mass number
M_u	molar mass
z	particle charge / e (in general ±1)
$\beta = \frac{v}{c}$ ve $\gamma = (1-\beta^2)^{-1/2}$	particle’s beta and gamma values

If density is measured in g/cm^3 then a basic form can be obtained:

$$-\frac{dE}{dx} \approx 0.307\frac{z^2\rho Z}{A\beta^2}\left(\ln\left[\frac{1.022\times 10^5\beta^2\gamma^2}{Z}\right]-\beta^2\right)\ \left[\frac{\mathrm{MeV}}{\mathrm{cm}}\right]$$

Common way to specify the energy loss is in MeV/cm or mass thickness in MeV/g/cm^2.

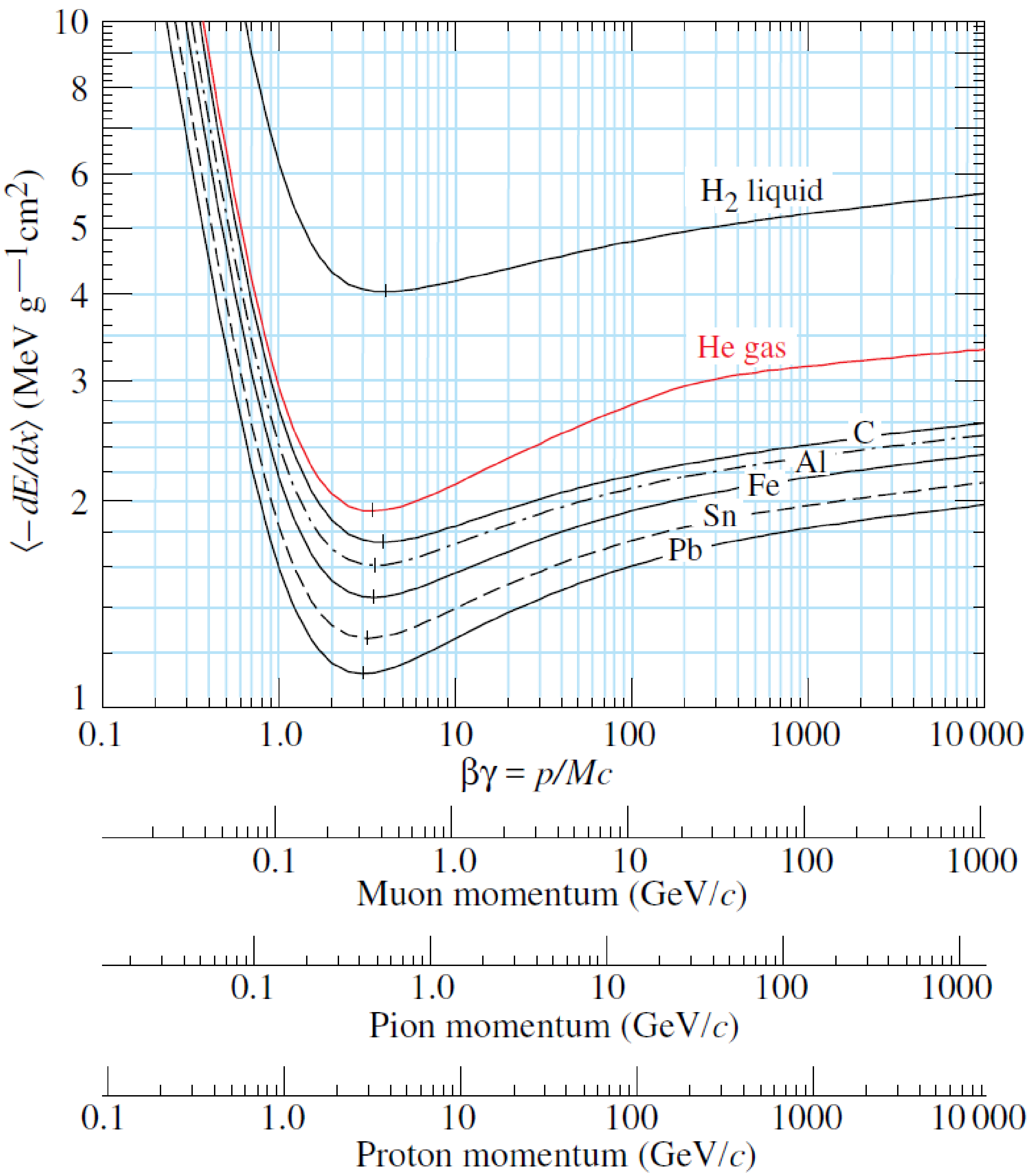
$$-\frac{1}{\rho}\frac{dE}{dx} \approx 0.307\frac{z^2 Z}{A\beta^2}\left(\ln\left[\frac{1.022\times 10^5\beta^2\gamma^2}{Z}\right]-\beta^2\right)\ \left[\frac{\mathrm{MeV}}{\mathrm{g/cm^2}}\right]$$

Example 5.1:

What is the average energy loss in a 2 cm iron block by a muon with 1 GeV energy as it passes through the block?

Answer: Look at the graph

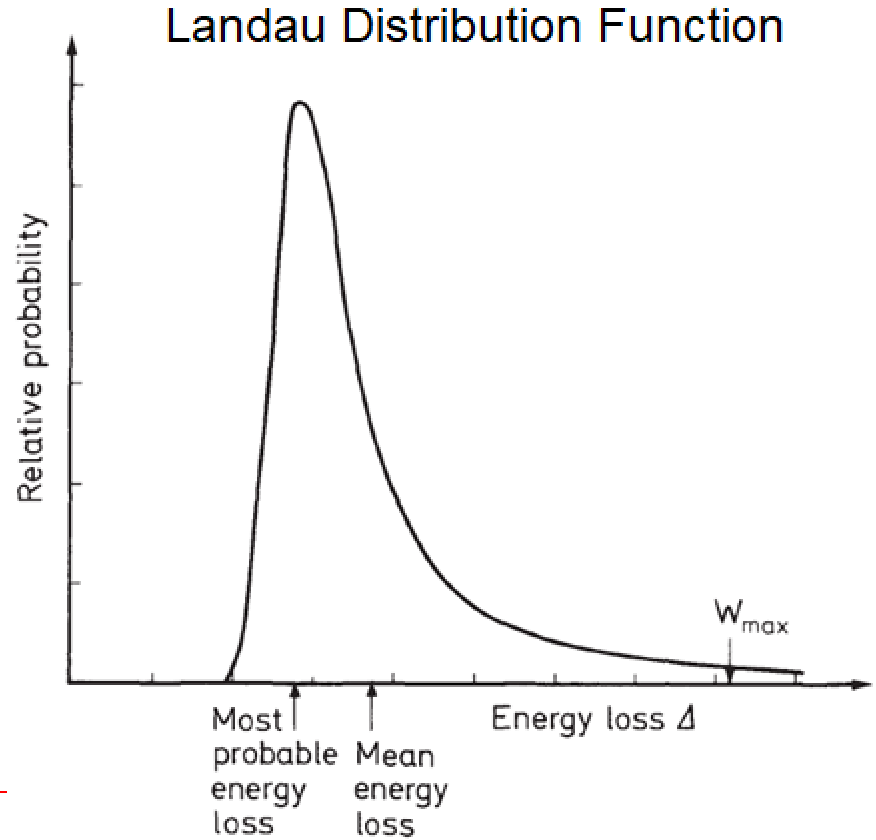
$$\Delta E \approx \left(1.8\frac{\text{MeV}}{g/cm^2}\right)/\left(7.9\frac{\text{g}}{\text{cm}^3}\right)(2\text{ cm}) \approx 0.5 \quad \text{MeV}$$



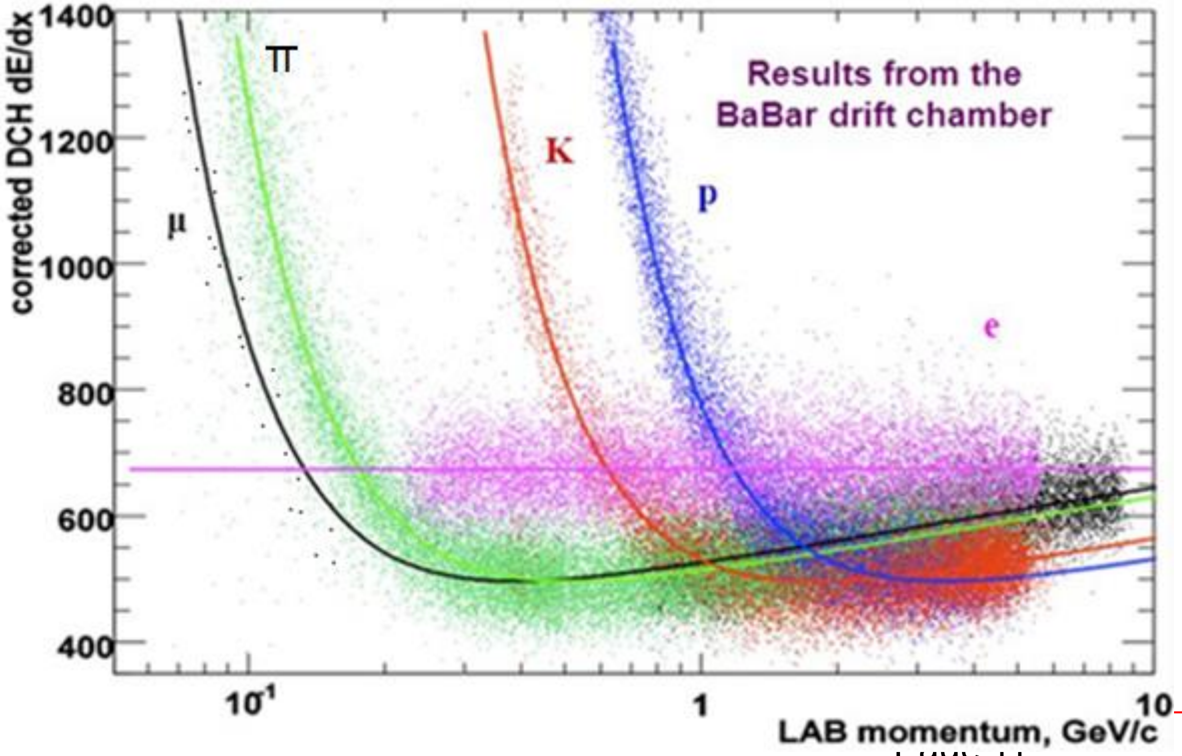
Landau Distribution

- Bethe-Bloch formula describes **average** energy loss for a charged particle (1932).
- Landau distribution describes the **fluctuations** in energy loss (1944).

$$f(x) = \frac{1}{\pi} \int_0^{\infty} e^{-u \ln u - xu} \sin(\pi u) du$$



Energy loss is used to identify particles at low energy.



Range and Bragg Curve

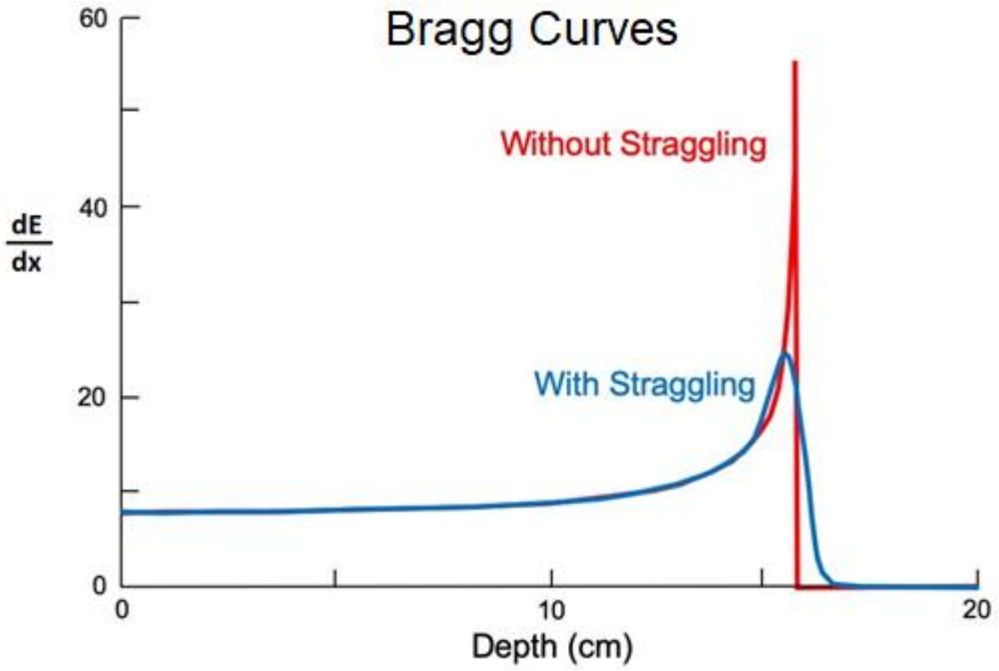
How much distance does a particle travel in a medium until it loses all of its energy?

Two identical particles cannot be subjected to the exact same interaction.
The range (distance) is different for both.
That is, the range shows a statistical distribution.
This phenomenon is called **Range Straggling** (Erim Dağınıklığı).

Theoretical calculation for range:

$$R = \int_0^{T_0} \left(\frac{dE}{dx}\right)^{-1} dE$$

T_0 = initial K.E. Of particle

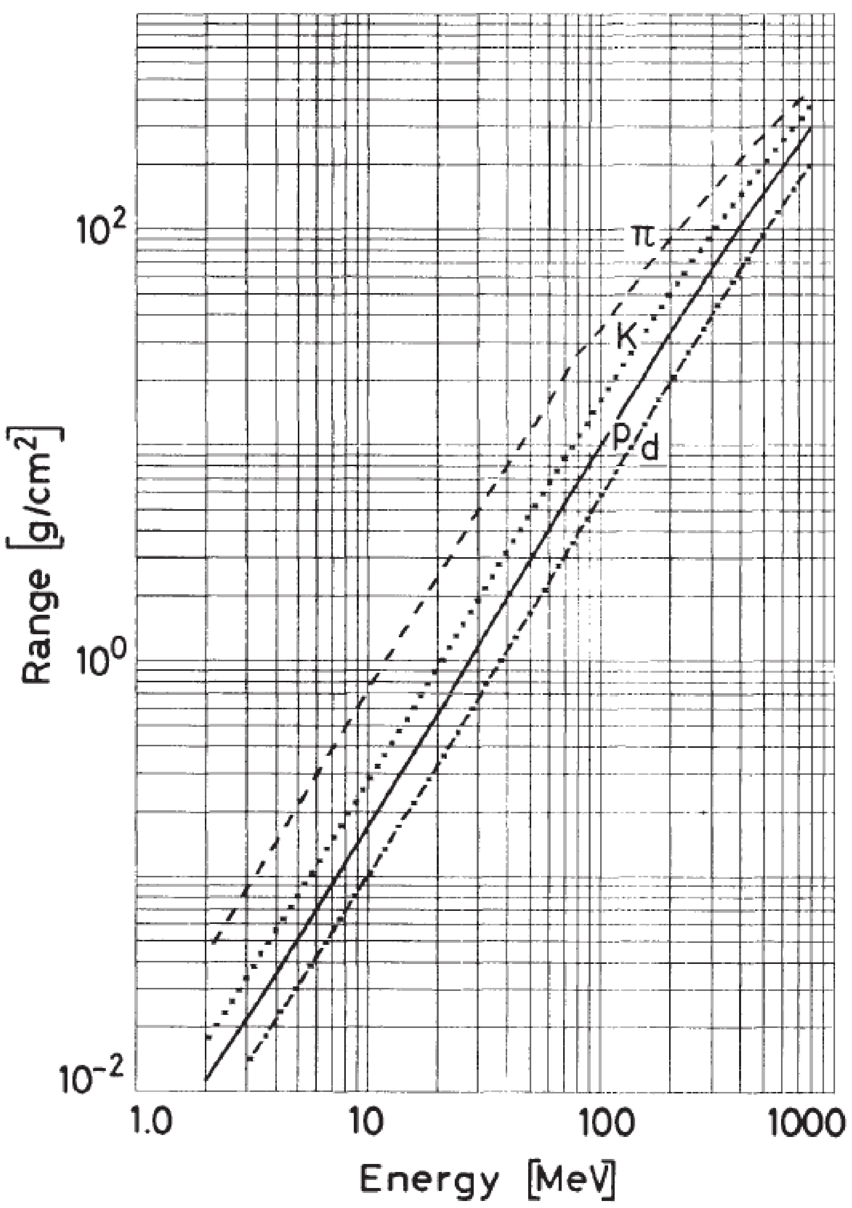


Example 5.2:

What is the range of a proton with T=100 MeV in Aluminum?

Ans:

$$R = \int_0^{100} \left(\frac{dE}{dx}\right)^{-1} dE \approx (10\ \mathrm{g/cm^2})\ /\ (2.7\ \mathrm{g/cm^3}) = 3.7\ \mathrm{cm}$$



Example 5.3

Write a ROOT Macro to compute average particle range by evaluating the integral:

$$R = \int_0^{T_0} \left(\frac{dE}{dx}\right)^{-1} dE$$

Inputs:

Particle info: T_0 and identity

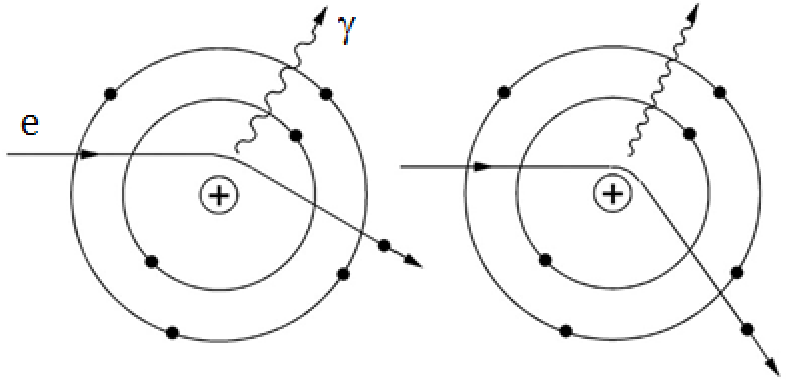
Material info: ρ, Z, A

Output:

Range R

Interaction of Electrons with Matter

- The Bethe-Bloch formula is only partially correct for electrons.
- Electrons are highly affected by the electric fields of charges in the medium (especially nuclei) and gain acceleration. As a result of this acceleration, they emit additional radiation (EM waves) called **bremsstrahlung** (braking radiation). Bremsstrahlung contributes significantly to energy loss, especially at high energies.



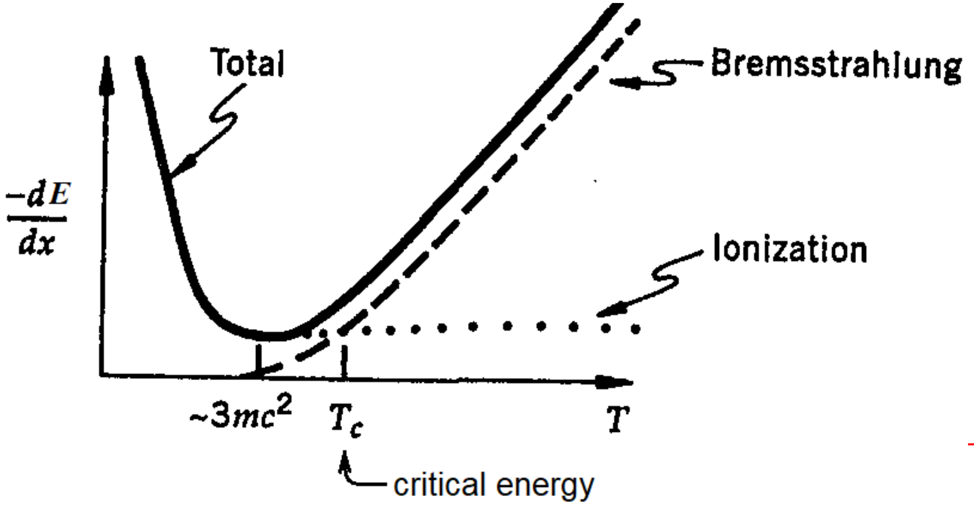
Total energy loss:

$$\left(\frac{dE}{dx}\right)_{tot} = \left(\frac{dE}{dx}\right)_{ion} + \left(\frac{dE}{dx}\right)_{brem}$$

$$\left(\frac{dE}{dx}\right)_{brem} = -\frac{E}{X_0} \qquad X_0 = \frac{716.4A}{Z(Z+1)\ln(287/\sqrt{Z})}.$$

X_0 = radiation length/density

value at which the electron energy drops to 1/e of its original value due to radiation.

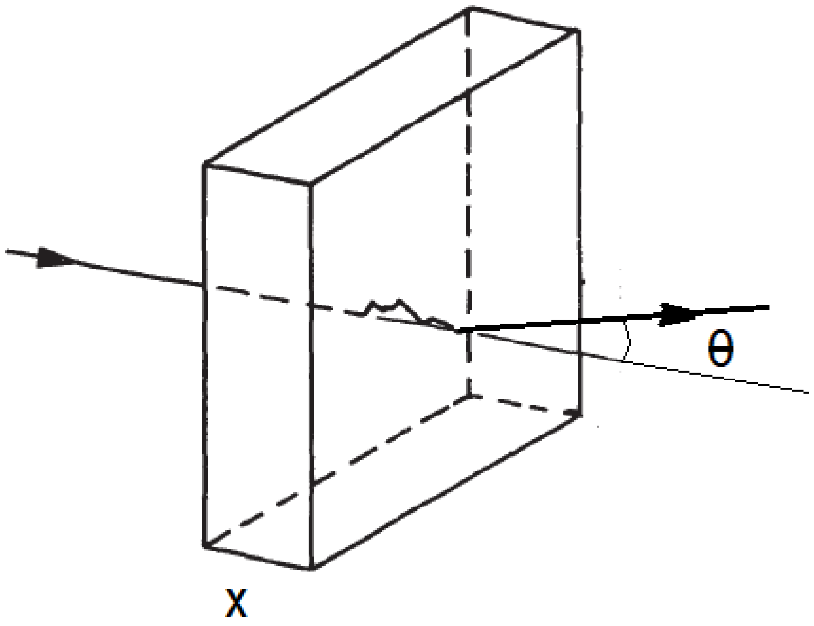


Multiple Coulomb Scattering

- Multiple scattering is a phenomenon in which particles interact with matter and are subjected to multiple collisions with atoms (or molecules)."

- RMS value of the scattering angle:

$$\theta_{RMS} \approx \frac{20\ \mathrm{MeV}}{\beta p c} z \sqrt{\frac{x}{X_0}}$$



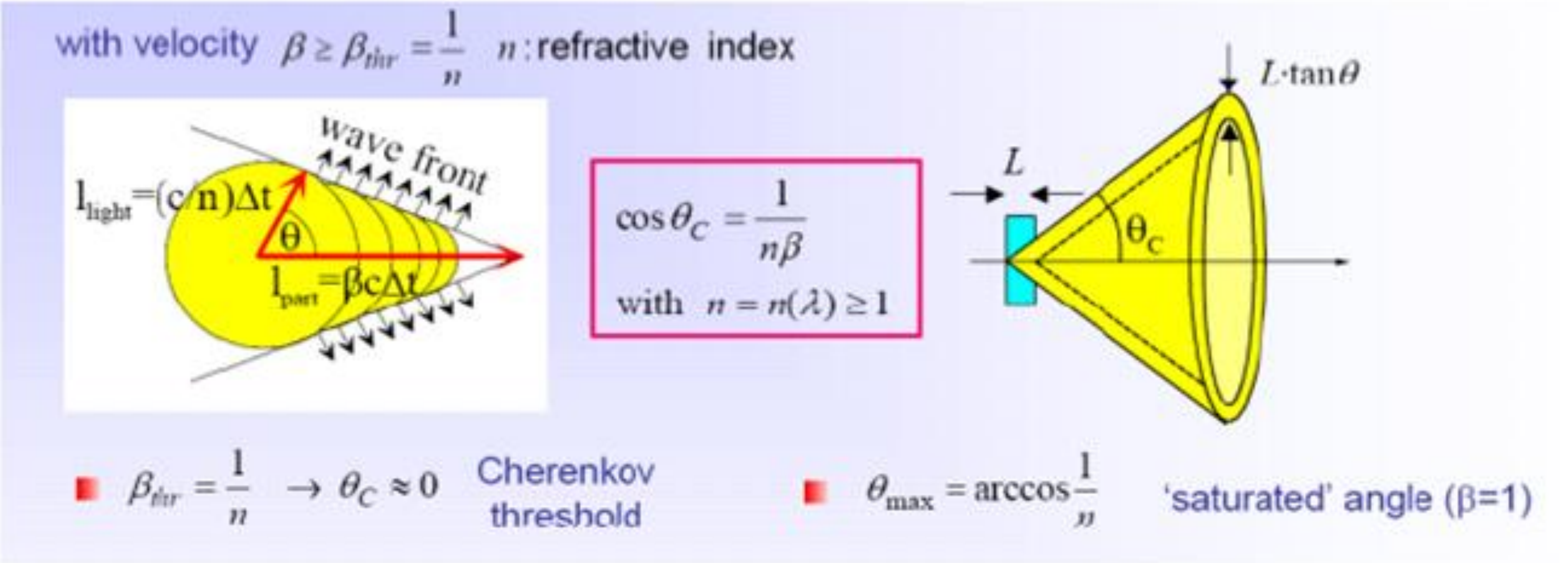
z = (charge) / e

$$X_0 = \frac{716.4A}{Z(Z+1)\ln(287/\sqrt{Z})}.$$

Cherenkov Radiation

- If the speed of a charged particle passing through an optical medium is greater than the speed of light in that medium (v>c/n, where n is the refractive index), the particle loses energy by emitting Cherenkov radiation. The radiation is emitted in a cone shape.
- Cherenkov cone angle:

$$\cos\theta_c = \frac{1}{n\beta}$$



- Average number of photons (N) generated can be calculated by:

$$\frac{d^2N}{dx d\lambda} = \frac{2\pi\alpha}{\lambda^2}\left(1 - \frac{1}{\beta^2 n^2}\right)$$

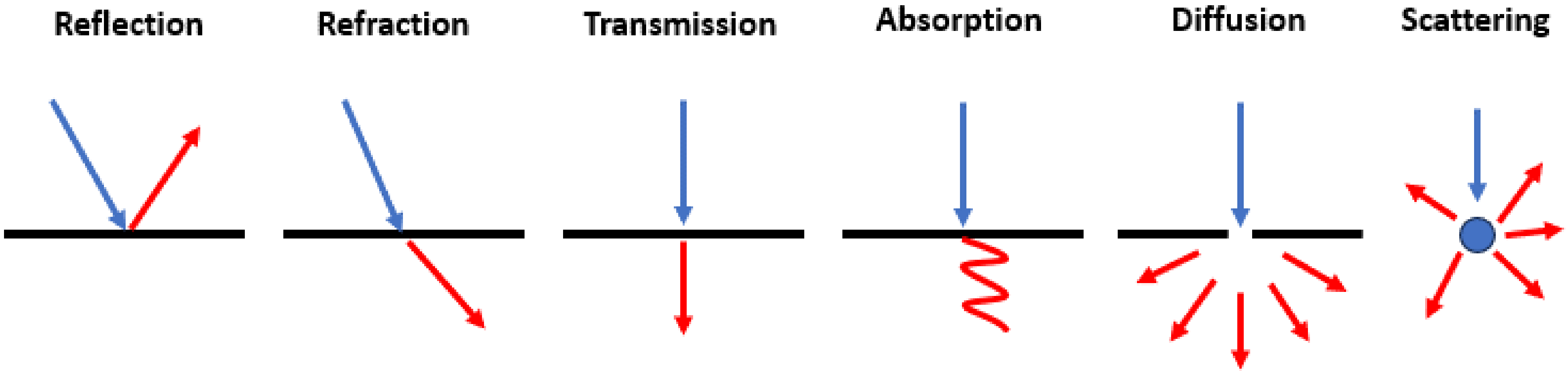
$\alpha = \frac{1}{137}$ and λ is the wavelength emitted.

Interaction of Photons with Matter

Photon-Matter Interactions

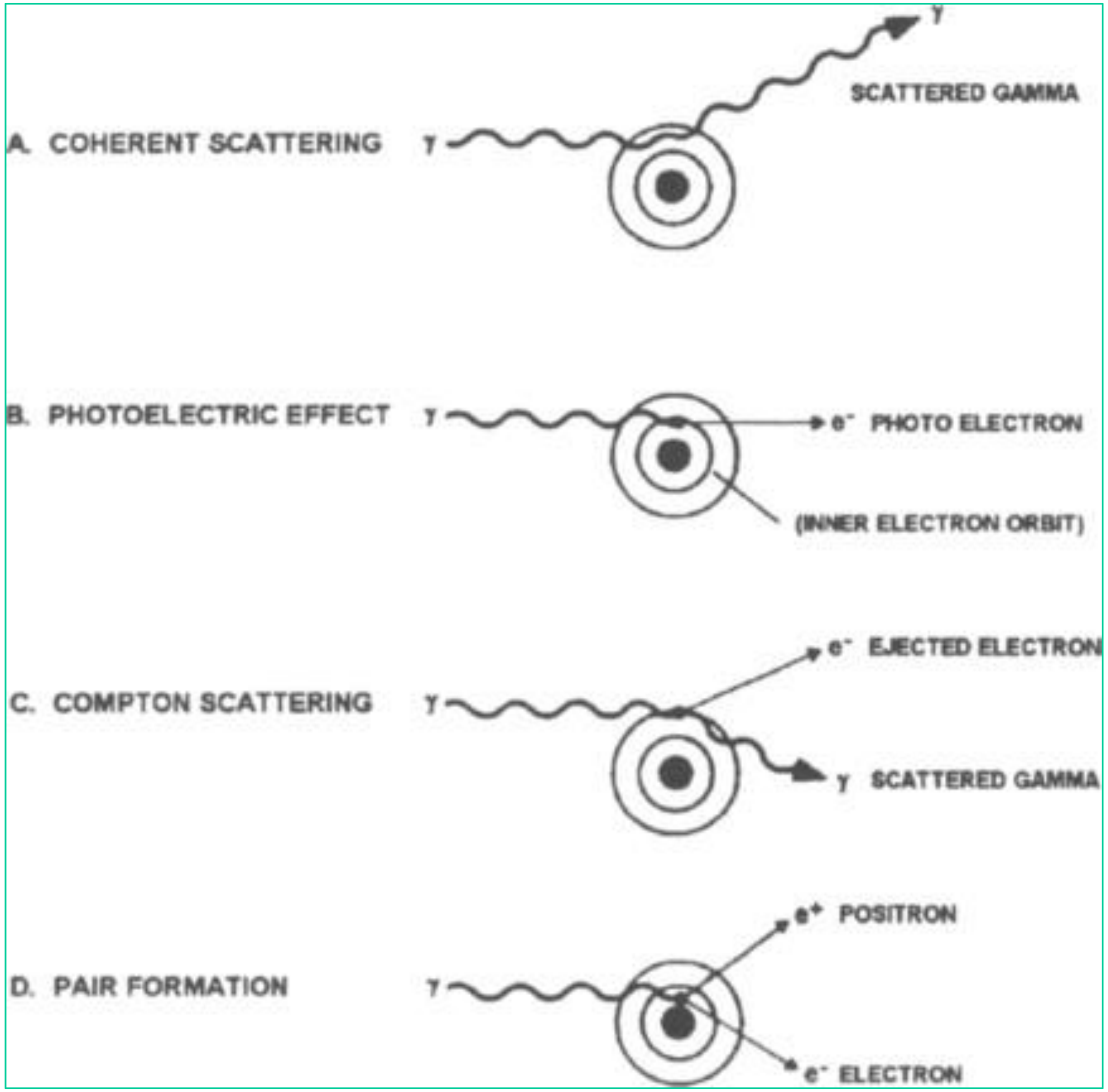
Since photons are uncharged, they are not subjected to the Coulomb force within matter. Because of this, one might get the impression that they cannot ionize the medium they travel through. However, since photons are the carrier particles of the EM (electromagnetic) field, they can engage in different interactions with matter.

Interaction of Optical Photos:



High energetic photons (X-rays ve γ-rays) can interact via

- Scattering
- Photoelectric Effect
- Compton Scattering
- Pair Production



Attenuation

Attenuation, can be described by absorption coefficient μ. ($\mu \propto \sigma$)

Initial intensity of photons: I_0

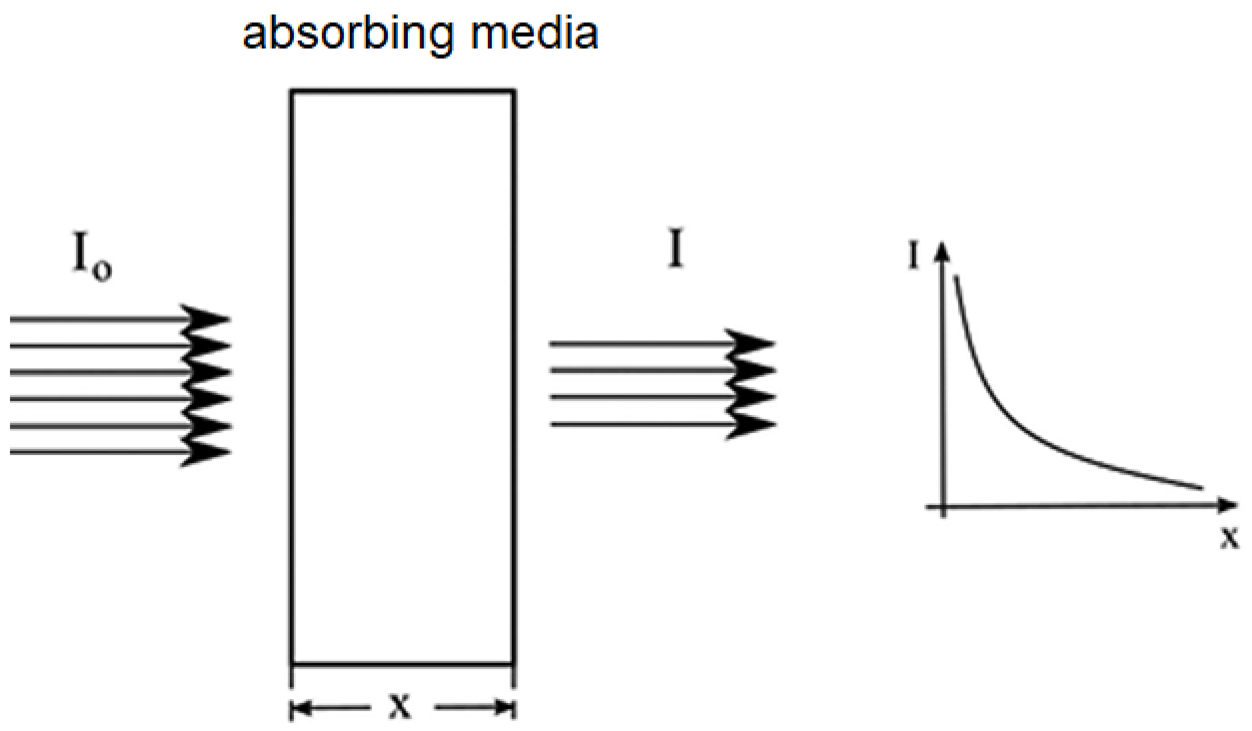
Final intensity: I

Thickness of material: x

$$I(x) = I_0 e^{-\mu x}$$

Radiation length: $X_0 = \frac{1}{\mu}$

Cross-section: $\sigma = \frac{\mu}{n} = \frac{A}{\rho N_A}\mu$



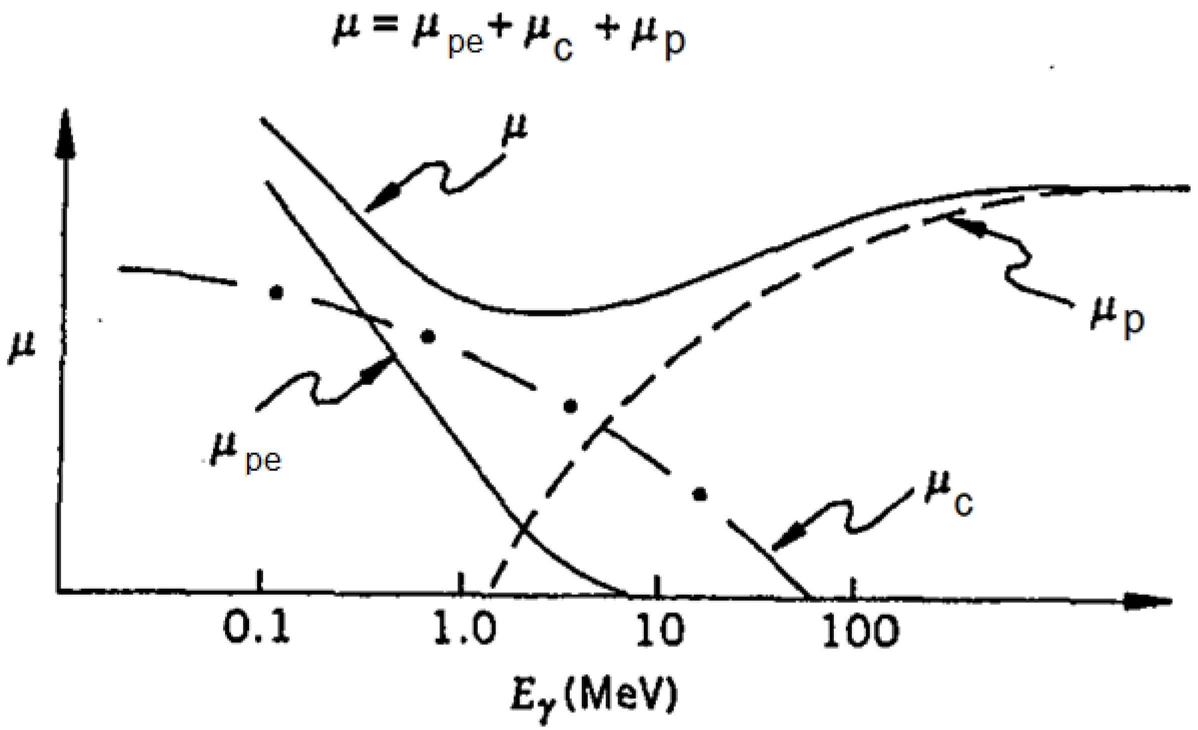
Total Absorption

Total absorption coefficient:

$$\mu = \mu_{\text{photoelectric}} + \mu_{\text{compton}} + \mu_{\text{pair}}$$

Total cross section:

$$\sigma = \frac{\mu}{n} = \frac{A}{\rho N_A}\mu$$

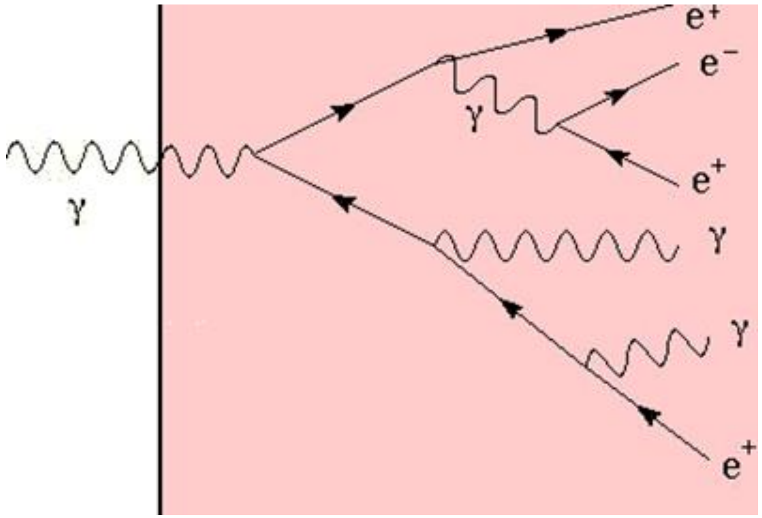


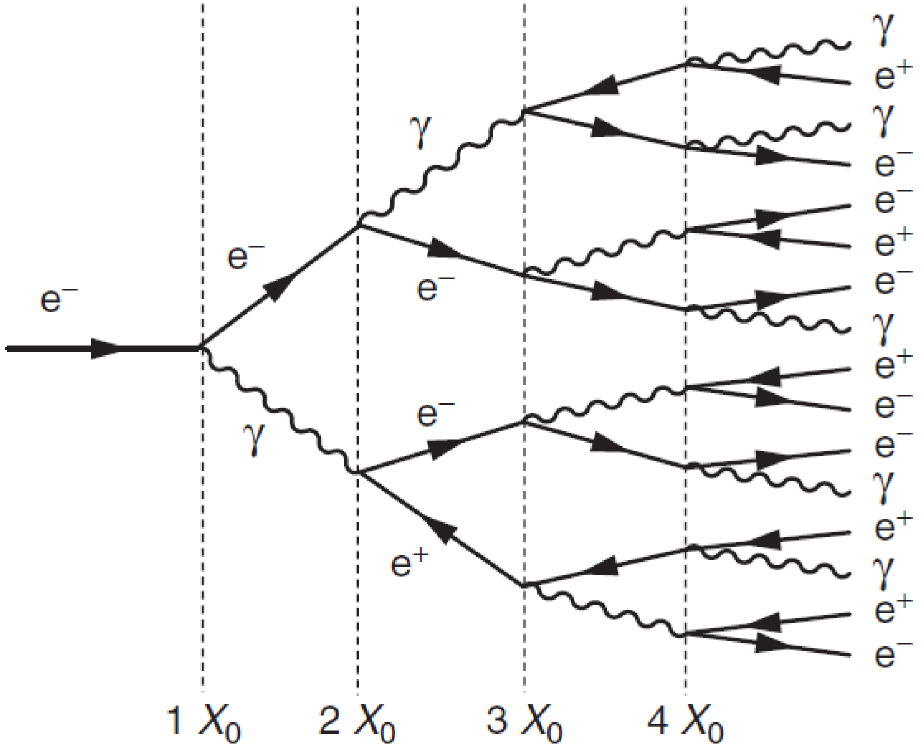
Electromagnetic and Hadronic Showers

EM Shower

For electrons and photons of high energy, a dramatic result of the combined phenomena of **bramsstrahlung** and **pair production** is the occurance of cascade showers.

A parent electron will radiate photons, which converts to pairs, which radiate and produce fresh pairs in turn, the number of particles increasing exponentially with depth in the medium.

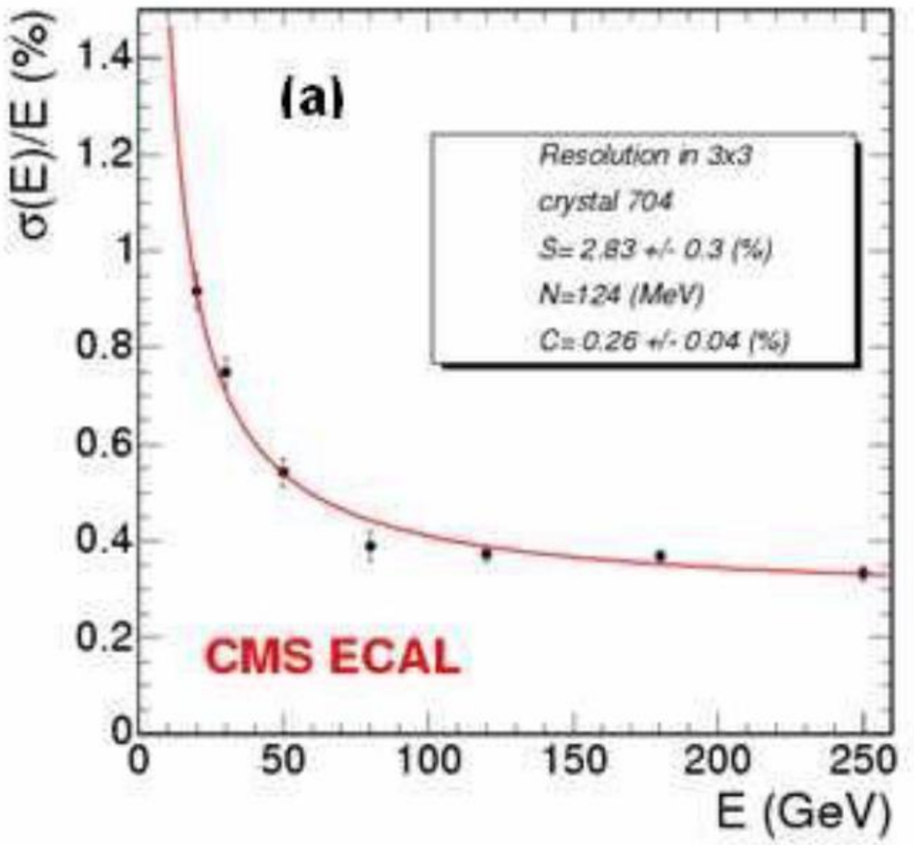




Number particles created is proportional to parent electron (or photon) energy, $N \propto E$.

The energy resolution of an electromagnetic calorimeter is modelled by:

$$\frac{\sigma_E}{E} = \frac{R}{\sqrt{E}} + k$$



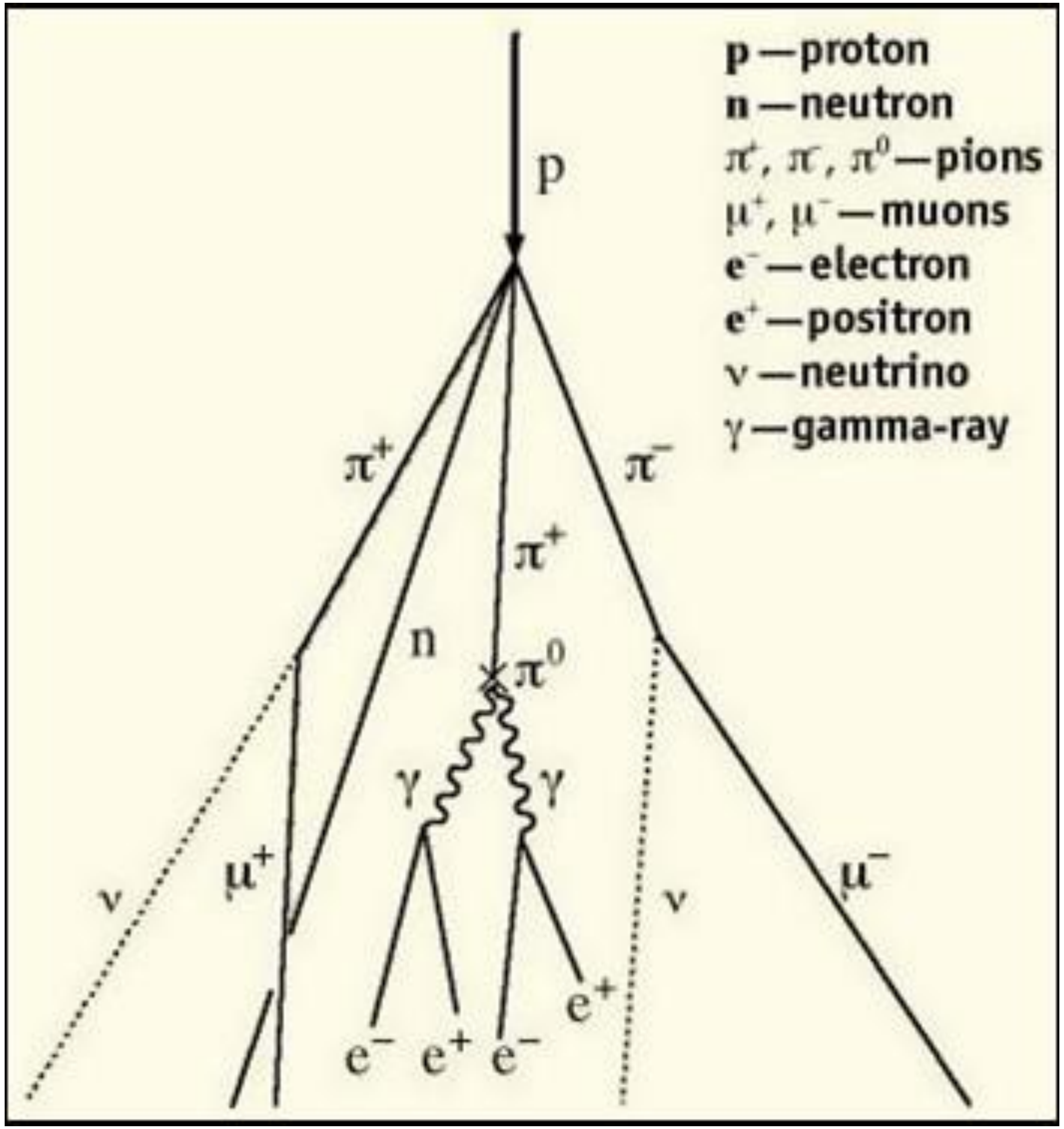
For ATLAS: R = 0.10 $GeV^{1/2}$.
For CMS: R = 0.05 $GeV^{1/2}$.
k is a small constant

Hadronic Shower

This is more complicated.

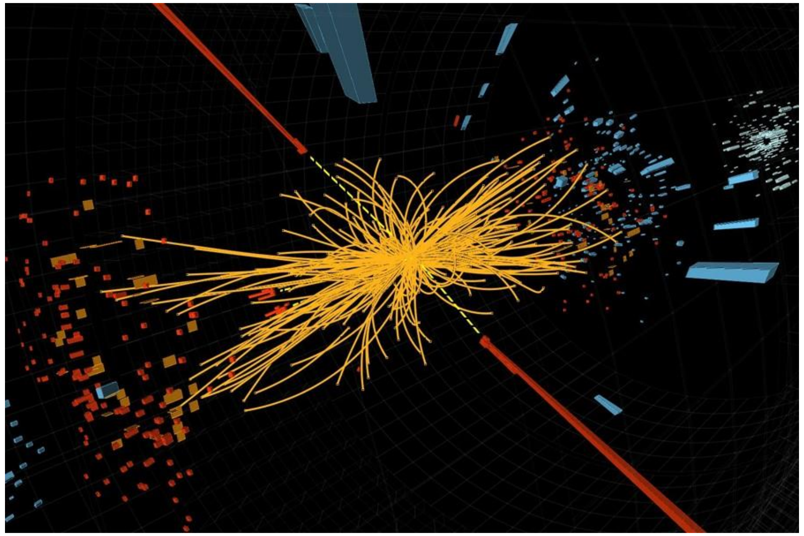
We can use same model.
Energy resolution of Hadronic Calorimeters is modelled as:

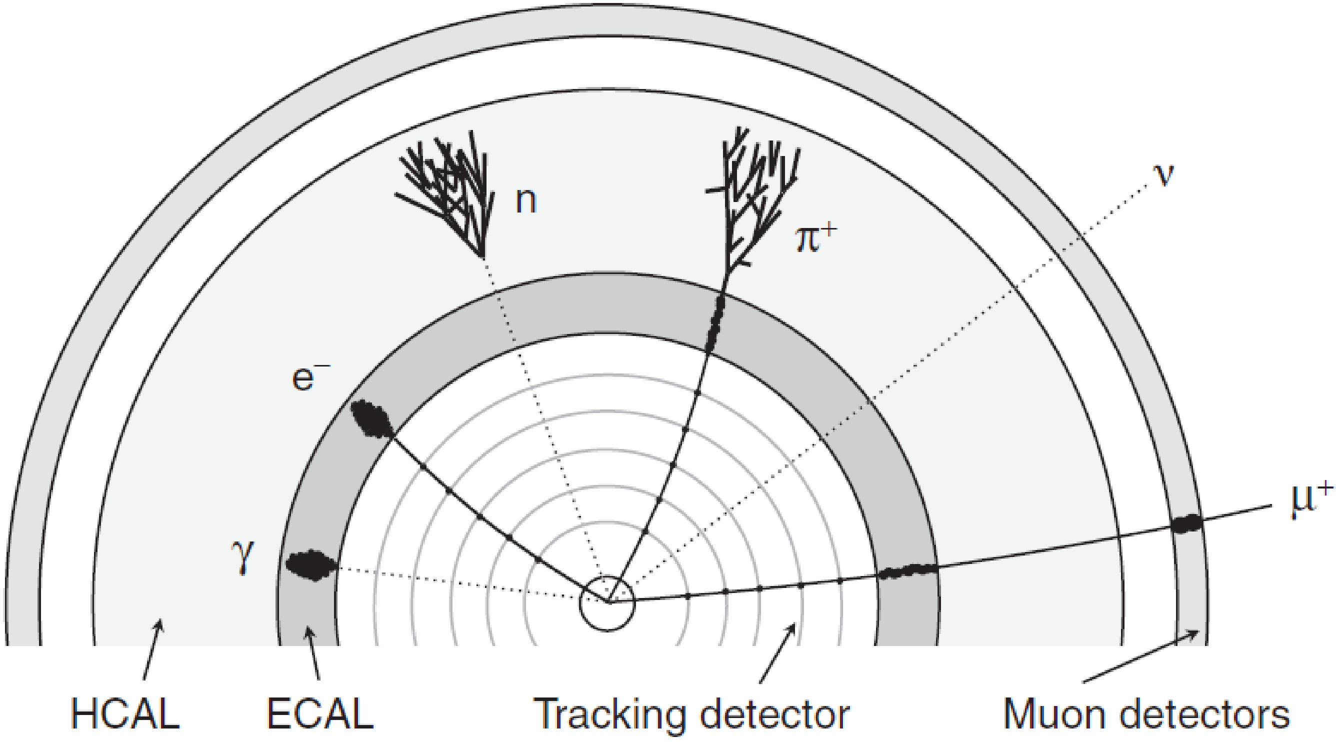
$$\frac{\sigma_E}{E} \geq \frac{0.50}{\sqrt{E}}$$



Collider Experiments

Different particle types interact differently with matter
(e.g. photons do not feel a magnetic field)
We need different types of detectectors
to measure different types of particles.





Problems

1. Protons and alpha particles of 20 MeV pass through 0.001 cm aluminum foil. How much do such particles deposit energy within the foil? Use Bethe-Block formula.

   Ans: $E_p^{dep} \approx 0.055\ \mathrm{MeV}$ and $E_\alpha^{dep} \approx 0.65\ \mathrm{MeV}$

2. Compare stopping power of p, e, μ, π, K particles in copper for particle momentum of 1 GeV.

3. What should be the thickness of a copper block used to reduce the energy of a proton from 600 MeV to 400 MeV?

4. What should be the thickness of a lead block used to stop a muon with K.E. of 2 GeV?

5. Estimate the scattering angle of a proton with a momentum of 100 MeV/c after traveling 1 cm in Argon gas. (For Argon gas, X_0=105 cm)

6. At high energies, the radiation length of lead is X_0 = 5.6 mm.
   Calculate the absorption coefficient and the e^+e^- pair production cross-section.

Assume that for pair-production: $\mu \approx \frac{7}{9X_0}$

7. Calculate number of generated photons/cm for the visible light (400-700 nm) in water (n = 1.33) for charged particle of veloctiy beta ~ 1.

8. Compare Cherenkov angles of the pions and kaons of momentum $p = 1$ GeV/c in glass ($n = 1.5$).

9. Write a ROOT Macro to draw Bragg Curve (without straggling) for protons in Aluminum. Input is the K.E. of the proton.

References

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