



Particle Data Analysis in High Energy Physics

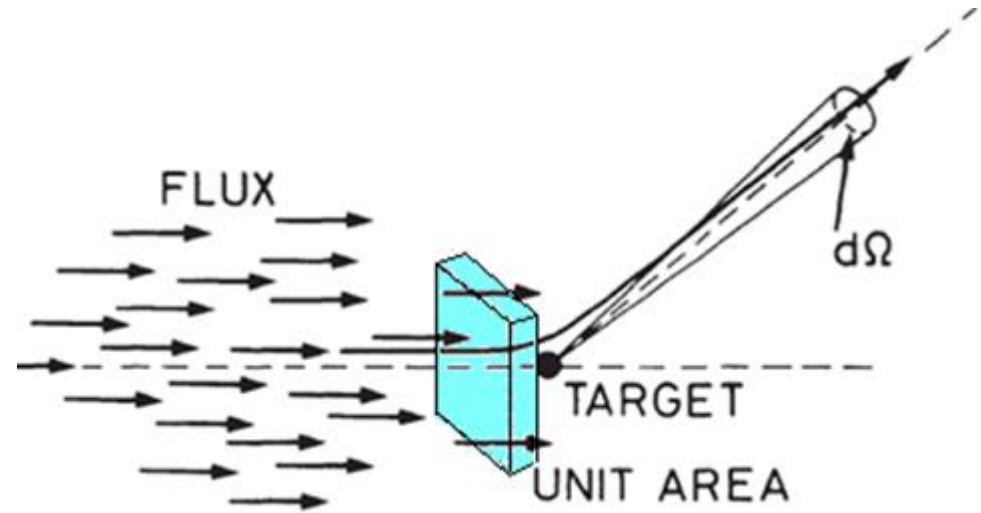
Lecture 4

Experimental Concepts

Ahmet Bingül

METU, Physics

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Introduction

In this chapter, some concepts used in experiments will be discussed.

We will discuss the following concepts:

Elementary Interactions

Cross Section

Mean Free Path

Luminosity and Reaction Rate

Decay Rate and Lifetime

Particle Width

Resonance Production

Measuring the Cross-Section

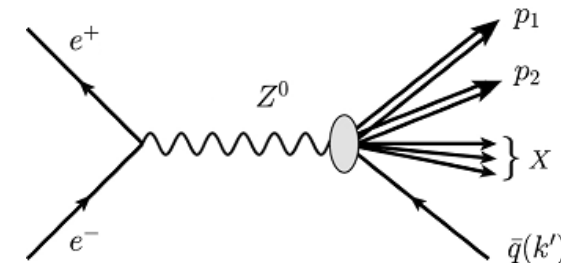
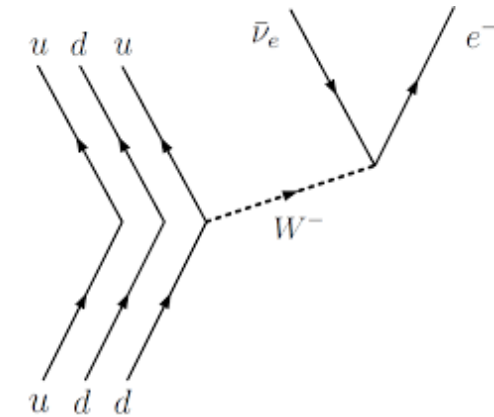
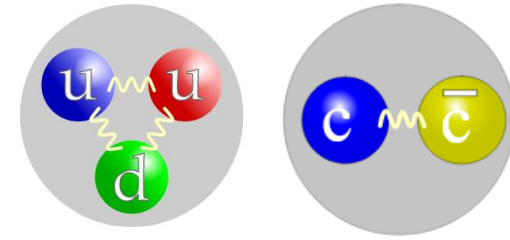
Elementary Interactions

There are three experimental probes of elementary particle interactions:

- Bound States $\pi^+(\bar{u}d)$, $p(uud)$, $J/\psi(c\bar{c})$

- Decays $\pi^+ \rightarrow \mu^+ + \nu_\mu$, $\pi^0 \rightarrow \gamma + \gamma$, $n \rightarrow p + e^- + \bar{\nu}_e$

- Scattering $p + p \rightarrow p + p + p + \bar{p}$, $e^- + e^+ \rightarrow Z^0 \rightarrow \text{hadrons}$



Nonrelativistic quantum mechanics (in Schrodinger's formulation) is particularly well adapted to handle bound states By contrast, the relativistic theory (in Feynman's formulation) is especially well suited to describe decays and scattering.

Cross Section (σ)

The collision or interaction of two particles is generally described in terms of the **cross section**. This is one of the important quantity that experimentalist measure and the theorist calculate. This quantity essentially gives a measure of the probability for a reaction to occur.

There are three categories:

- Interaction Cross-section
- Scattering Cross-section
- Production Cross-section

Interaction Cross Section

Consider that we fire beam of particles at definite energy to a target.

Beam refers to a large number of particles moving in the same direction.

Interaction probability of beam with target can be defined as:

$$\text{interaction probability} \approx \frac{\# \text{ interacting particles}}{\# \text{ of incoming particle}} \approx \frac{\text{total cross sectional area}}{\text{total geometric area}}$$

$$p = \frac{N' - N}{N} = \frac{n(S\Delta x)\sigma}{S} \Rightarrow -\frac{\Delta N}{N} = n\sigma\Delta x$$

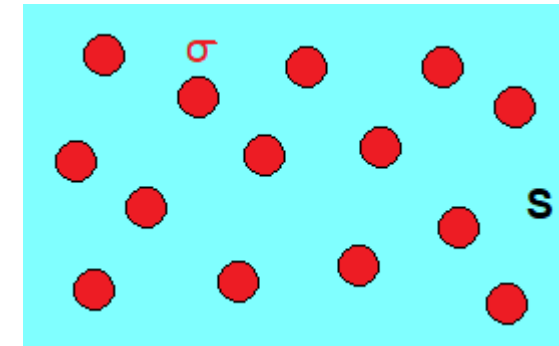
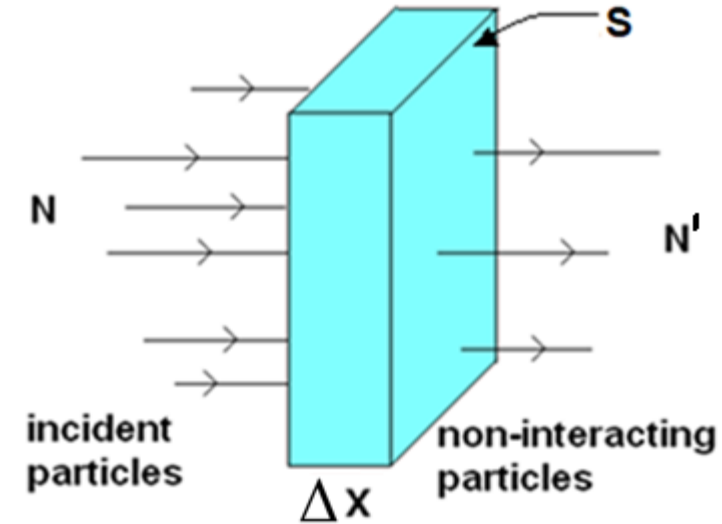
Hence, number of surviving particles at any distance x:

$$N(x) = N_0 \exp(-n\sigma x)$$

N_0 : Initial number of particles at $x = 0$

$n = \rho N_A / A$: number of atoms per unit volume

$\sigma = \sigma(E, Z, A)$: cross section (has a unit barn and $1 \text{ b} = 10^{-28} \text{ m}^2$)



Red circles represents atoms with cross sectional area σ . A random particle is fired into the rectangle. What is the probability of hitting a red circle?

Mean Free Path

Mean free path (λ) is the average distance a particle travels before interacting with the surrounding medium. For a single element, it is defined by:

$$\lambda = \frac{\int xN(x)dx}{\int N(x)dx} = \frac{1}{n\sigma} = \frac{A}{N_A\rho\sigma}$$

Interaction probability at distance x : $P_{\text{int}}(x) = 1 - \exp(-x/\lambda)$

Proof:

probability of not having interaction after a distance x : $P(x)$

probability of having interaction in dx : $n\sigma dx = dx/\lambda$

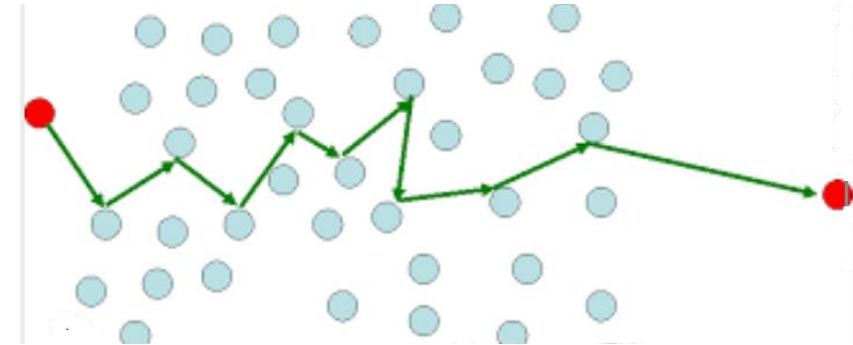
probability of having not interaction between x and $x+dx$

$$P(x + dx) = P(x)\left(1 - \frac{dx}{\lambda}\right)$$

...

$$P(x) = \exp(-x/\lambda)$$

$$P_{\text{int}}(x) = 1 - \exp(-x/\lambda)$$



Example 4.1

Total cross-section for a neutrino interacting with iron is about $\sigma = 10^{-47} \text{ m}^2$.

Calculate the mean free path of neutrinos in an iron block. ($M_{\text{Fe}} = 55.9 \text{ g/mole}$, $d_{\text{Fe}} = 7.8 \text{ g/cm}^3$)

[Ans $\lambda = 1.2 \times 10^{18} \text{ m} = 130 \text{ ly}$]

Scattering Differential Cross Section

Most of our knowledge of particle physics has been obtained from scattering experiments.

Let F be incoming particle flux (luminosity) in $\text{m}^{-2} \cdot \text{s}^{-1}$.

Differential cross section for scattering can be defined as

$$\frac{d\sigma}{d\Omega}(E, \Omega) = \frac{1}{F} \frac{dN}{d\Omega} \equiv \frac{\text{number of scattered particles}}{\text{sr} \cdot \text{m}^2 \cdot \text{s}}$$

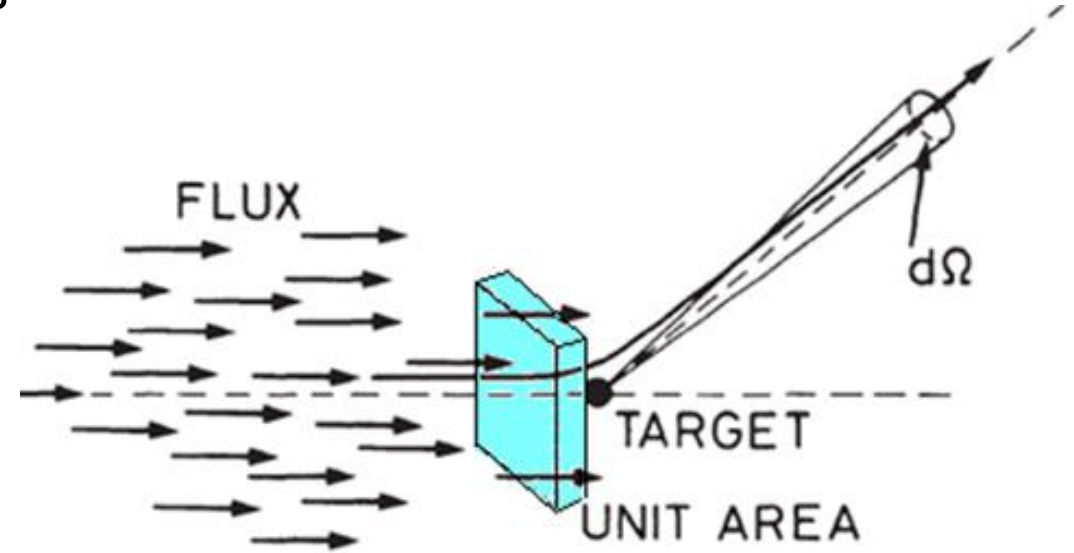
Solid angle: $d\Omega = \sin\theta d\theta d\phi \rightarrow \Omega = \int \int \sin\theta d\theta d\phi$

Total scattering cross section (depends on energy):

$$\sigma(E) = \int \frac{d\sigma}{d\Omega} d\Omega$$

Note that by scattering here, we mean any reaction in which an outgoing particle is emitted into Ω .

The incident particle need not retain its identity!



Production Cross Section

In a typical scattering experiment, a mono-energetic beam of particles (such as pions) are directed on to a target (such as liquid Hydrogen) and production rates of various particles are measured.

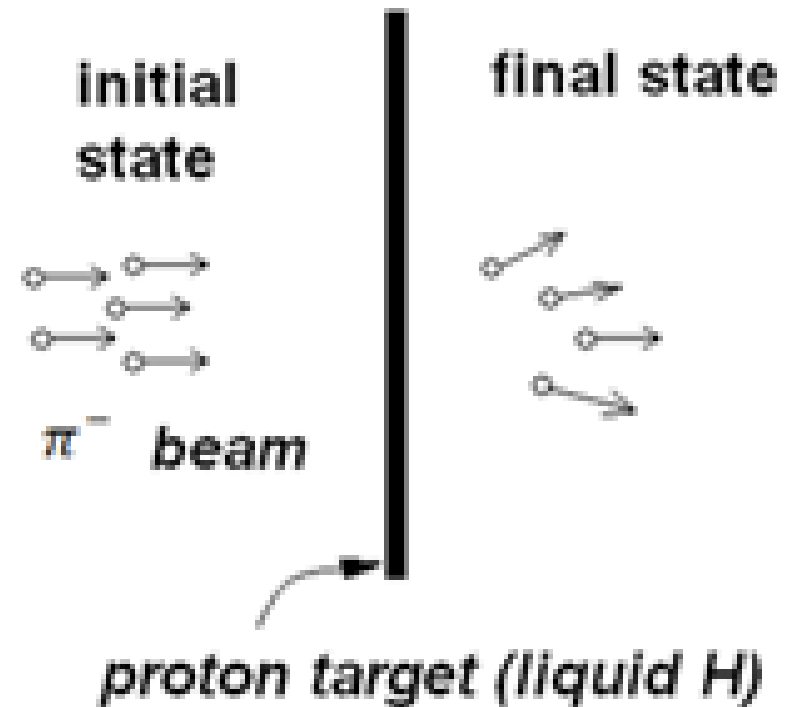
For each initial state a number of final state is possible:

For example:

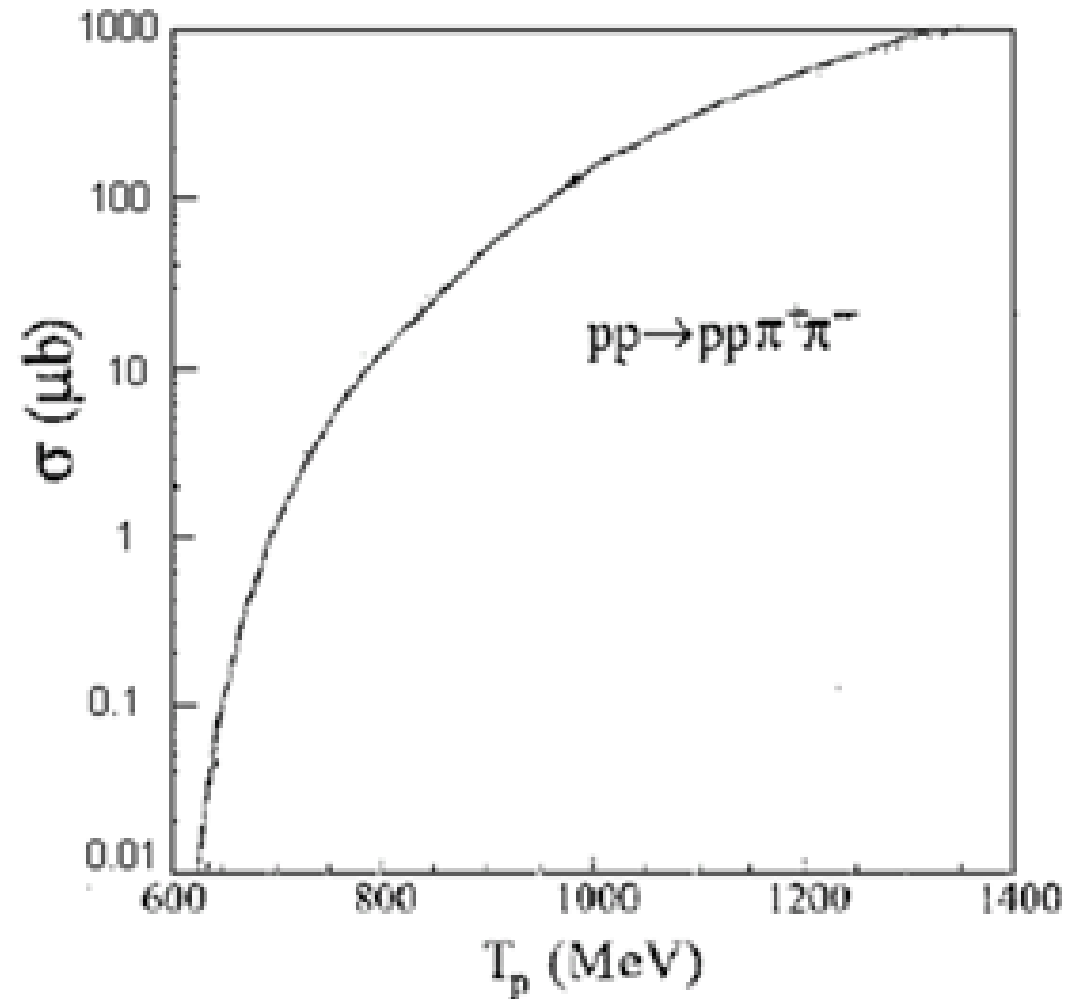
- $\pi^- + p \rightarrow \pi^- + p$
- $\pi^- + p \rightarrow K^0 + \Lambda$
- $\pi^- + p \rightarrow \pi^- + \pi^+ + n$
- $\pi^- + p \rightarrow \pi^- + \pi^+ + \pi^0 + n$

The production rate will be proportional to:

- Number of particles in the target
- Number of particles in beam (per unit area per unit time)



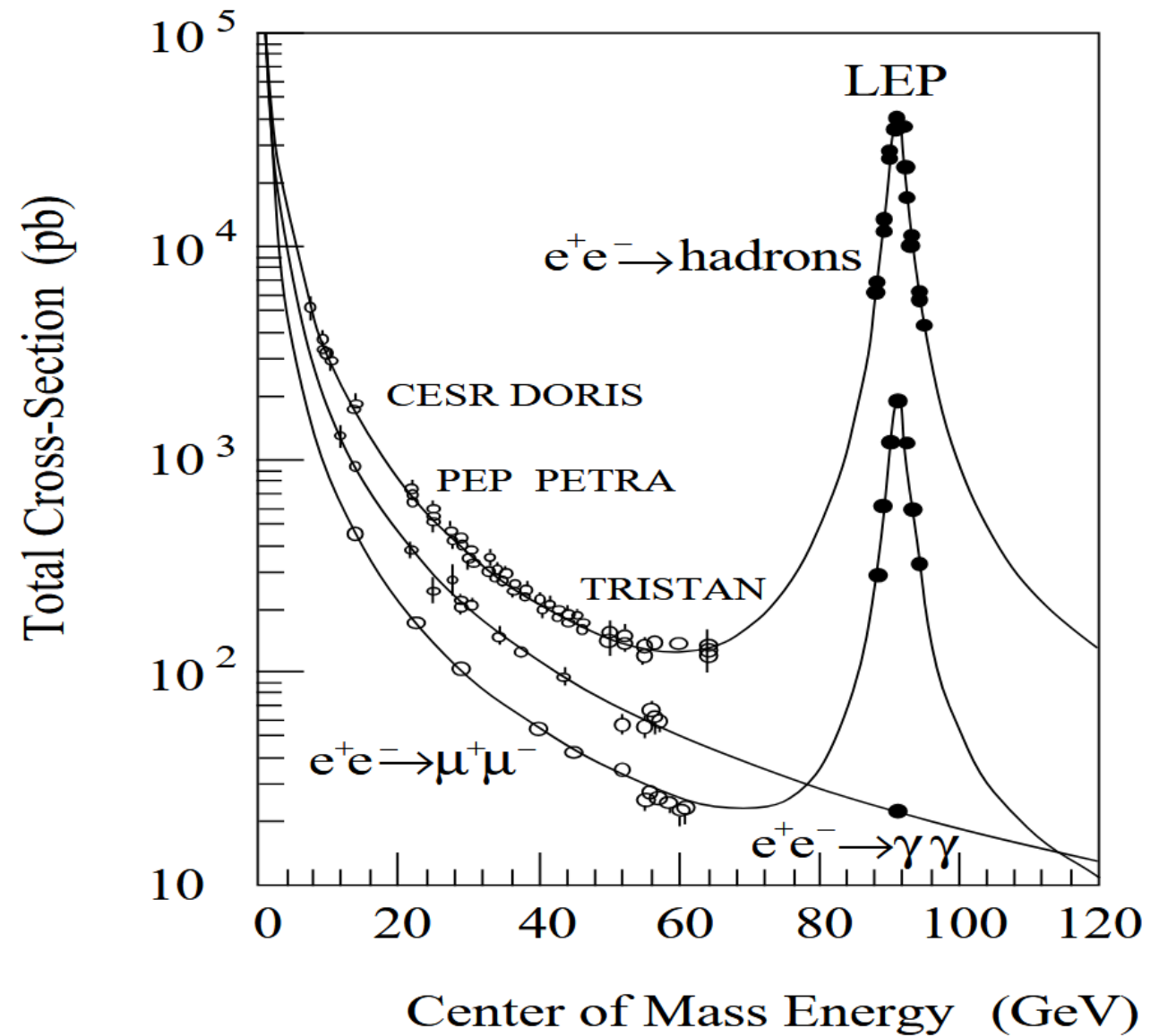
The cross section of a reaction can be a function of the incident energy. Figure below shows the production cross section as a function of energy for the reaction $p + p \rightarrow p + p + \pi^+ + \pi^-$.



Production of Z Boson at LEP:

$$e^- + e^+ \rightarrow Z \rightarrow q\bar{q} \rightarrow \text{hadrons.}$$

$$e^- + e^+ \rightarrow \mu^+ + \mu^- .$$



Luminosity and Reaction Rate

Luminosity (L) is defined as the number of beam particles per unit area per unit time.
The production rate (R) of an event is the number of events observed (N) per unit time.

Definition:

$$\frac{dN}{dt} = R = \sigma L$$

Total number of particles produced can be found by integration:

$$N = \sigma \int L dt = \sigma \mathcal{L}$$

where \mathcal{L} is known as the **integrated luminosity** and has unit of 1/barn (usually nb^{-1} , pb^{-1} is used)

Luminosity for Colliding Beams

Consider two beams of particles consisting of n -bunches with N_1 and N_2 particles in each. The bunches traverse a circular path and collide head on.

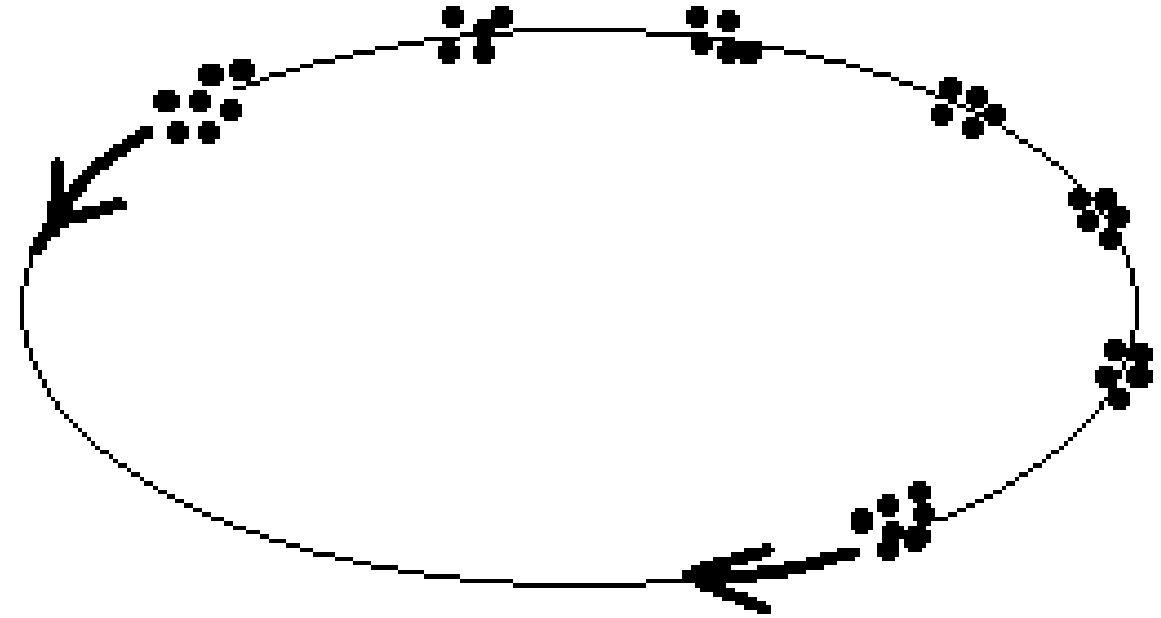
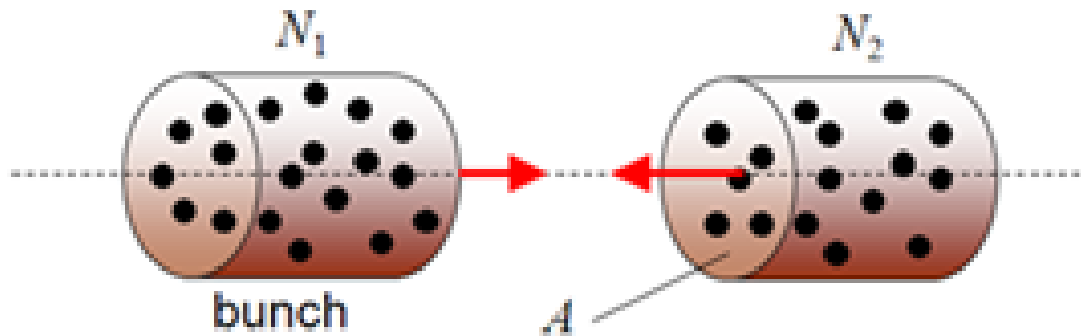
In this case luminosity is given by:

$$L = nf \frac{N_1 N_2}{A}$$

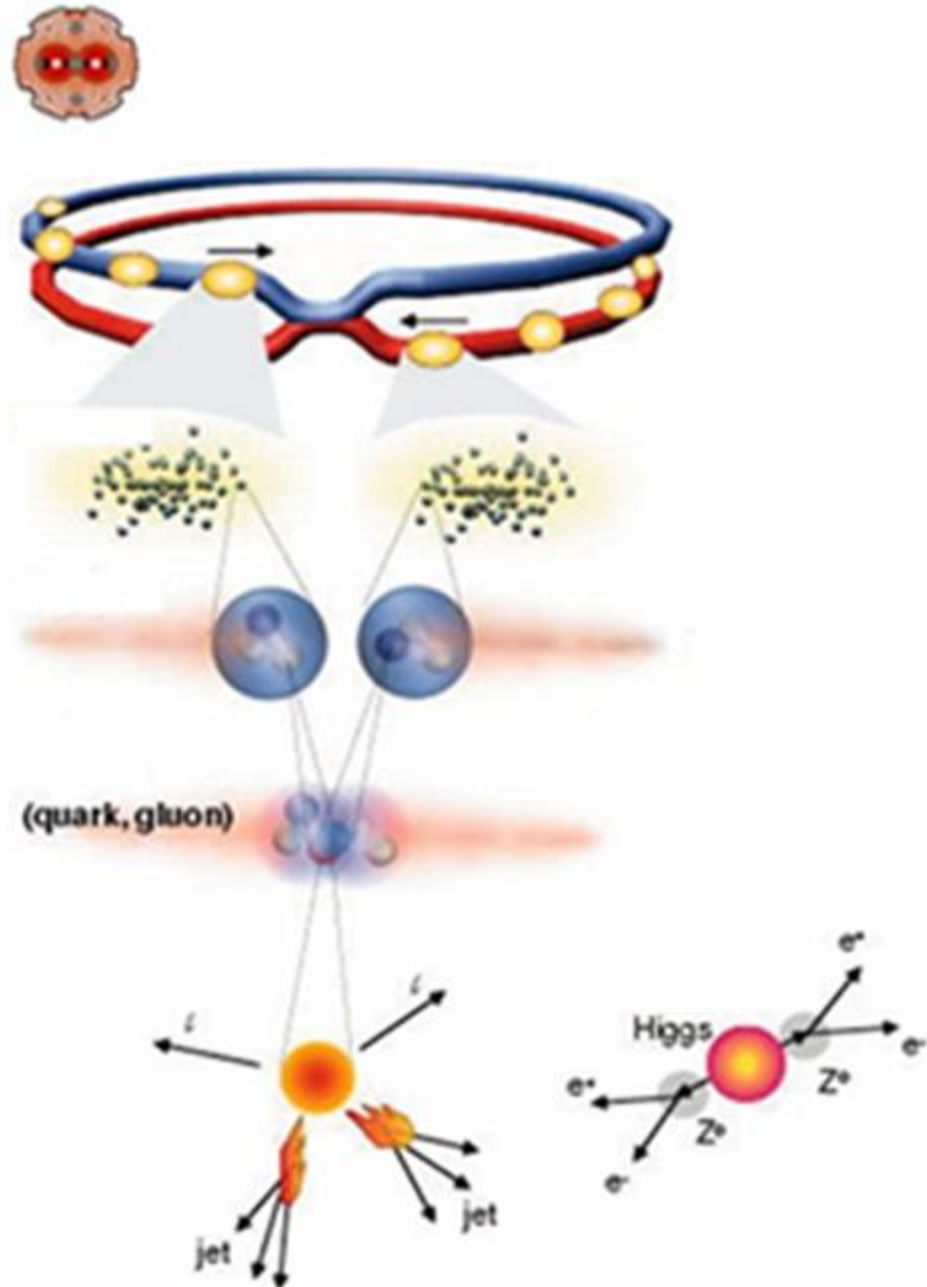
where

f = collision frequency

A = cross-sectional area of beams



LHC



proton-proton

circumference: 27 km

bunches: 3564 + 3564

protons / bunch: 10^{11}

beam energy: 7 TeV

luminosity: 10^{33} - 10^{34} $\text{cm}^{-2}\text{s}^{-1}$

bunch spacing: 25 ns

collision rate: 10^8 - 10^9 Hz

dipole field: 8.4 T

number of dipoles: ~ 1200

heavy ions (Pb-Pb)

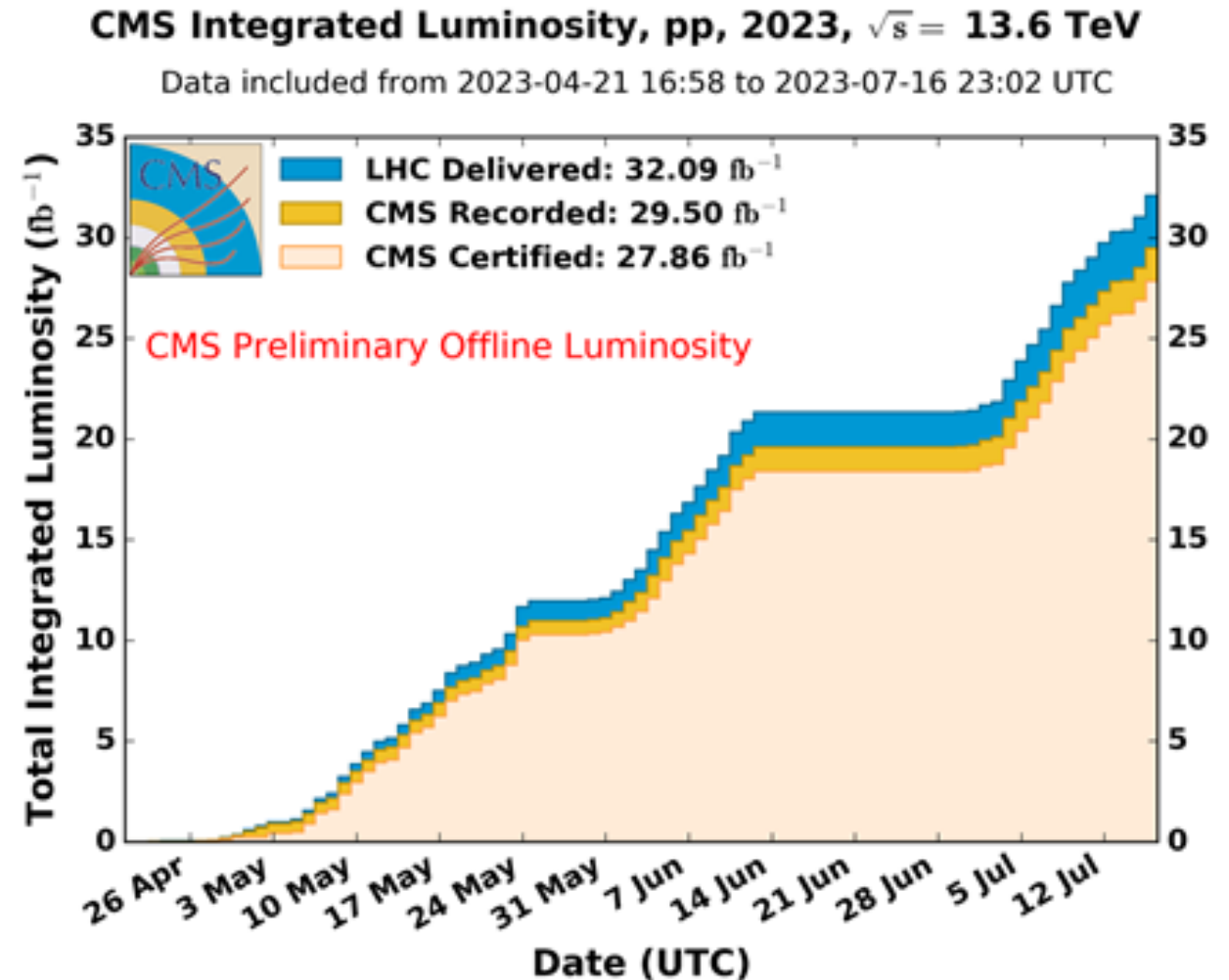
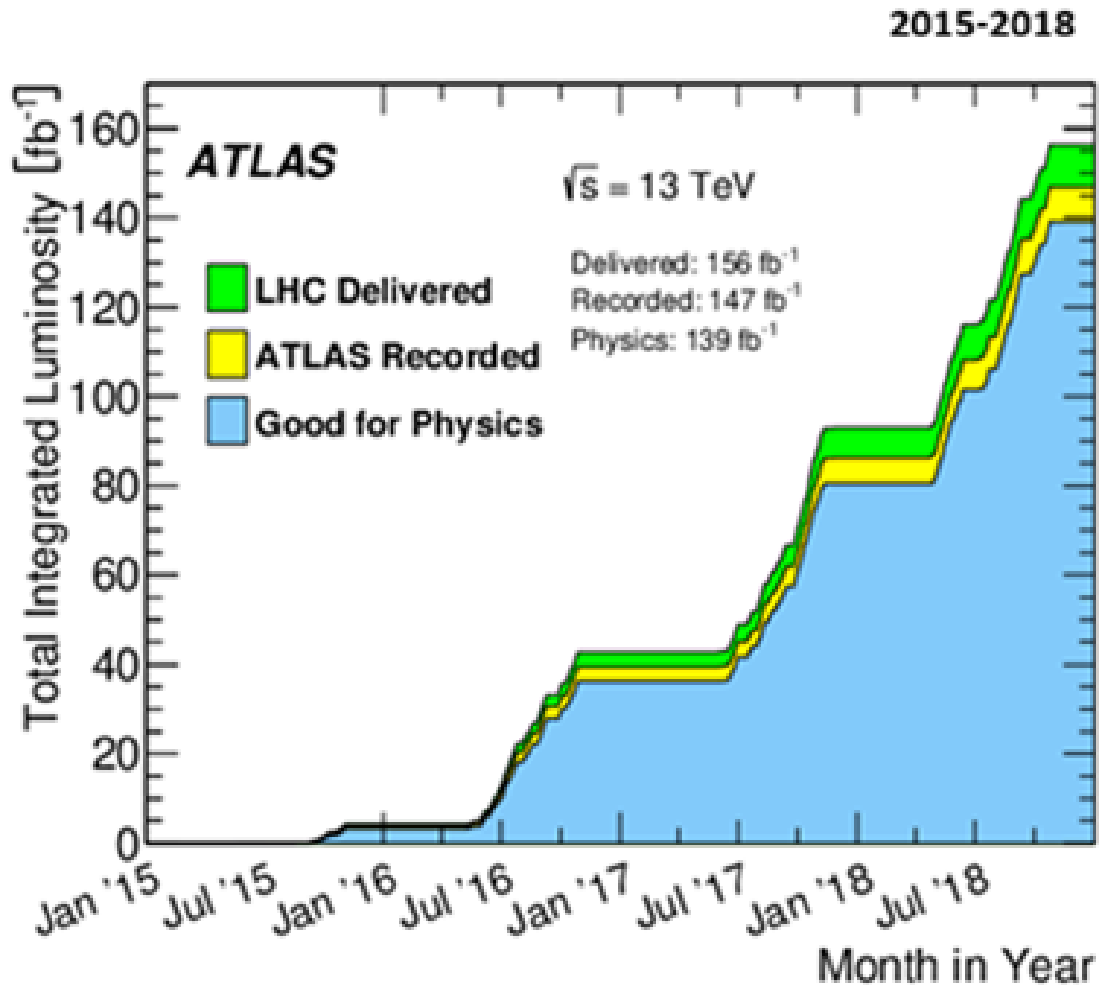
beam energy:

2.8 (5.5) TeV / nucleon pair

luminosity: 10^{27} $\text{cm}^{-2}\text{s}^{-1}$

Cumulative Integrated Luminosity

Cumulative integrated luminosity delivered to and recorded by ATLAS and CMS during stable beam pp collision data-taking at 13 TeV.



Decay Rate and Lifetime

Consider a particle (say muon) is produced at a given space-time.

Lifetime of the muon is the time elapsed before it disintegrates.

We define the decay rate, Γ , the probability per unit time that any given particle will decay.

Imagine we have a large collection of muons say $N(t)$, at time t , at rest.

Then on average $N \Gamma dt$ of them will decay in the next instant dt .

Hence the number remaining muons are $dN = -N \Gamma dt$.

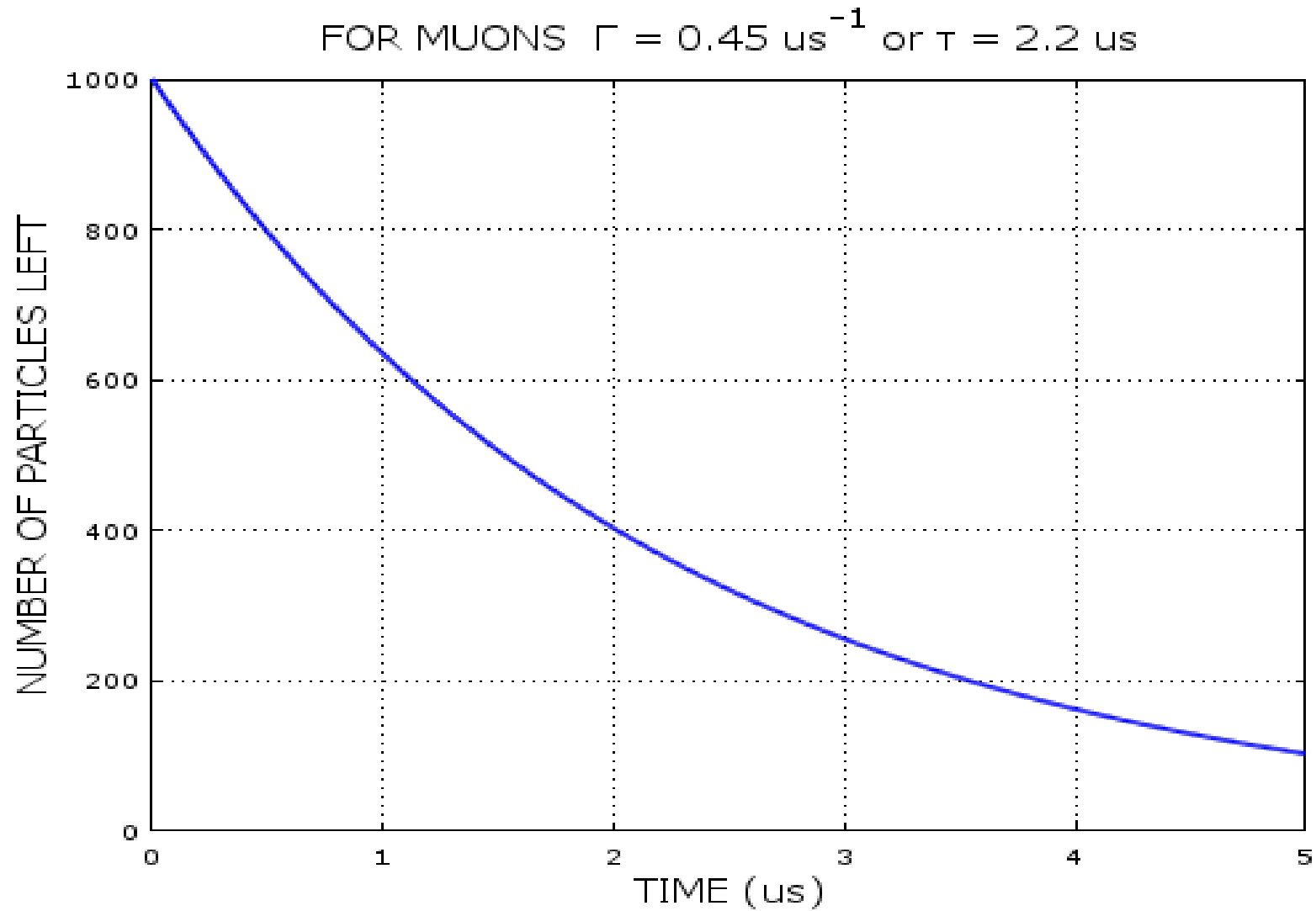
Solving for N yields: $N(t) = N_0 \exp(-\Gamma t) = N_0 \exp(-t/\tau)$

Here N_0 is the number of muons at $t = 0$ and τ is called the mean lifetime.

The mean lifetime of muons can be found from:

$$\tau = \frac{\int_0^{\infty} t N dt}{\int_0^{\infty} N dt} = \frac{\int_0^{\infty} t N_0 e^{-\Gamma t} dt}{\int_0^{\infty} N_0 e^{-\Gamma t} dt} = \frac{1}{\Gamma}$$

Number of muons left decrease exponentially with time.



Most particles can decay by several different ways.

For example, D^{*+} (2010) meson decays to three different channels as follows:

Mode	Channel	Fraction Γ_i / Γ
Γ_1	$D^{*+} \rightarrow D^0 \pi^+$	0.6770 ± 0.005
Γ_2	$D^{*+} \rightarrow D^+ \pi^0$	0.3070 ± 0.005
Γ_3	$D^{*+} \rightarrow D^+ \gamma$	0.0160 ± 0.004

The total decay rate is the sum of the individual decay rates:

$$\Gamma = \sum \Gamma_i$$

and lifetime

$$\tau = \sum \tau_i = \frac{1}{\Gamma}$$

Hence, the **Branching Ratio** can be defined as:

$$\mathcal{B}_i = \Gamma_i / \Gamma$$

Example 4.2: Measuring the Lifetime of Muons

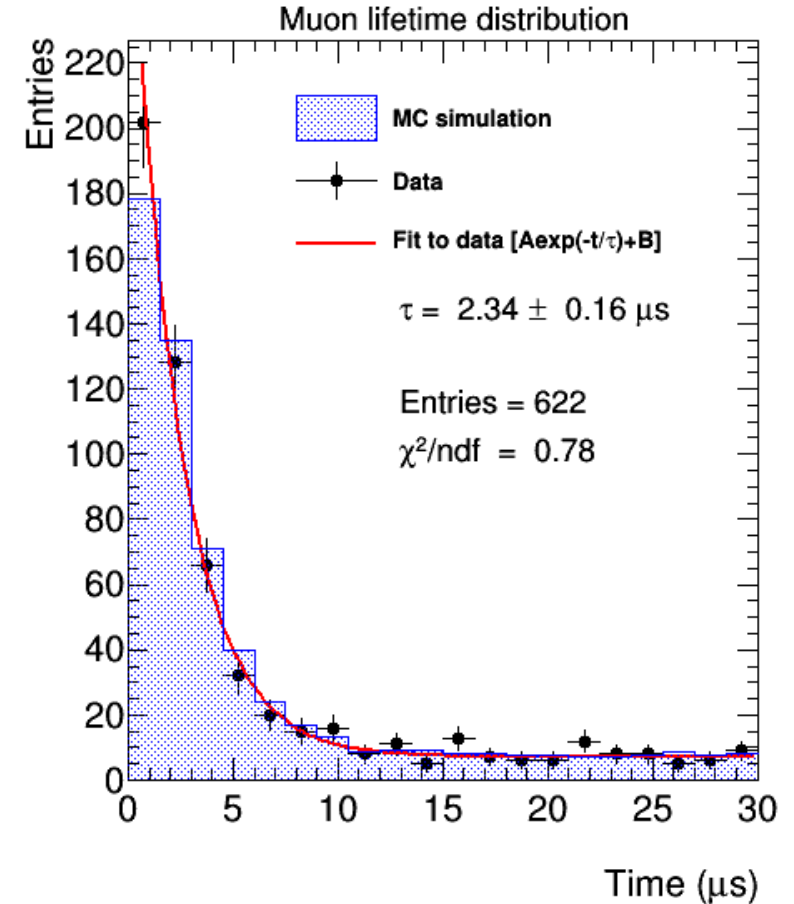
See Run 2 data at:

<http://www1.gantep.edu.tr/~bingul/muon>

We have one column of decay time data in microsec.

We have recorded

- signal events (exponential function) and
- background events (uniformly distributed)



A short-lived particle decays in mean time τ in own rest frame (CM frame).

Suppose now it is moving in lab system with a speed v in $+x$ -direction.

Where and when does the decay take place in lab? \rightarrow We need to use Lorentz Transformation.

$$\begin{pmatrix} c\tau_{lab} \\ x_{lab} \\ y_{lab} \\ z_{lab} \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c\tau \\ 0 \\ 0 \\ z \end{pmatrix}$$

or

$$\tau_{lab} = \gamma\tau \quad \text{and} \quad x_{lab} = \beta\gamma\tau c$$

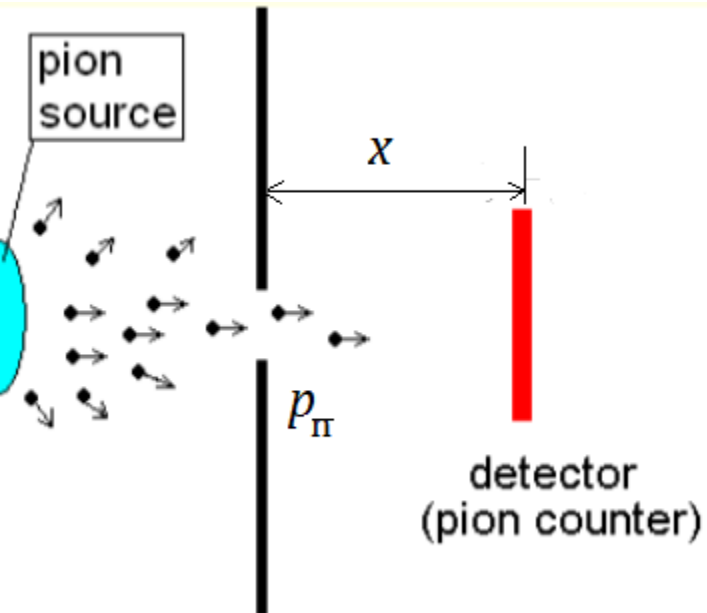
where $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$

Here t and x are the observed **mean lifetime** and **mean decay length** for a moving particle.

For a muon moving at $v = 0.99c \rightarrow \tau_{lab} = 15.6 \mu\text{s}$ and $x_{lab} = 4.63 \text{ km}$.

Example 4.3: Measuring the Lifetime of Charged Pions

The most precise measurement of lifetimes of charged pions was done in an experiment reported by Ayres et al. [Phys. Rev. D 3, 1051 (1971)]. Here is the setup.



A pion counter is moved along the pion beam and measured the number of pions at various distances.

Assume that in lab frame
mean decay time is $\tau_{lab} = \gamma\tau$
and decay length is $x_{lab} = v\tau_{lab} = \beta c\gamma\tau$

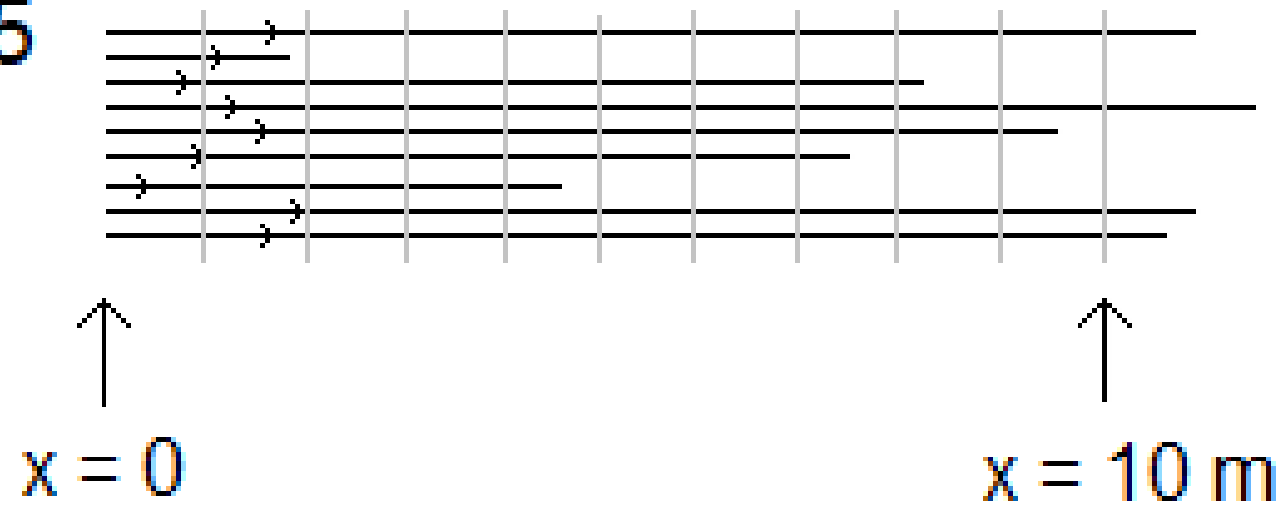
Number of pions surviving at a distance x :

$$N = N_0 \exp(-t/\tau_{lab}) = N_0 \exp(-x/x_{lab})$$

For the given pion momentum p_π in lab, we have:

$$\gamma = \sqrt{p_\pi^2 + m_\pi^2}/m_\pi \quad \text{and} \quad \beta = \sqrt{1 - 1/\gamma^2}$$

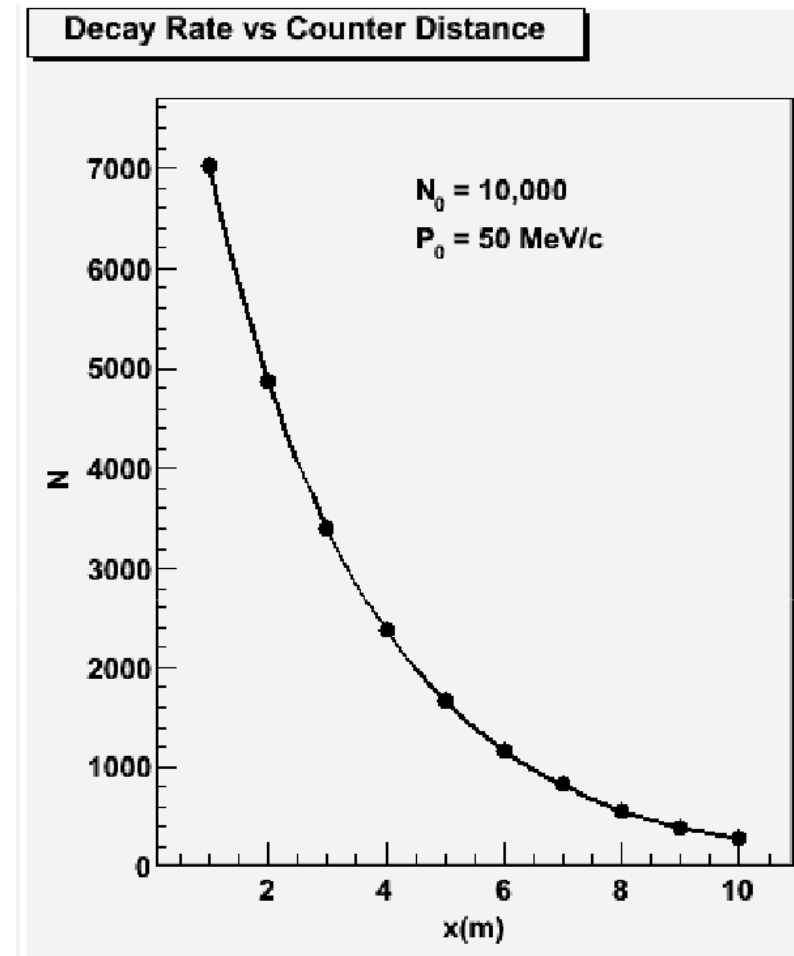
$N(0) = 10^5$
pions



For a pion beam momentum $p_\pi = 50 \text{ MeV}$, assume that following data is collected.
 Using curve fitting tool, write a ROOT program to determine mean lifetime of pions in its rest frame?
 Compare your result with the PDG value.

<u>Distance x (m)</u>	<u># pions counted</u>
1	6912
2	4852
3	3382
4	2391
5	1724
6	1211
7	832
8	574
9	398
10	270

Same data formatted for programming:
 1,2,3,4,5,6,7,8,9,10
 6912,4852,3382,2391,1724,1211,832,574,398,270



Particle Width

While particles are assigned a well-defined mass in theory, unstable particles do not have a single, sharp mass value in practice. Due to their brief lifetimes, quantum mechanics imposes a fundamental limit: the **energy-time uncertainty principle**. This leads to an intrinsic spread in the observed mass of unstable particles. As a result, the mass of an unstable particle is not seen as sharp value but rather as a distribution, typically described by a **Breit-Wigner** profile.

[See Appendix for the derivation of Breit-Wigner line shape]

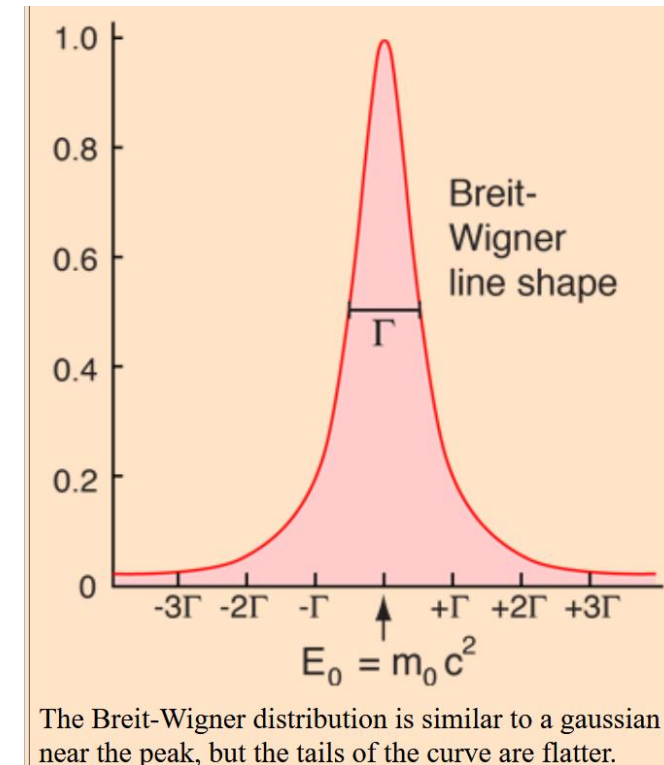
Uncertainty principle:
$$\Delta E \Delta t \sim \frac{\hbar}{2}$$

The particle width (Γ) represents the range of energies a particle can possess and is directly related to its lifetime (τ) through the equation:

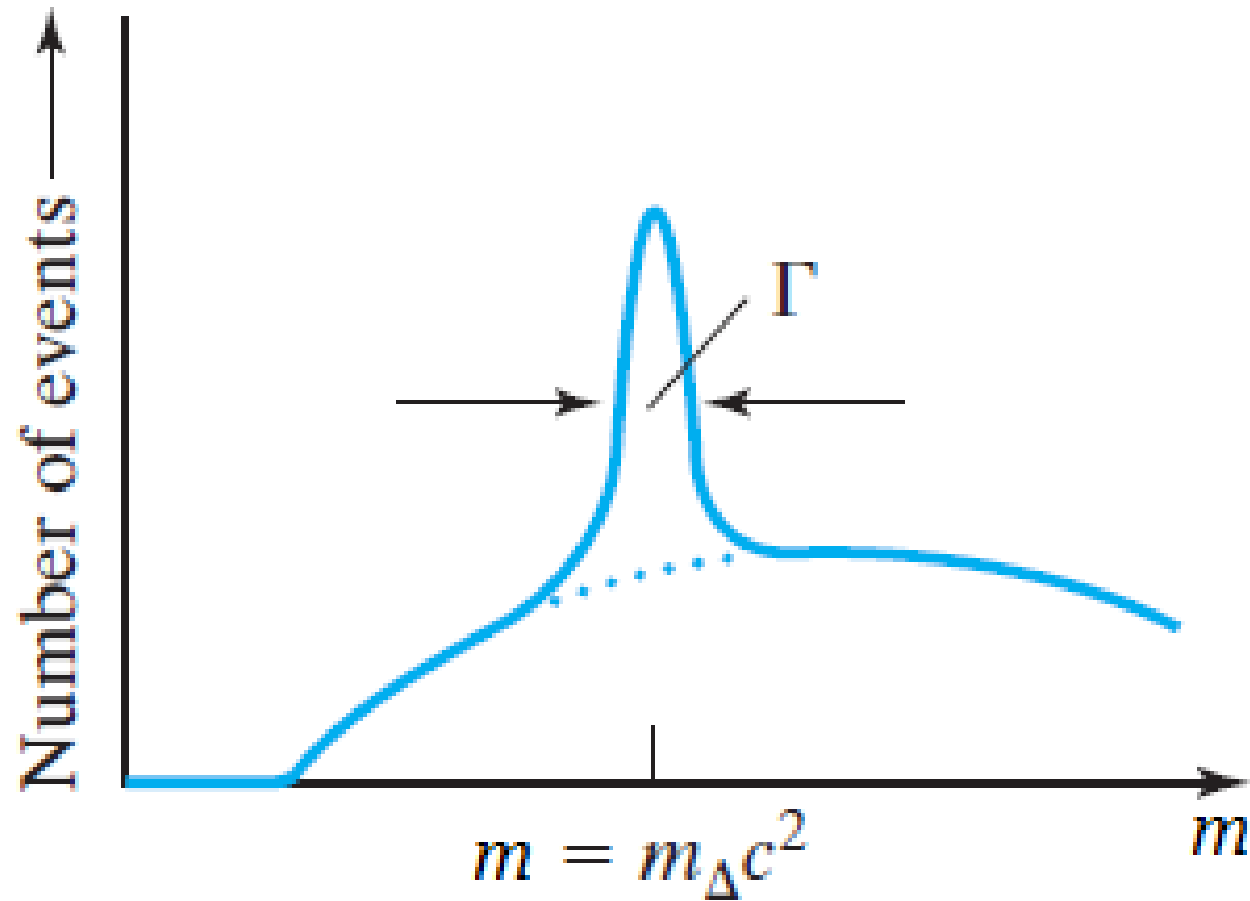
$$\Gamma = \frac{\hbar}{\tau}$$

Note 1: We use the same symbol (Γ) for both decay rate and particle width.

Note 2: PDG provides either Γ or τ value.



Resonance Production



Measurement of Cross-section from mass spectra

We saw number of signal particles can be found from: $N_S = \sigma \mathcal{L}$

Or, for the simple case the cross section is given by: $\sigma = N_S / \mathcal{L}$

We should also consider the efficiency of the detector and branching ratio.

Therefore, the corrected cross-section can be computed from:

$$\sigma = \frac{N_S}{\mathcal{L} \times \varepsilon \times \mathcal{B}}$$

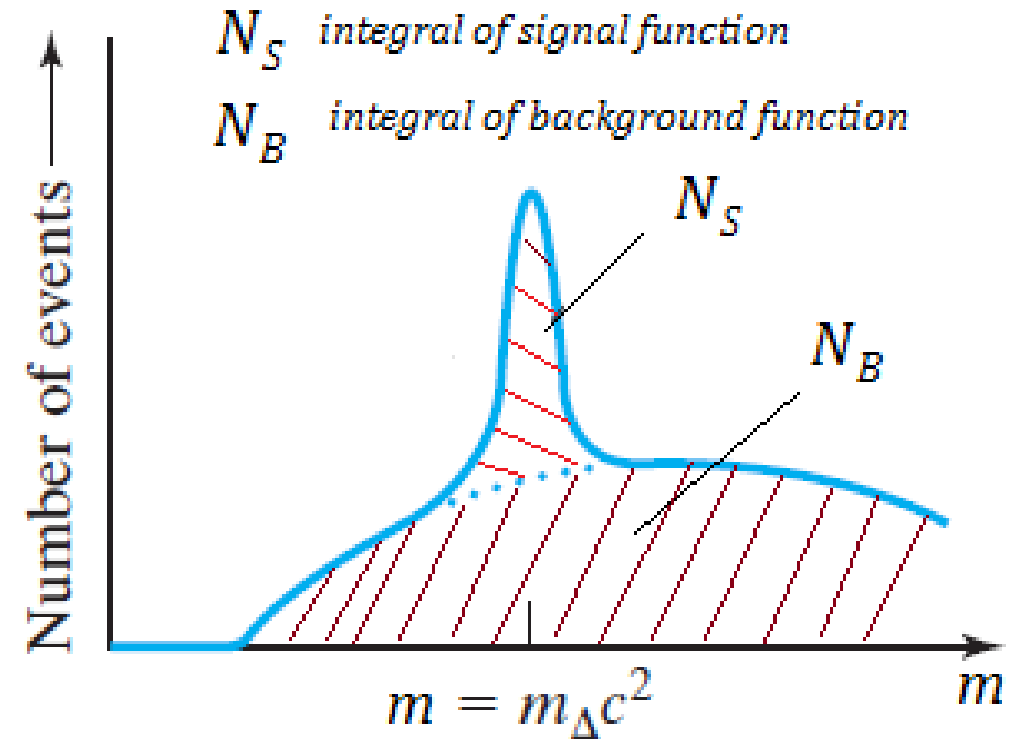
\mathcal{L} = integrated Luminosity

ε = efficiency in the range: (0,1)

\mathcal{B} = branching ratio in the range: (0,1)

Note that if signal and background events are counted together, then:

$$\sigma = \frac{N_{\text{Total}} - N_B}{\mathcal{L} \times \varepsilon \times \mathcal{B}}$$

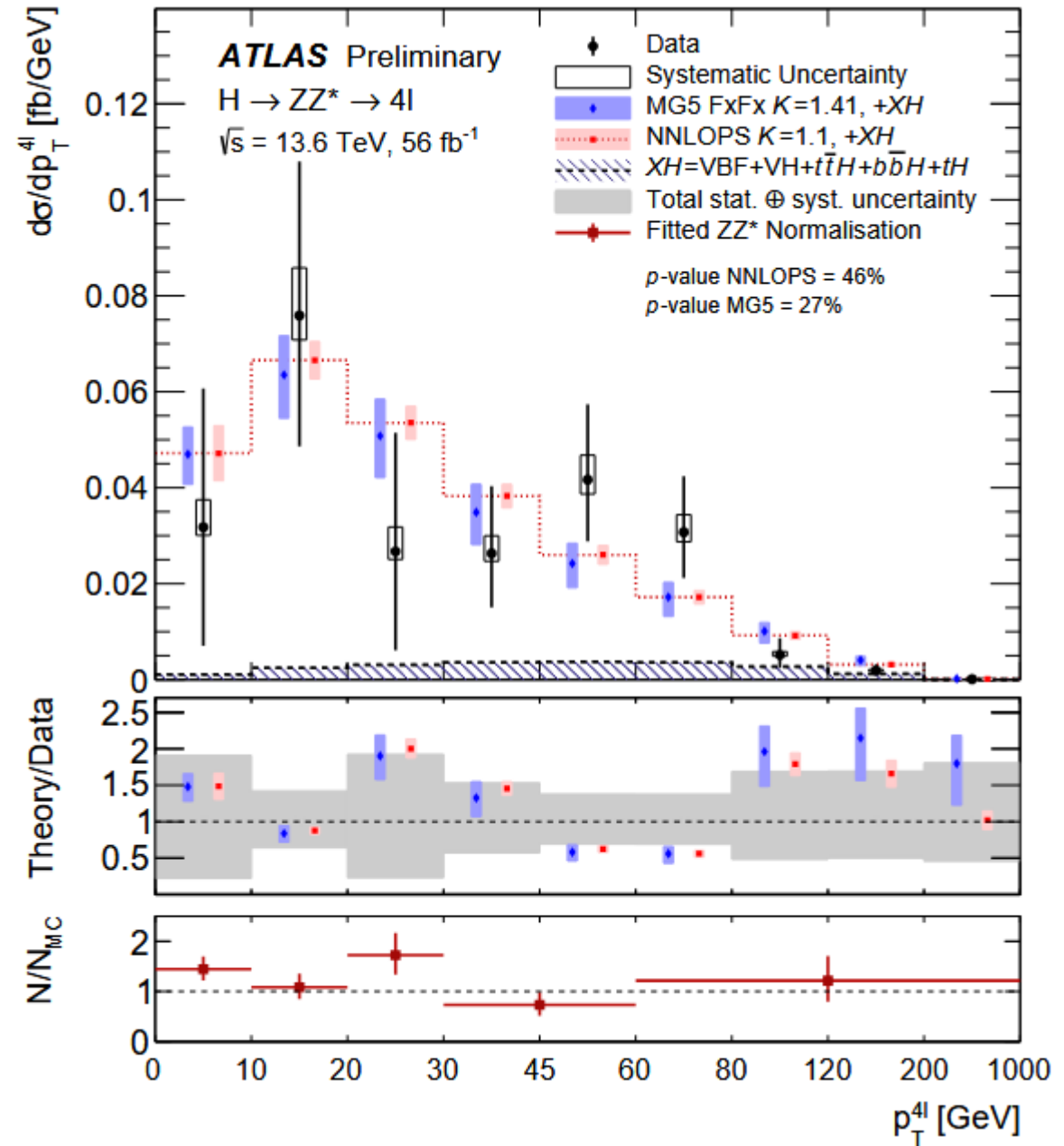
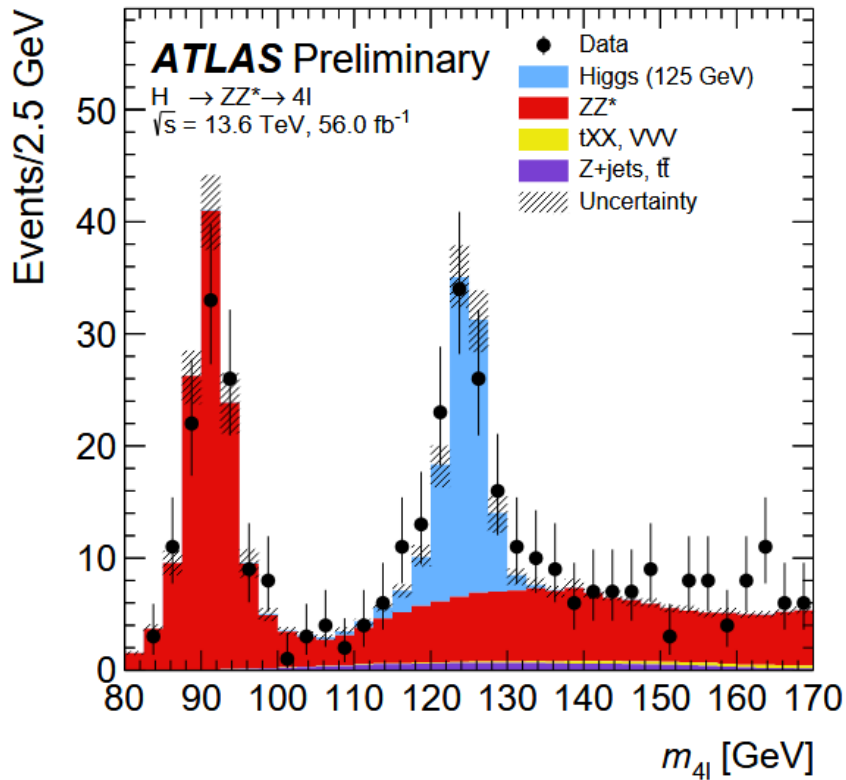


Production of Higgs Boson in ATLAS at LHC:

<https://arxiv.org/pdf/2503.19420>

$$p + p \rightarrow H \rightarrow ZZ^* \rightarrow 4\ell .$$

$$(H \rightarrow ZZ^* \rightarrow 4\mu \text{ and } H \rightarrow ZZ^* \rightarrow 4e)$$

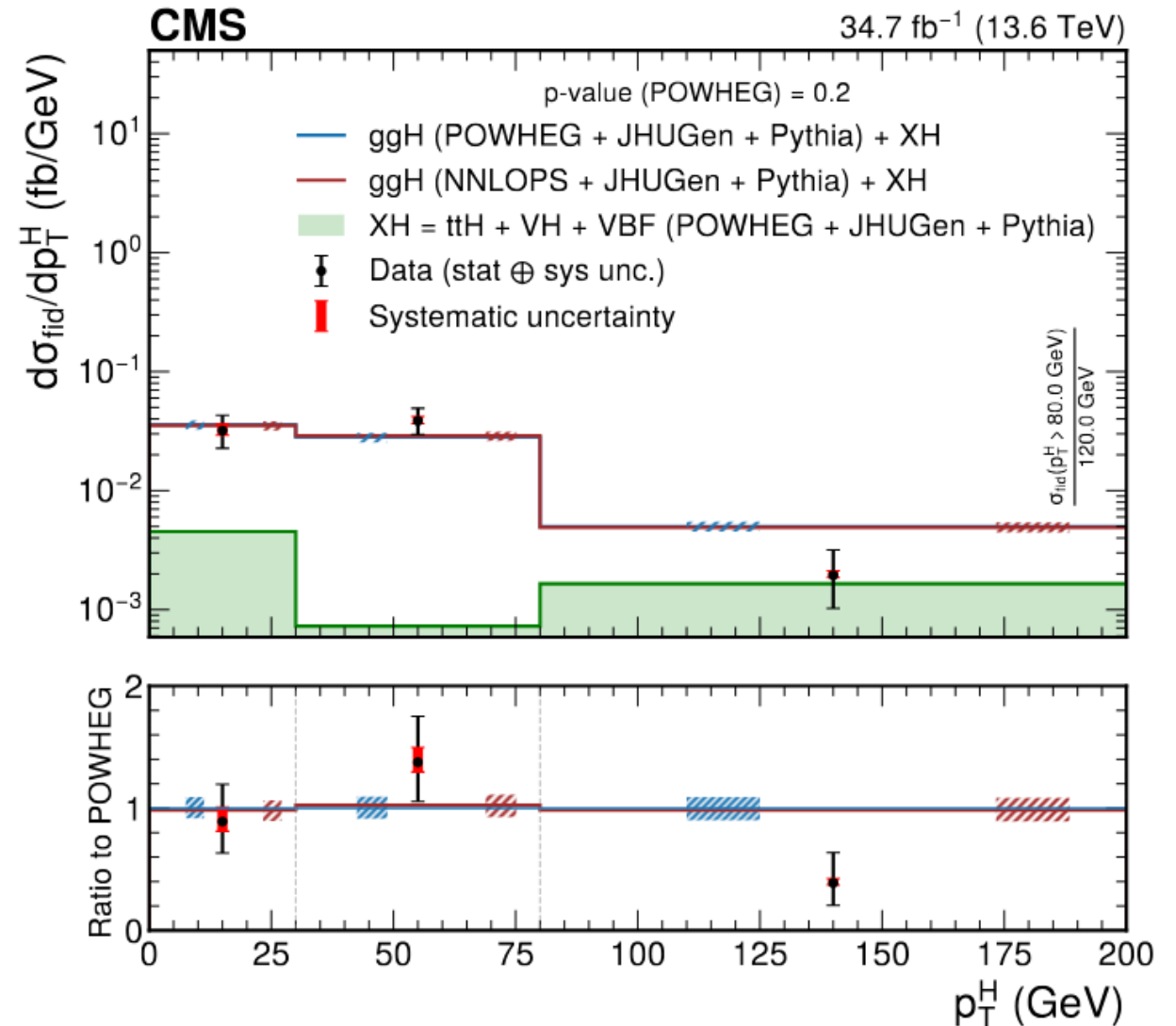
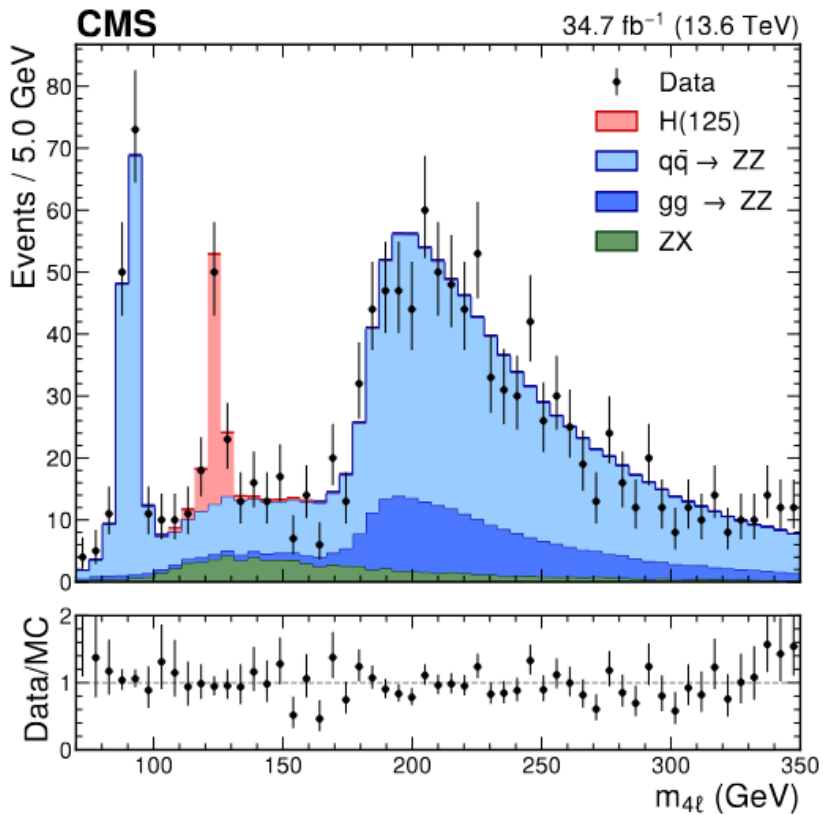


Production of Higgs Boson in CMS at LHC:

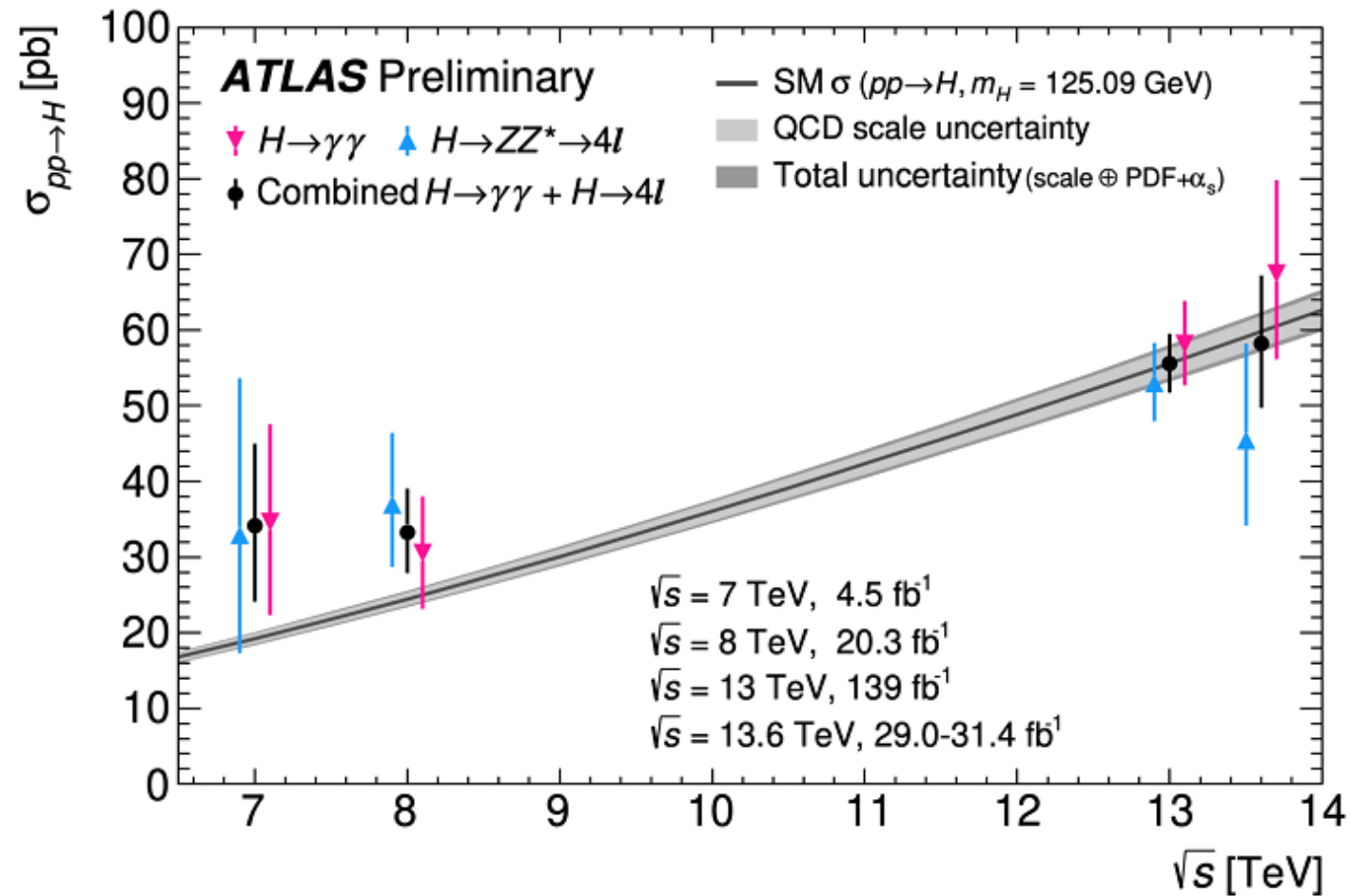
<https://arxiv.org/pdf/2501.14849>

$$p + p \rightarrow H \rightarrow ZZ^* \rightarrow 4\ell .$$

$$(H \rightarrow ZZ^* \rightarrow 4\mu \text{ and } H \rightarrow ZZ^* \rightarrow 4e)$$



Total cross-section extracted by ATLAS as a function of center of mass energy in pp collisions:



Appendix: Origin of Breit Wigner (Lorentzian) Function

In standard quantum mechanics, we use Hermitian operators because they yield real eigenvalues (energies). For a decaying state, we introduce a complex energy to account for the loss of probability over time:

$$E = E_0 - i\frac{\Gamma}{2}$$

When we plug this into the time-dependent Schrödinger equation, the wavefunction evolves as:

$$\Psi(t) \propto e^{-iEt/\hbar} = e^{-iE_0t/\hbar} e^{-\Gamma t/2\hbar}$$

If we look at the probability density $|\Psi(t)|^2$, we see the classic exponential decay law:

$$P(t) = |\Psi(t)|^2 \propto e^{-\Gamma t/\hbar} = e^{-t/\tau}$$

The Breit-Wigner function is the energy-domain representation of this exponential decay. To find the distribution of energies, we take the Fourier transform of the time-dependent wavefunction $\Psi(t)$:

$$A(E) = \int_0^\infty \Psi(t) e^{iEt/\hbar} dt \propto \int_0^\infty e^{i(E-E_0+i\Gamma/2)t/\hbar} dt \quad \longrightarrow \quad A(E) \propto \frac{1}{(E - E_0) + i\Gamma/2}$$

The actual probability of observing a particle at a specific energy E (the spectral line shape or cross-section σ) is the square of the amplitude's magnitude:

$$\sigma(E) \propto |A(E)|^2 = \frac{1}{(E - E_0)^2 + (\Gamma/2)^2}$$

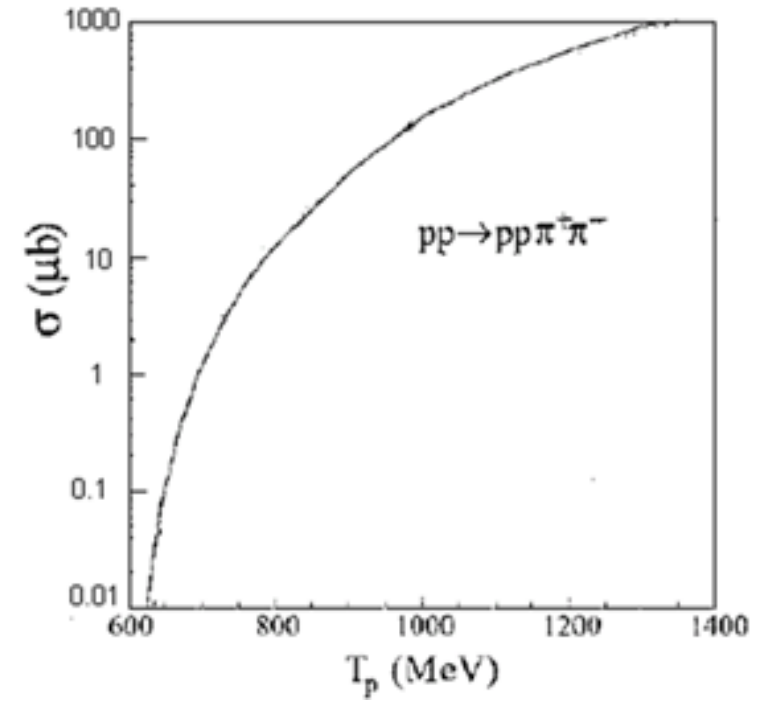
Here $E = M$ the particle mass.

Problems

1. The mean free path of fast neutrons in lead is about 5 cm. Find the total neutron cross section of lead. (atomic mass number ~ 200 , density $\sim 10 \text{ g/cm}^3$). [Ans: $\sigma = 6.64 \text{ b}$].

2. Figure shows the cross section for the reaction: $p + p \rightarrow p + p + \pi^+ + \pi^-$

(a) For $L = 10^{26} / \text{cm}^2\text{s}$, determine the production rate of this reaction if incident proton has KE of $T = 1000 \text{ MeV}$.



(b) For case (a) compute how many reactions are produced per hour? and per day?

3. The LHC at CERN is the largest particle accelerator with circumference of 27 km. It collides opposing protons bunches (each contains 1.0×10^{11} protons) at the center of mass energy of 14 TeV. For the operation, the collision frequency is designed to be $nf = 40$ MHz and each bunch has the geometric cross-sectional area of $A = \pi (35 \mu\text{m})^2$. The primary task of the LHC will be the detection and the study of the Higgs Boson (H) whose mass is $m_H = 125 \text{ GeV}/c^2$. Assume that the cross-section of the Higgs production is 3 fb (1 fb = 10^{-15} b).

(a) Calculate the luminosity of LHC in $1/(\text{s}\cdot\text{cm}^2)$ unit.

(b) Calculate the Higgs production rate at LHC at the luminosity in part (a).

(c) How many Higgs bosons will be produced per day at LHC at the luminosity in part (a)?

4. At LEP, there were 8 bunches of electrons and 8 bunches of positrons in circulation (circumference of 27 km). Each bunch contains $N = 1.7 \times 10^{11}$ particles. If the beams have the cross-sectional area of $A = 4 \times 10^{-3} \text{ mm}^2$

(a) Calculate the velocity of the circulating electrons (or positrons) if its energy is 45.6 GeV [Ans: $v = c$]

(b) Calculate the frequency of the collision and the cross-over rate of the beams [Ans: $f = 11 \text{ kHz}$, $t = 11 \text{ } \mu\text{s}$]

(c) Calculate the luminosity [Ans: $6.4 \times 10^{31} \text{ /cm}^2\text{/s}$]

(d) Calculate the production rate of the Z bosons if the cross section $\sigma = 30 \text{ nb}$