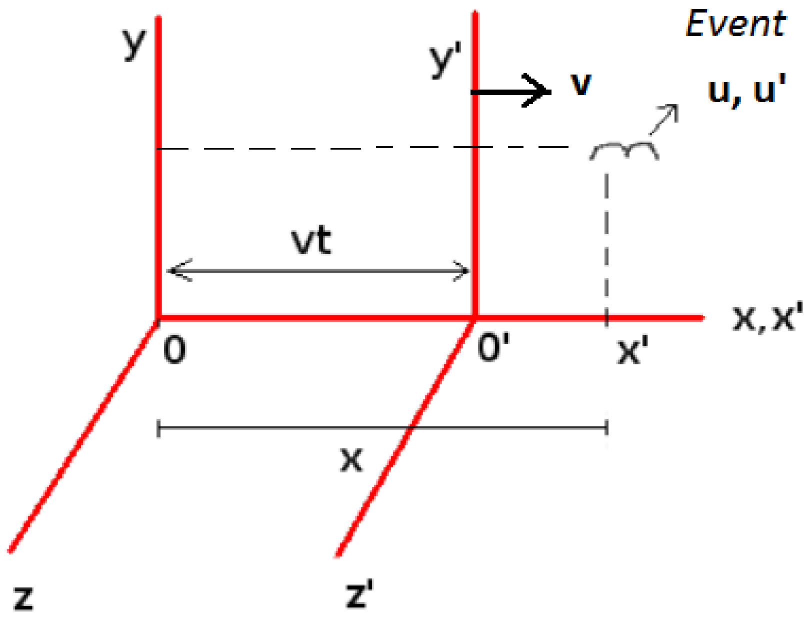
MIDDLE EAST TECHNICAL UNIVERSITY

Particle Data Analysis in High Energy Physics

Lecture 3

Relativistic Kinematics



Ahmet Bingül

METU, Physics

Feb 2026

Introduction

This chapter gives a brief overview of the basic concepts of **special relativity**, with the emphasis on the definition and application of four-vectors in data analysis. We will discuss the following

- Unit Systems
- Lorentz Transformation
- Invariant Mass
- Pseudorapidity
- Four-Vector Notation
- Feynman Diagrams
- Problems

SI Units

In SI unit system, the base quantities are mass, length and time.

Units for these are chosen to be on a human scale.

Quantity	Dimensions	Units
Mass	[M]	1 kg
Length	[L]	1 m
Time	[T]	1 s

Other quantities are derived, e.g.

Quantity	Dimensions	Units
Velocity	[L/T]	1 m /s
Ang.Mom.	[M L^2/T]	1 kg m^2 / s
Energy	[M L^2/T^2]	1 J

SI units are not convenient for particle physics!

Mass of proton: $m_p = 1.67 \times 10^{-27}$ kg

Radius of proton: $r_p = 0.8 \times 10^{-15}$ m

Natural Units

Particle physics relies on Special Relativity and Quantum Mechanics.

In SR, speed of light: $c = 3 \times 10^8\ \mathrm{m/s}$

In QM, Planck constant: $\hbar = h/2\pi = 1.055 \times 10^{-34}\ \mathrm{J.}s$

In natural units and we set $\hbar = c = 1$

This simplifies algebraic expressions

- Energy-momemtum-mass relation: $E^2 = p^2c^2 + m^2c^4 \quad \rightarrow \quad E^2 = p^2 + m^2$
- Energy of photon: $E = \hbar\omega \quad \rightarrow \quad E = \omega$
- Momentum: $\mathbf{p} = \hbar\mathbf{k} \quad \rightarrow \quad \mathbf{p} = \mathbf{k}$
- Charge radius of proton: $r_p = 0.8 \times 10^{-15}\ \mathrm{m} = 4.1\ \mathrm{GeV}^{-1}$
- Rest mass of proton: $m_p = 1.67 \times 10^{-27}\mathrm{kg} = 0.938\ \mathrm{GeV}$

Note that:

$\hbar c = 0.197\ \mathrm{GeV.fm}$, $1\ \mathrm{GeV} = 1.602 \times 10^{-10}\ \mathrm{J}$ and $1\ \mathrm{fm} = 1.0 \times 10^{-15}\ \mathrm{m}$

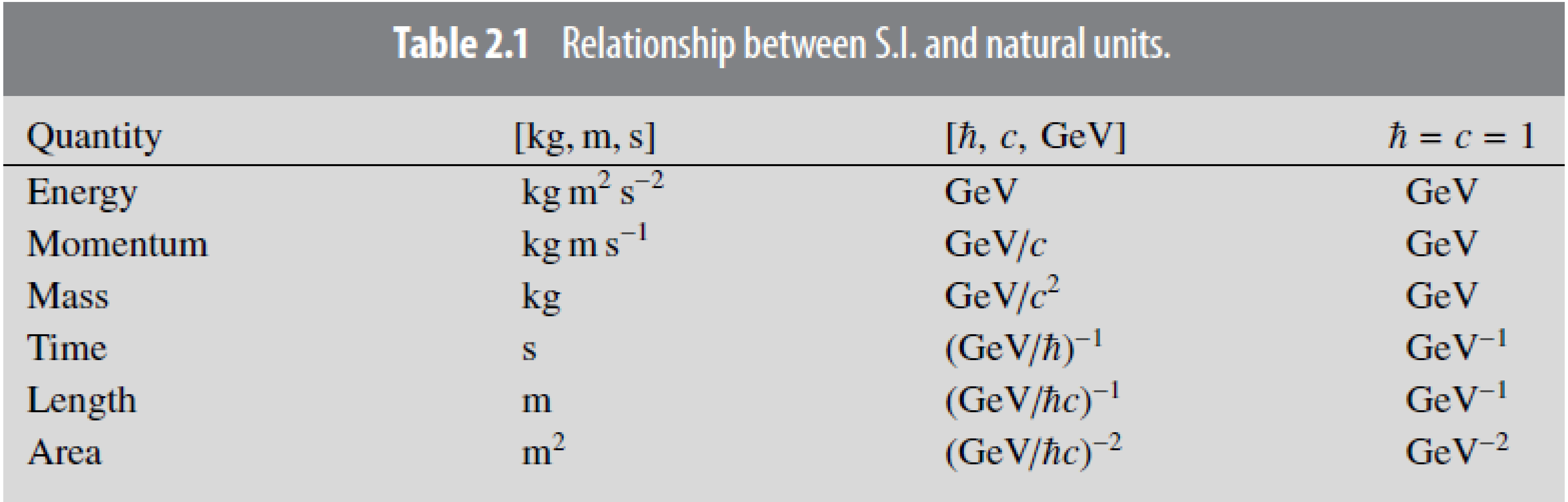
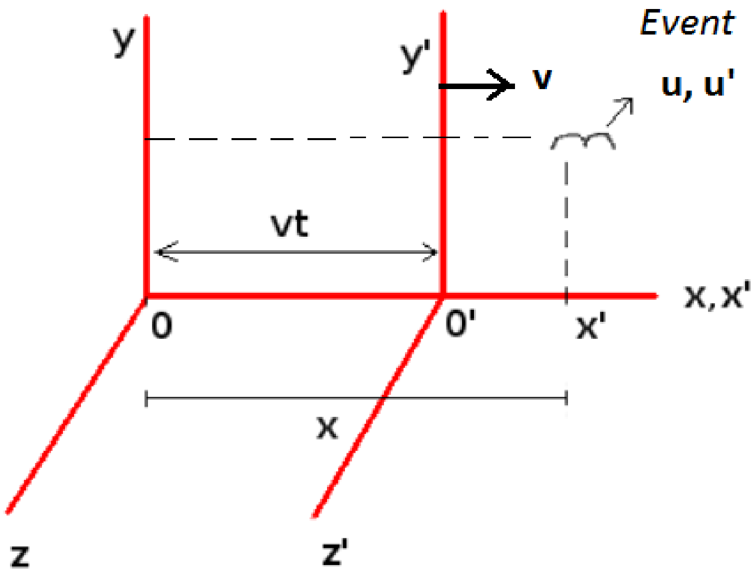


Table 2.1 Relationship between S.I. and natural units.

Quantity	[kg, m, s]	[$\hbar$, c, GeV]	$\hbar = c = 1$
Energy	$\mathrm{kg\,m^2\,s^{-2}}$	GeV	GeV
Momentum	$\mathrm{kg\,m\,s^{-1}}$	GeV/c	GeV
Mass	kg	GeV/c^2	GeV
Time	s	$(\mathrm{GeV}/\hbar)^{-1}$	GeV^{-1}
Length	m	$(\mathrm{GeV}/\hbar c)^{-1}$	GeV^{-1}
Area	$\mathrm{m^2}$	$(\mathrm{GeV}/\hbar c)^{-2}$	GeV^{-2}

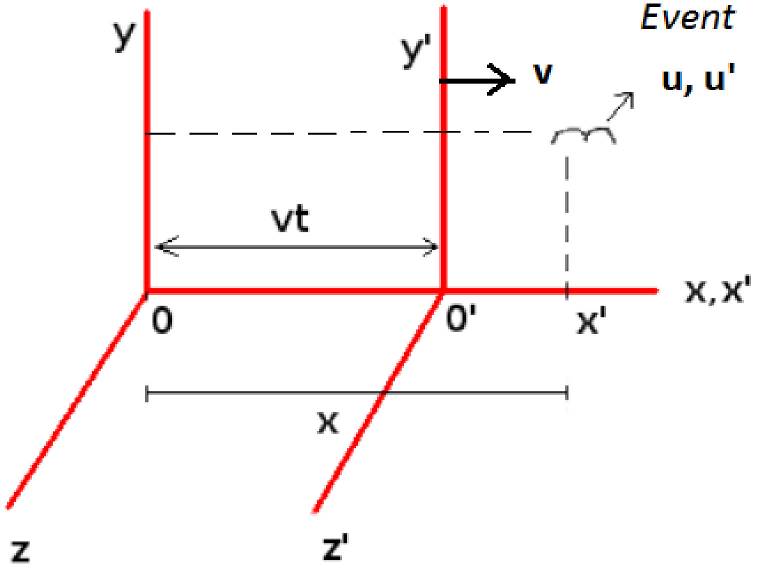
Lorentz Transformation

Special relativity is based on the **space-time transformation** properties of Physical observables as measured in two or more inertial frames moving relative to each other.



Galilean Transformations of Coordinates

An event is a four-dimensional coordinate in space-time.
Consider two inertial (non-accelerating) frames whose origins are O and O’ respectively. O is at rest and O’ is moving at a constant speed v in the direction of +x-axis.
An event may be represented by four-element vectors:
(x, y, z, t) in O
(x’, y’, z’, t’) in O’.



The connection between these two coordinates are given by Galilean Transformations when v<<c as follows:

Galilean coordinate transformations in 1D.	*Galilean velocity transformations in 1D.*	*Galilean transformations in 3D.*
$x' = x - vt$	${u_x}' = u_x - v$	$\mathbf{r}' = \mathbf{r} - \mathbf{v}t$
$y' = y$	${u_y}' = u_y$	$\mathbf{u}' = \mathbf{u} - \mathbf{v}$
$z' = z$	${u_z}' = u_z$	$t' = t$
$t' = t$		

Lorentz Transformations of Coordinates

LT can be extracted from Einstein’s “Theory of Special Relativity” which is based on two axioms:

Postulate 1: The laws of physics are the same for all inertial observers.

Postualte 2: Speed of light as measured by all observers is *c* = 3x10^8 m/s independent of motion of the source.

Lorentz coordinate transformations in 1D.

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma(t - \frac{vx}{c^2})$$

Lorentz velocity transformations in 1D.

$$u_x{}' = \frac{u_x - v}{1 - (v/c^2)u_x}$$

$$u_y{}' = \frac{u_y/\gamma}{1 - (v/c^2)u_x}$$

$$u_z{}' = \frac{u_z/\gamma}{1 - (v/c^2)u_z}$$

Lorentz coordinate transformations in 3D.

$$\mathbf{r}' = \mathbf{r} + \mathbf{v}(\frac{(\gamma - 1)\mathbf{v}\cdot\mathbf{r}}{v^2} - \gamma t)$$

$$t' = \gamma(t - \mathbf{v}\cdot\mathbf{r}/c^2)$$

where

$$\beta = v/c, \quad \boldsymbol{\beta} = \mathbf{v}/c \quad \text{and} \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1-\beta^2}}$$

Transformations of Energy and Momentum (in Natural Units)

Particle momentum and energy at O: $\mathbf{p} = (p_x, p_y, p_z)$, E

Particle momentum and energy at O’: $\mathbf{p}' = (p_x', p_y', p_z')$, E'

Speed of coordinate O’: $\boldsymbol{\beta} = (\beta_x, \beta_y, \beta_z)$

$$\boldsymbol{\beta}^2 = \beta_x^2 + \beta_y^2 + \beta_z^2$$

$$\gamma = (1 - \boldsymbol{\beta}^2)^{-1/2}$$

1D Lorentz Transformation

$$p_x{}' = \gamma(p_x - \beta_{\boldsymbol{x}} E)$$

$$p_y{}' = p_y$$

$$p_z{}' = p_z$$

$$E' = \gamma(E - \beta_x p_x)$$

1D Inverse Lorentz Transformations

$$p_x = \gamma(p_x{}' + \beta_{\boldsymbol{x}} E')$$

$$p_y = p_y{}'$$

$$p_z = p_z{}'$$

$$E = \gamma(E' + \beta_{\boldsymbol{x}} p_x{}')$$

3D Lorentz Transformations

$$\mathbf{p}' = \mathbf{p} + \gamma\boldsymbol{\beta}\left(\frac{\gamma}{\gamma+1}\boldsymbol{\beta}\cdot\mathbf{p} - E\right)$$

$$E' = \gamma(E - \boldsymbol{\beta}\cdot\mathbf{p})$$

3D Inverse Lorentz Transformations

$$\boldsymbol{p} = \mathbf{p}' + \gamma\boldsymbol{\beta}\left(\frac{\gamma}{\gamma+1}\boldsymbol{\beta}\cdot\mathbf{p}' + E'\right)$$

$$E = \gamma(E' + \boldsymbol{\beta}\cdot\mathbf{p}')$$

Some Useful Equations

Now consider a particle with mass m and speed u. Then,

	SI Units	Natural Units
Energy when $u = 0$ (rest energy):	$E = mc^2$	$E = m$
Relativistic Momentum:	$p = \gamma m u = \frac{mu}{\sqrt{1-u^2/c^2}} = \frac{mu}{\sqrt{1-\beta^2}}$	$p = \gamma m \beta$
Total energy:	$E = \gamma m c^2$	$E = \gamma m$
Relativistic kinetic energy:	$T = (\gamma - 1)mc^2 = E - mc^2$	$T = (\gamma - 1)m = E - m$
Energy-momentum relation:	$E^2 = p^2c^2 + m^2c^4$	$E^2 = p^2 + m^2$
Particle speed:	$u = \beta c = pc^2 / E$	$u = \beta = p/E$
For massless particles ($m = 0$, like photon):	$T = E = pc$ and $u = c$	$T = E = p$ and $u = \beta = 1$

Define proper time t_0 in coordinate O. Time t ellapsed in O' → $t = \gamma t_0$ (time dilation)

Define proper length L_0 in coordinate O. Length L in O' → $L = L_0/\gamma$ (length contraction)

Invariant Mass

Invariant mass (*M*) has the same value in any inertial frame.

Invariant mass for single particle:

$$s = M^2 = E^2 - p^2 = E'^2 - p'^2$$

Invariant mass for *n* particles:

$$M^2 = \left(\sum_i E_i\right)^2 - \left(\sum_i \mathbf{p}_i\right)^2$$

or

$$M^2 = (E_1 + E_2 + \cdots + E_n)^2 - (\mathbf{p}_1 + \mathbf{p}_2 + \cdots + \mathbf{p}_n)^2$$

We can define a coordinate system where total momentum of particles is zero.
This coordinate system is known as **center of mass**. In this case

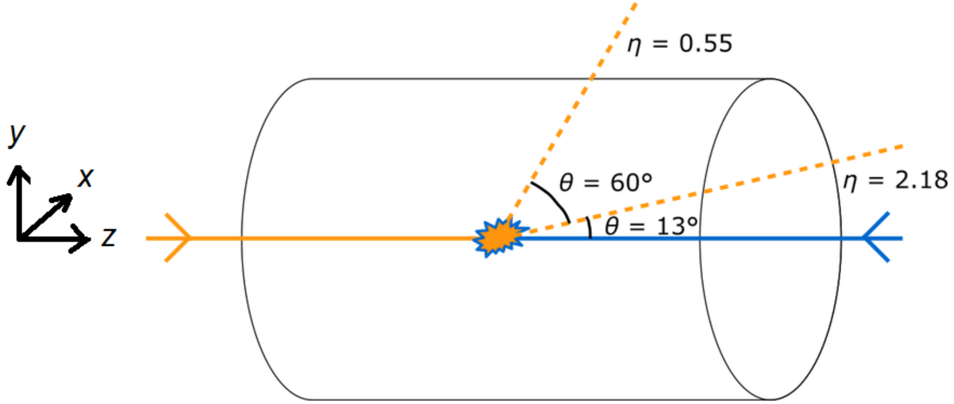
$$M^2 = (E'_1 + E'_2 + \cdots + E'_n)^2 - \underbrace{(\mathbf{p}'_1 + \mathbf{p}'_2 + \cdots + \mathbf{p}'_n)^2}_{\text{zero}} = (E'_1 + E'_2 + \cdots + E'_n)^2$$

So $\boldsymbol{M = E_{cm} = \sqrt{s}}$

Pseudorapidity

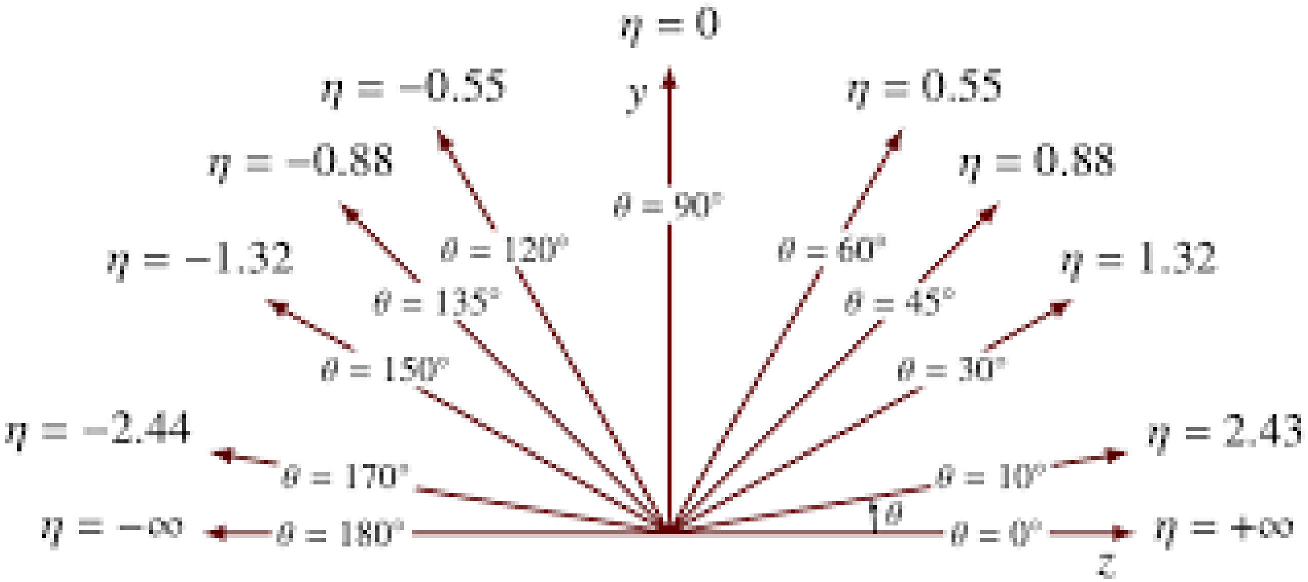
In a collider, the "beam pipe" is the longitudinal axis (z-axis). Most of the interesting physics happens at very small angles relative to this pipe.

Pseudorapidity, η, is a commonly used spatial coordinate representing the angle of a particle relative to the beam axis. It is defined as:



$$\eta = -\ln[\tan(\theta/2)]$$

where θ is the angle between the particle three-momentum **p** and the positive direction of the beam axis.



One can also use rapidty:

$$y = \frac{1}{2}\ln\left(\frac{E + p_z}{E - p_z}\right)$$

Hadron colliders measure physical momenta in terms of transverse momentum p_T, polar angle in the transverse plane ϕ and pseudorapidity η. (See next page)

To obtain Cartesian momenta (p_x , p_y , p_z) the following conversions are used:

$$\begin{aligned} p_\mathrm{x} &= p_\mathrm{T} \cos\phi \\ p_\mathrm{y} &= p_\mathrm{T} \sin\phi \\ p_\mathrm{z} &= p_\mathrm{T} \sinh\eta, \end{aligned}$$

$$p_T = \sqrt{p_x^2 + p_y^2} = |\mathbf{p}|\sin\theta$$

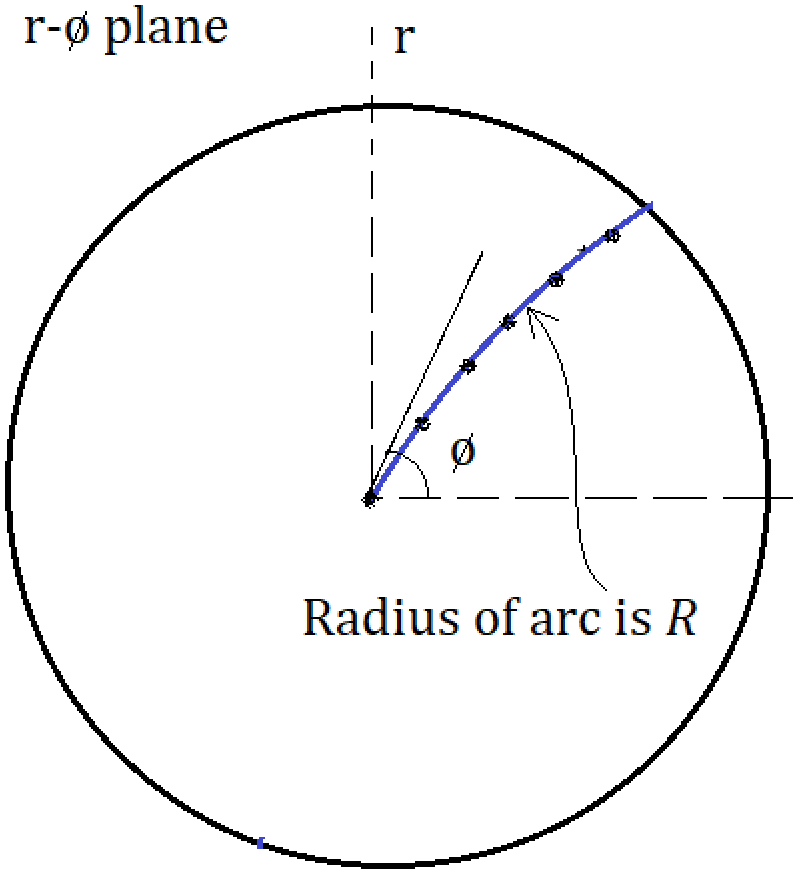
$$|\mathbf{p}| = \sqrt{p_x^2 + p_y^2 + p_z^2} = p_T\cosh\eta$$

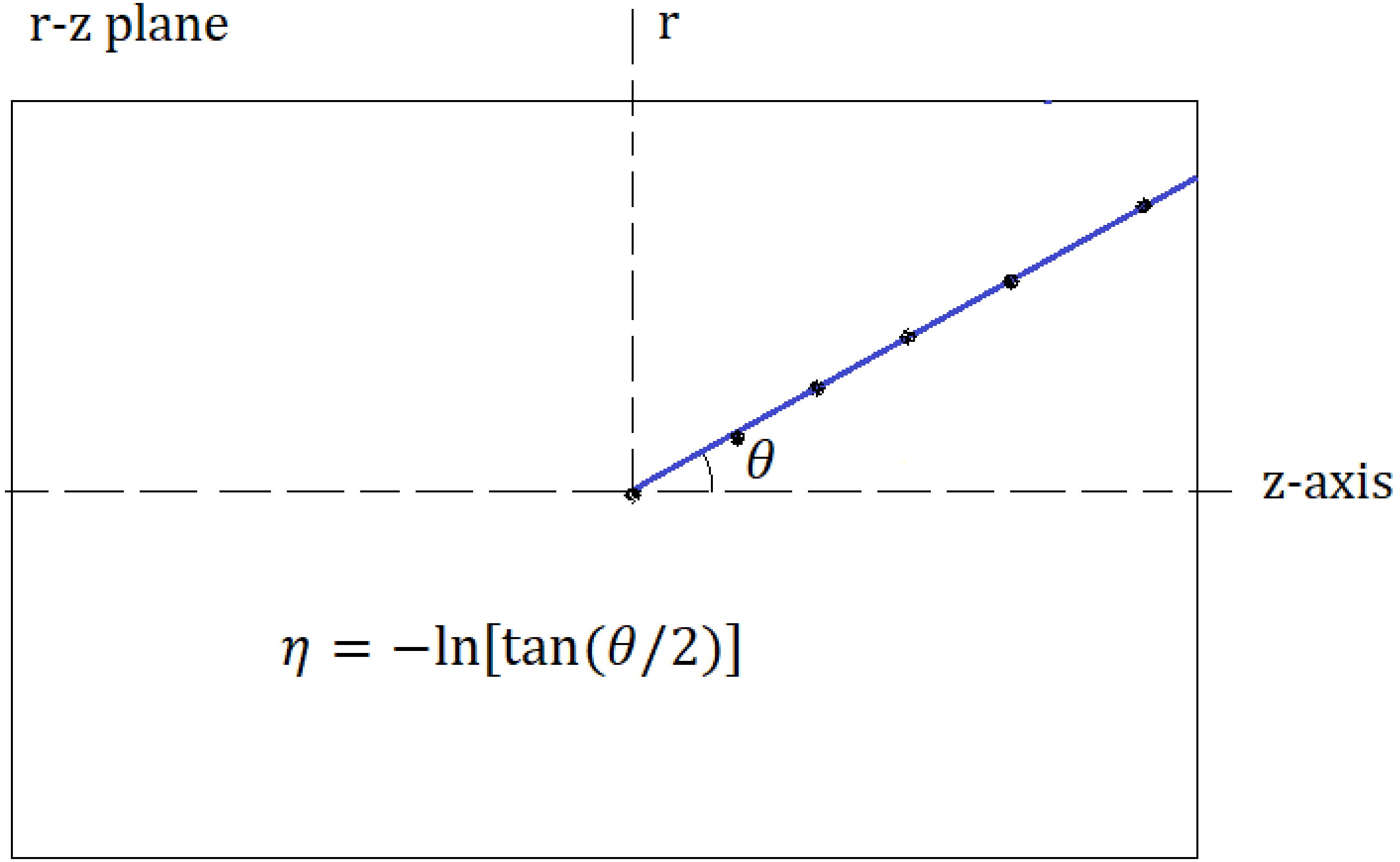
At LHC, the number of particles produced per unit of pseudorapidity (dN/dη) is roughly **constant**.

By designing detectors with uniform segments in η (rather than θ), physicists ensure that each sensor "sees" roughly the same amount of activity. So, a useful measure is the angular distance between two patricles:

$$\Delta R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} = \sqrt{(\eta_2 - \eta_1)^2 + (\phi_2 - \phi_1)^2}$$

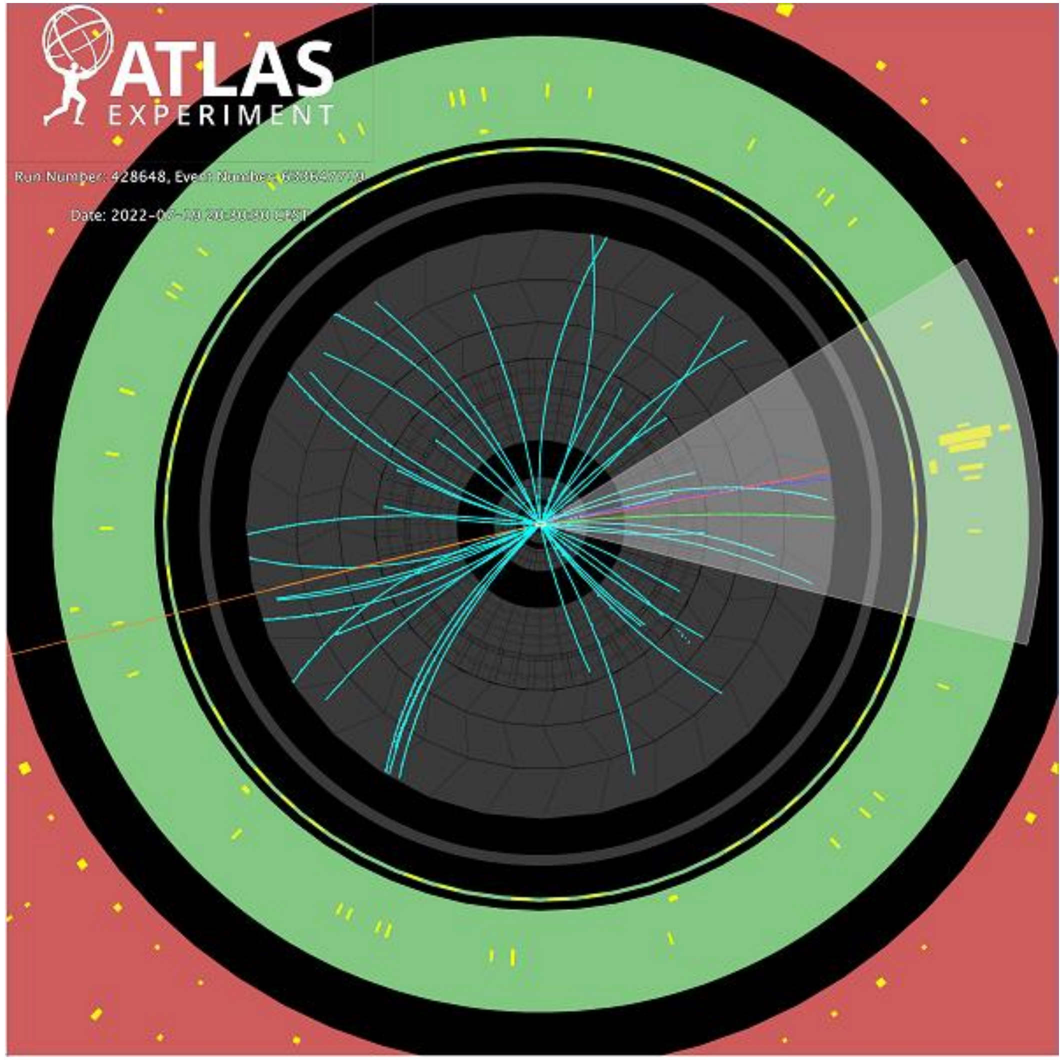
that defines a cone with the tip at the origin of the coordinate system

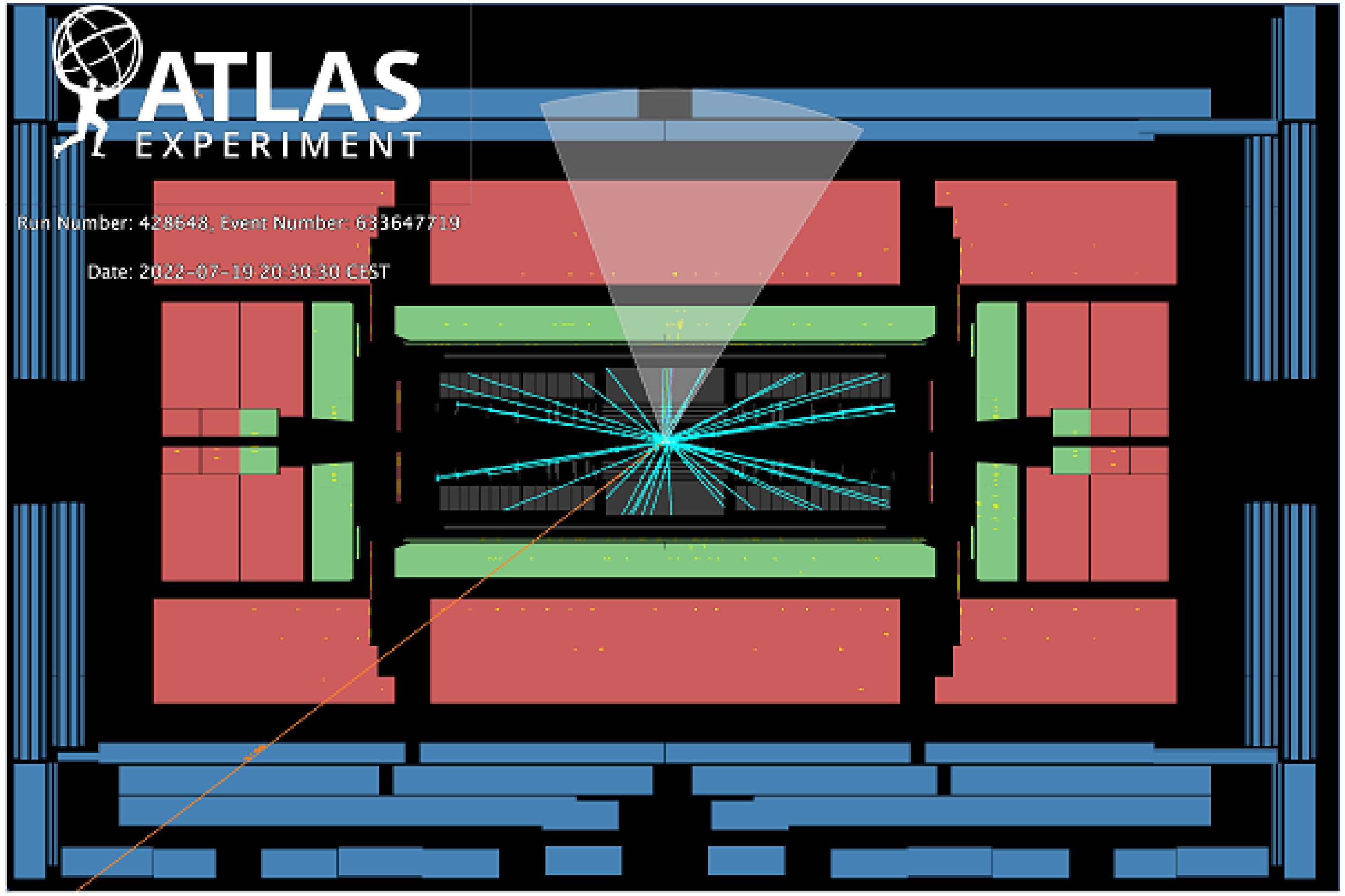




$$\eta = -\ln[\tan(\theta/2)]$$

B magentic field intensity (in Tesla)
R radius of curvature (in meter)
Transverse momentum in GeV of a charged track is: $p_T = 0.3BR$


ATLAS
EXPERIMENT
Run Number: 428648, Event Number: 633647719
Date: 2022-07-19 20:30:30 CEST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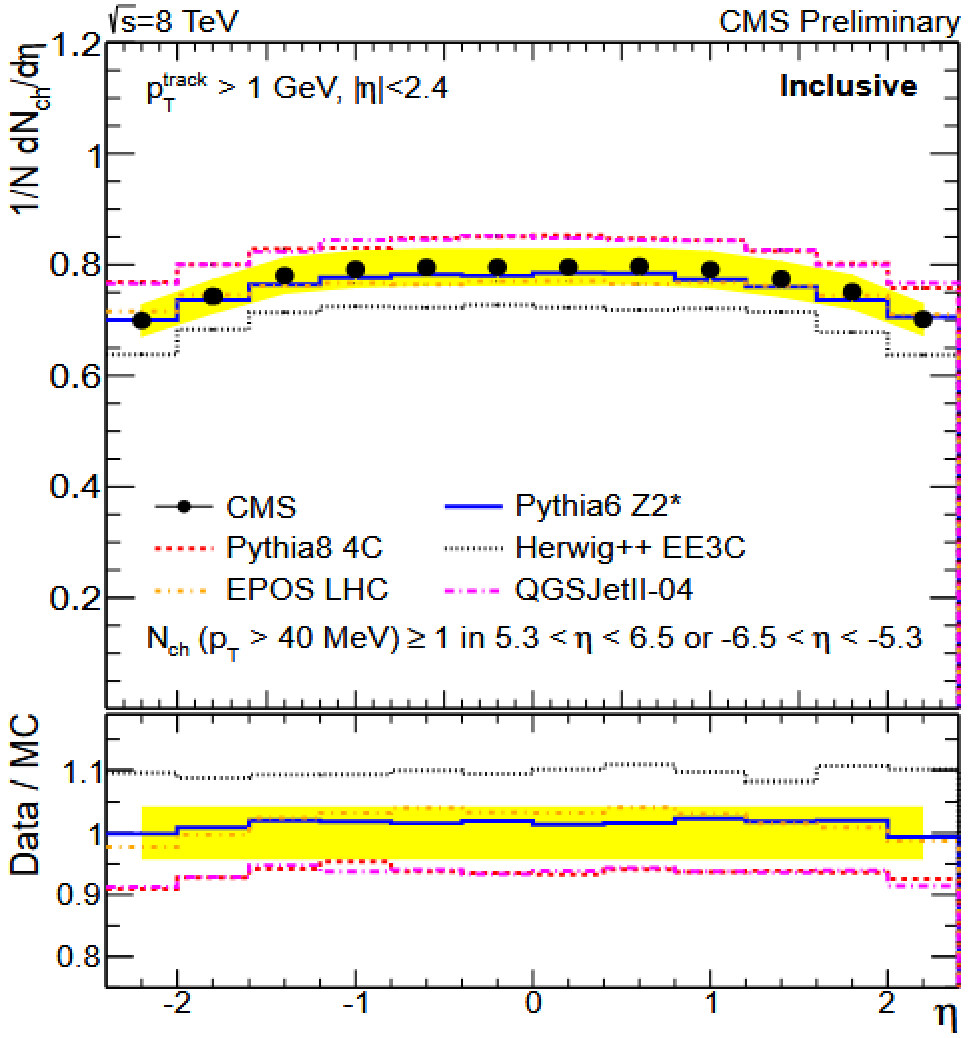
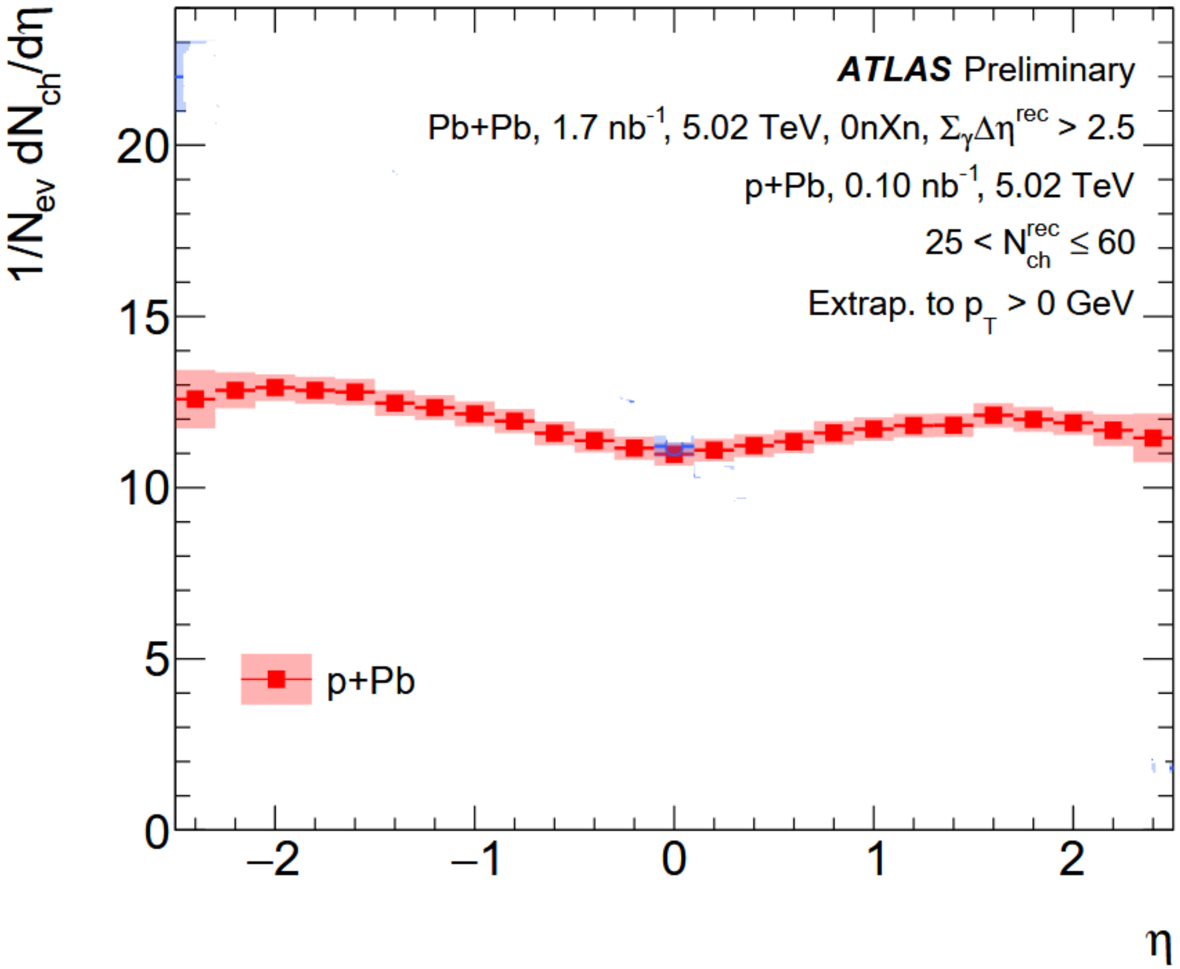


√s=8 TeV
CMS Preliminary
$p_T^{track} > 1$ GeV, $|\eta|<2.4$
Inclusive
$1/N\ dN_{ch}/d\eta$
1.2
1
0.8
0.6
0.4
0.2
CMS
Pythia6 Z2*
Pythia8 4C
Herwig++ EE3C
EPOS LHC
QGSJetII-04
N_{ch} (p_T > 40 MeV) ≥ 1 in 5.3 < η < 6.5 or -6.5 < η < -5.3
Data / MC
1.1
1
0.9
0.8
-2
-1
0
1
2
η



$1/N_{ev}\ dN_{ch}/d\eta$
ATLAS Preliminary
Pb+Pb, 1.7 nb^{-1}, 5.02 TeV, 0nXn, $\Sigma_\gamma \Delta\eta^{rec} > 2.5$
p+Pb, 0.10 nb^{-1}, 5.02 TeV
$25 < N_{ch}^{rec} \leq 60$
Extrap. to $p_T > 0$ GeV
p+Pb
20
15
10
5
0
−2
−1
0
1
2
η

Four-Vector Notation

Special relativity mixes up the spatial and time components specifying the coordinates of a particle. Thus, we can define a 4-vector $x^{\mu} = (ct, x, y, z)$ or in natural units:

$$x^{\mu} = (x^0, x^1, x^2, x^3) = (t, x, y, z)$$

More formally, we define contavariant 4-vector: $x^{\mu} = (t, \mathbf{x}) = (t, x, y, z)$

and covariant 4-vector: $x_{\mu} = (t, -\mathbf{x}) = (t, -x, -y, -z)$

and metric tensor $g_{\mu\nu}$

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$x_{\mu} = g_{\mu\nu}\, x^{\nu}$ and $x^{\mu} = g^{\mu\nu} x_{\nu}$

where $\mu = 0{,}1{,}2{,}3$ and $\nu = 0{,}1{,}2{,}3$

Length of a 4-vector: $x^{\mu} x_{\mu} = t^2 - x^2 - y^2 - z^2$

This is an invariant (scalar) quantity in special relativity.

Four-Vector Notation

Dot product of two vectors:

$$x \cdot y = g_{\mu\nu} x^\mu y^\nu = x_\nu y^\nu = x_0 y^0 + x_1 y^1 + x_2 y^2 + x_3 y^3 = x_0 y^0 - \mathbf{x} \cdot \mathbf{y}$$

Four momentum:

$$p^\mu = (E, \mathbf{p}) = (E, p_x, p_y, p_z)$$
$$p_\mu = (E, -\mathbf{p}) = (E, -p_x, -p_y, -p_z)$$

and its length (dot product):

$$\begin{aligned} p^\mu p_\mu = p_\mu p^\mu &= E^2 - \mathbf{p} \cdot \mathbf{p} = E^2 - p^2 = m^2 \\ &= (\text{rest mass})^2 \\ &= \text{scalar} \end{aligned}$$

Four-Vectors in ROOT

`TLorentzVector` is a general four-vector class, which can be used either for the description of position and time (x, y, z, t) or momentum and energy (px, py, pz, E).

```
TLorentzVector a;      // initialized by (0., 0., 0., 0.)
TLorentzVector b(1.0, 1.0, 1.0, 1.0);
TLorentzVector c(a);
TLorentzVector d(TVector3(1.0, 2.0, 3.0),4.0);
```

One can use special methods:

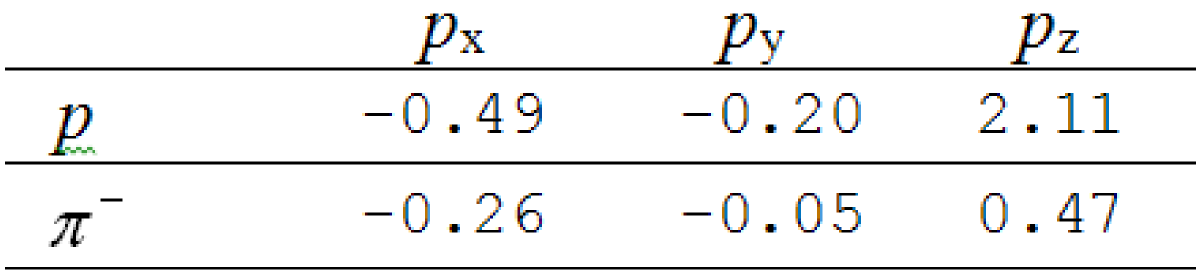
```
// python
from ROOT import TLorentzVector
particle = TLorentzVector()
particle.SetXYZM(1,2,3,0)
particle.SetPxPyPzE(1,2,3,5)
particle.SetPtEtaPhiM(0.75, -1.1, 0.4, 140)
print(" mass = ",particle.M());
```

```
// C++

TLorentzVector particle;
particle.SetXYZM(1,2,3,0);
particle.SetPxPyPzE(1,2,3,5);
particle.SetPtEtaPhiM(0.75,-1.1,0.4,140);
cout << " mass = " << particle.M() << endl;
```

Example 3.1

Consider a neutral particle X^0 decays as $X^0 \rightarrow p + \pi^-$.
The measured momentum components of the decay products in GeV are given in the table.



	p_x	p_y	p_z
p	−0.49	−0.20	2.11
π^-	−0.26	−0.05	0.47

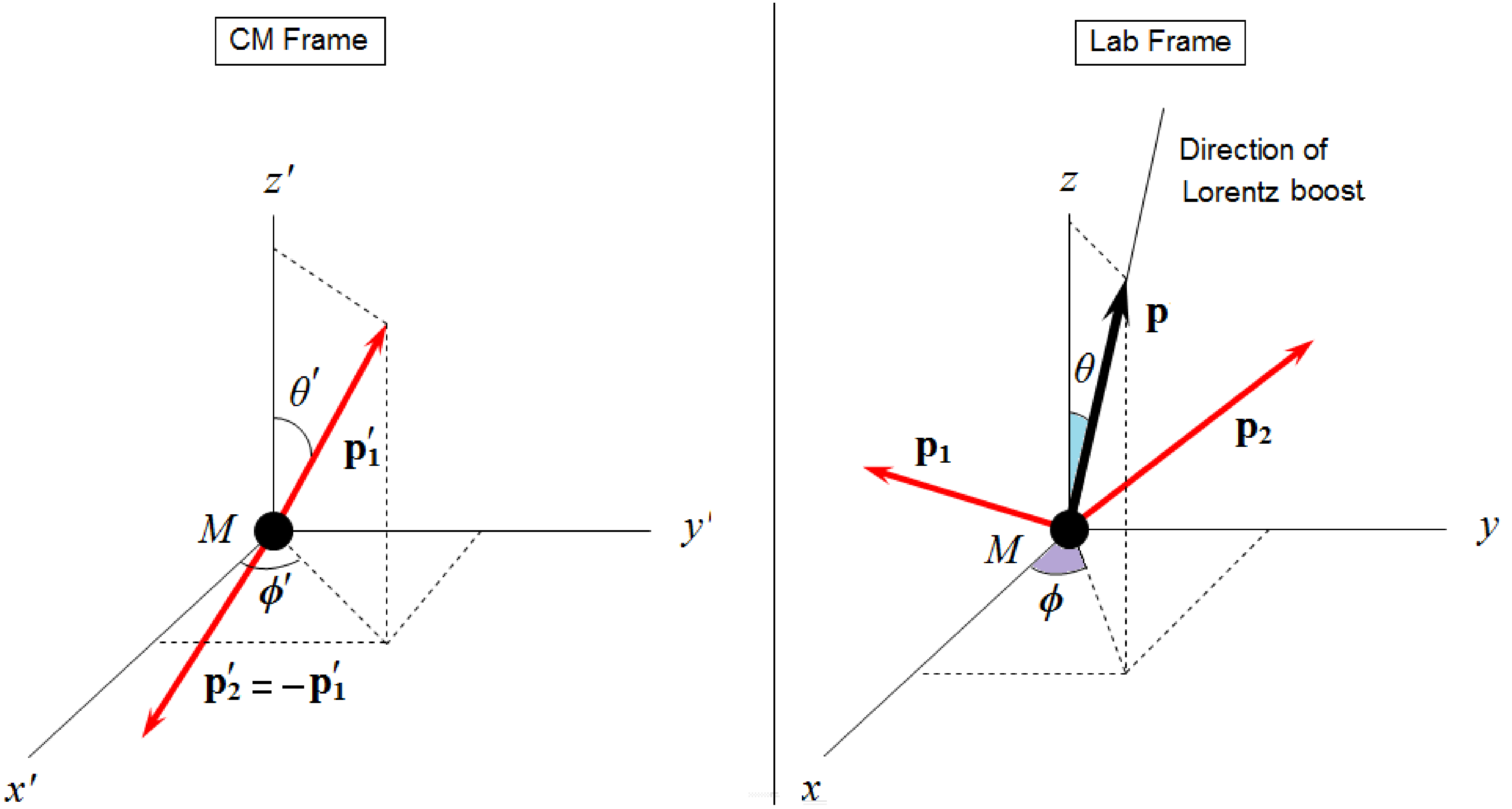
Write a python script or ROOT macro to determine

(a) Identity the particle X
(b) (p_T, η, ϕ) values of the mother (parent) particle
(c) opening angle between decay products
(d) kinetic energy of each doughter
(e) opening angle between decay products in CM frame of the mother particle *X*
(f) kinetic energy of each doughter in CM frame of the mother particle *X*

Two-Body Decays

Consider the decay $M \to m_1 + m_2$.

Usually, we analyse the process in two different frames as follows:



In any frame, 4-vector:

$$p^{\mu} = p_1^{\mu} + p_2^{\mu}$$

its length

$$p^{\mu} p_{\mu} = (p_1^{\mu} + p_2^{\mu})\left(p_{1\mu} + p_{2\mu}\right)$$

or

$$M^2 = m_1^2 + m_2^2 + 2(E_! E_2 - \mathbf{p_1} \cdot \mathbf{p_2})$$

In CM frame, $p = |\mathbf{p_1}| = |\mathbf{p_2}|$ and $\mathbf{p_1} + \mathbf{p_2} = 0$, Hence angle between vectors is 180°.
Solving for p from last eqn and putting into enegy momentum relation, we obtain energies and magnitude of momenta of particles in CM as follows:

$$E_1 = \frac{M^2 + m_1^2 - m_2^2}{2M} \qquad p_1 = \sqrt{E_1^2 + m_1^2}$$

$$E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M} \qquad p_2 = \sqrt{E_2^2 + m_2^2}$$

Example 3.2

For the decay $\pi^0 \rightarrow \gamma + \gamma$, assume that pion moves at momentum p_0 along z-axis.

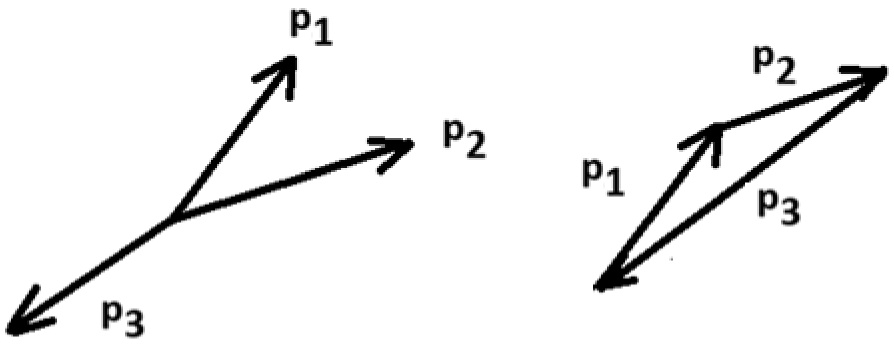
For the given input p_0 value in MeV, write a program to compute

(a) the photon energies in the CM frame and

(b) the maximum and minimum photon energies in lab frame.

Three-Body Decays

Now consider the case of one particle decaying into three particles, $M \to m_1 + m_2 + m_3$.
The energies of the final particles are not predictable even in the rest frame of the decaying particle. energy is shared in different ways among the particles. We can find the maximum and the minimum possible energy that any of the three decay products might possess.
e.g. Assume the decay occurs at rest. Consider the Particle 3.



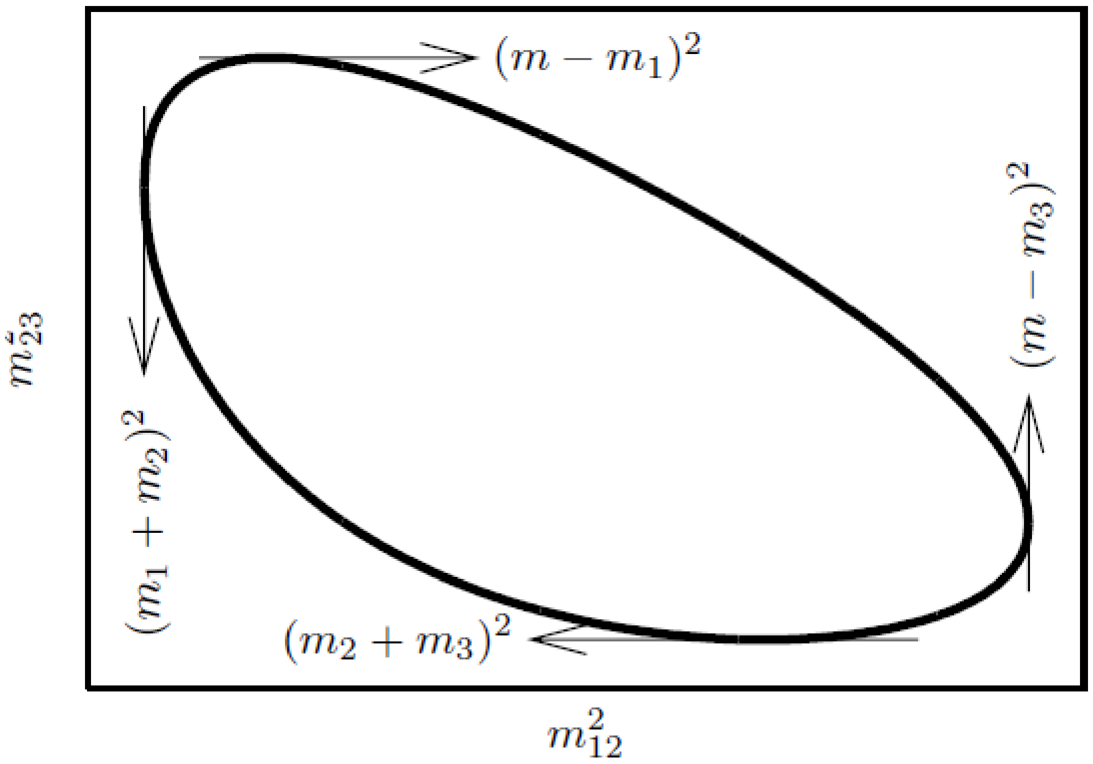
Its minimum energy must be is mass energy (particle 3 produced at rest) namely

$$(E_3)_{\min} = m_3$$

and its maximum energy must be:

$$(E_3)_{\max} = \frac{M^2+m_3^2-(m_1+m_2)^2}{2M}$$

which corresponds the case that particle 1 and 2 parallel to each other and particle 3 has its maximum momentum.



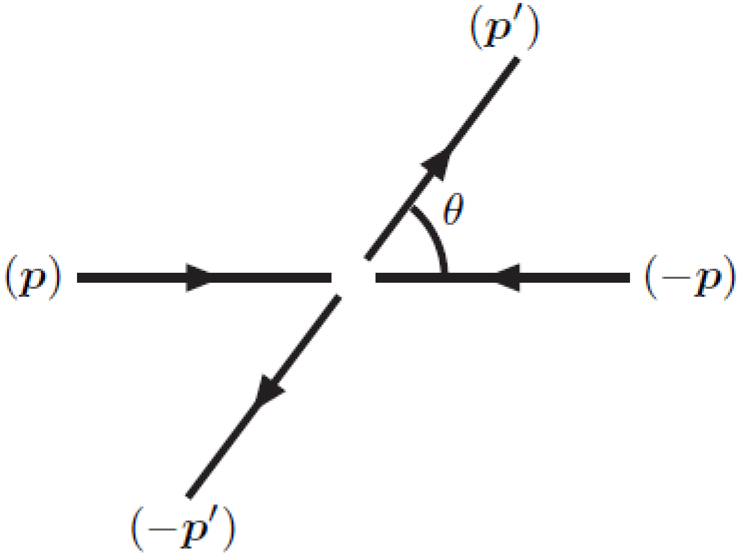
Dalitz Plot represents kinematically allowed region in a three-body decay.

m_{12} is invariant mass of particle 1 and 2.

Kinematics of Scattering

Elastic scattering the final state contains the same particles as in the initial state.

Inelastic scattering the particle contents of the initial state and the final state are not the same.



Consider the process:

$$m_1 + m_2 \rightarrow m'_1 + m'_2$$

We can calculate invariant mass of the system (in any frame):

$$M^2 = (E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2$$

Energies of products in CM frame can be computed via the following equation:

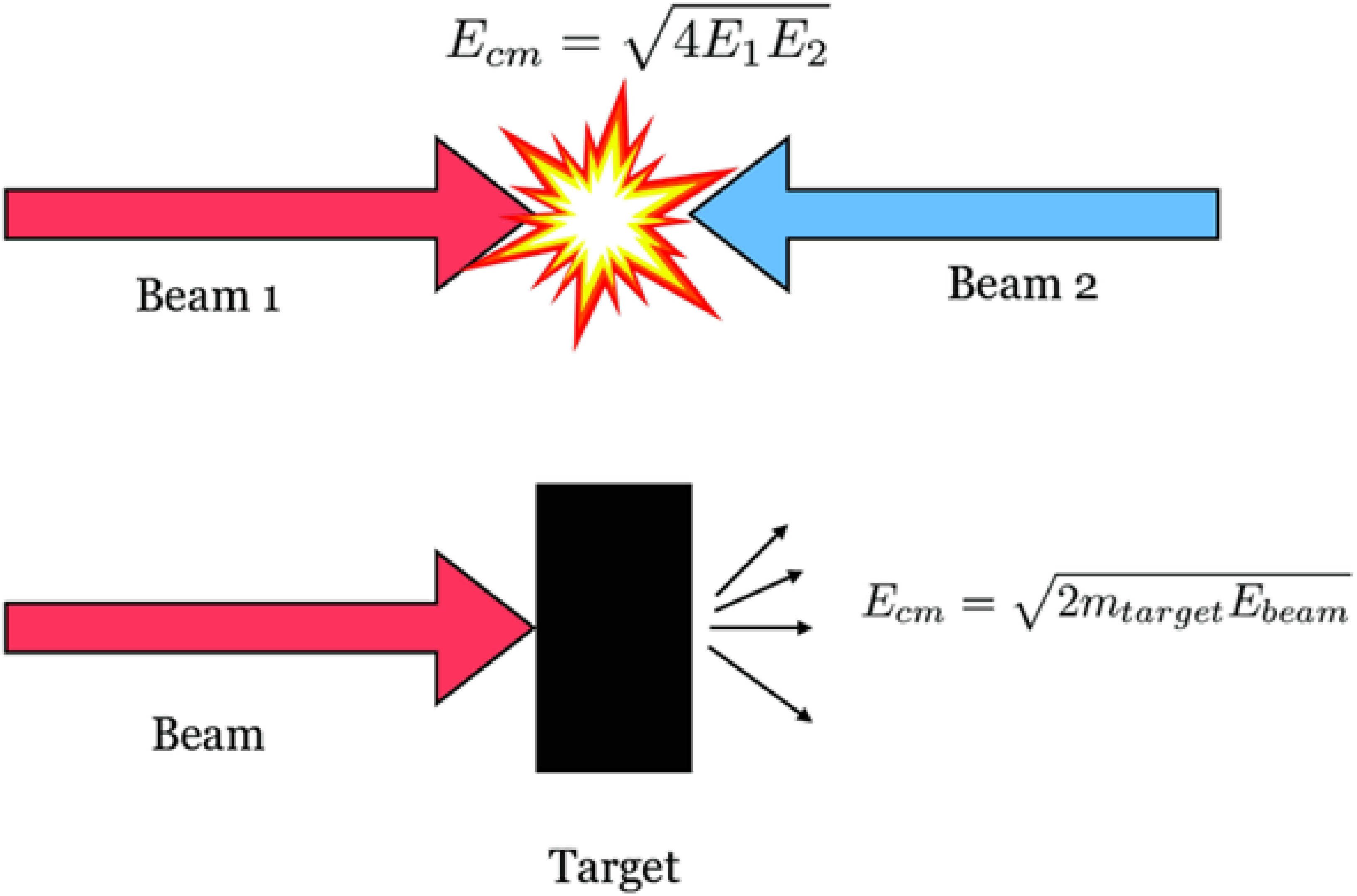
$$E'_1 = \frac{M^2 + m'^2_1 - m'^2_2}{2M} \qquad E_2 = \frac{M^2 - m'^2_1 + m'^2_2}{2M}$$

Fixed Target vs Colliding Beam Experiments

In nuclear and particle physics, most of our knowledge have been obtained either *from the bombardment of the stable target nuclei with energetic incident beam of particles* or *from the head on collision of particles*. There are two cases:

- Fixed target experiments where center of mass energy is $E_{cm} \propto \sqrt{E}$
- Colliding beam experiments where center of mass energy $E_{cm} \propto E$

Here the *center of mass energy* $\sqrt{s} = E_{cm}$ is the energy can be used for generation of new particles.

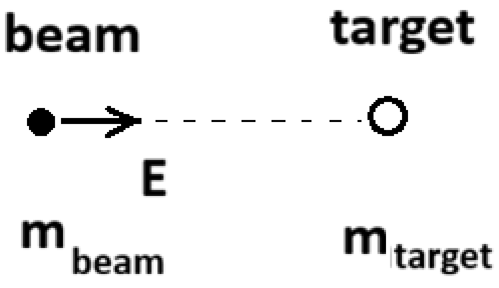


$$E_{cm} = \sqrt{4E_1E_2}$$

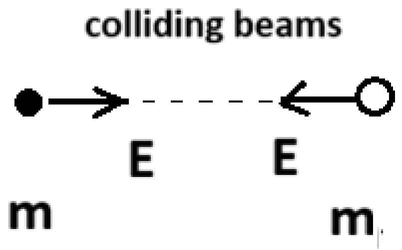
$$E_{cm} = \sqrt{2m_{target}E_{beam}}$$

Example 3.3

(a) Show that the center of mass energy in a Fixed Target Experiment is $E_{cm} \propto \sqrt{E}$ where the target particle is at rest and the projectile particle has energy E. (Hint compute invariant mass in lab and cm frames).



(b) Show that the center of mass energy in a Colliding Beam experiment is $E_{cm} \propto E$ where both particles (each has energy E) are they make head on collision.



Threshold Energy

This is the energy required to start a reaction. Consider the reaction $p + p \rightarrow p + p + p + \bar{p}$ where one of the proton is at rest. Assume that the mass of each proton is m_p and the reaction is a typical fixed target experiment.

Invariant mass for initial protons in lab frame:

$$M^2 = \left(E + m_p\right)^2 - p_p^2$$

At the absolute minimum energy, all four resulting protons are at rest relative to each other. Invariant mass for final protons for which all of them are at rest:

$$M^2 = (E_1 + E_2 + E_3 \ + E_4)^2 = \left(4m_p\right)^2$$

where *E* is the threshold energy. Solving for *E*, we get

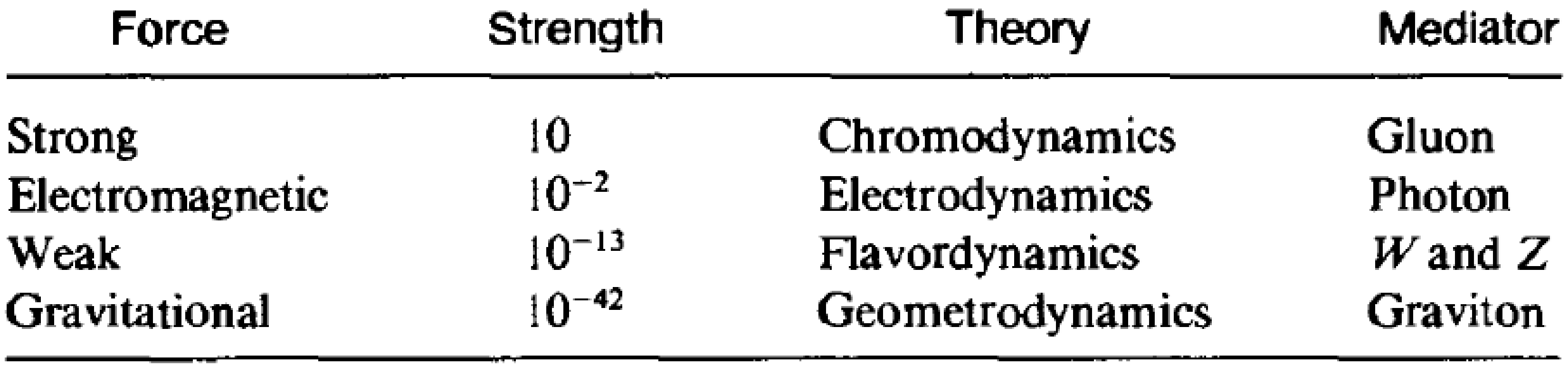
$$E = 7m_p \approx 6.566 \text{ GeV}$$

That is the threshold kinetic energy the incident proton must be at least around 5.628 GeV.

Feynman Diagrams

In this section, we will revise the fundamental forces by which elementary particles interact, and discuss the Feynman diagrams at lowest level used to represent these interactions.
See also: http://hst-archive.web.cern.ch/archiv/HST2002/feynman

As far as we know, there are just four fundamental forces in nature:



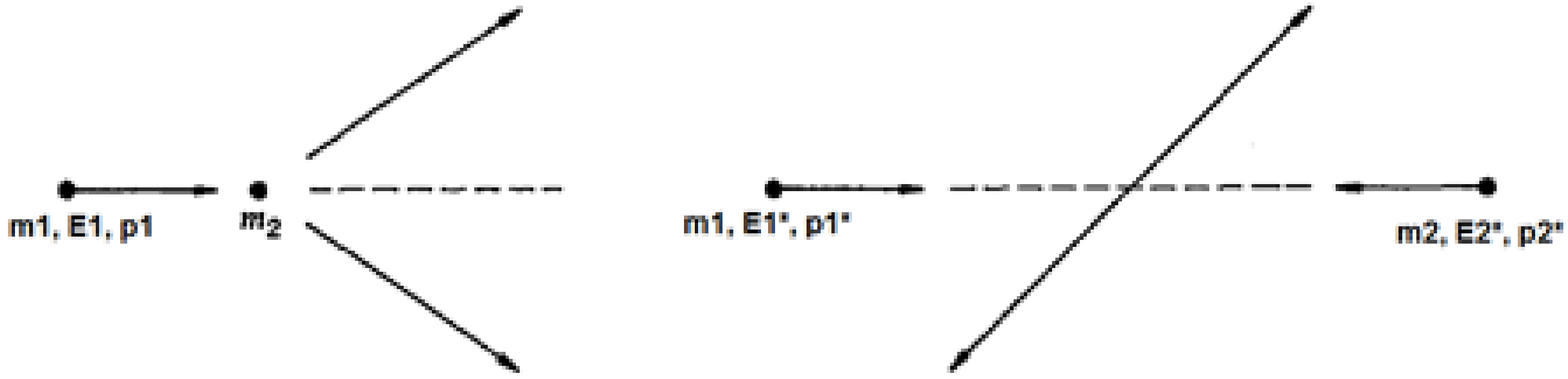
Force	Strength	Theory	Mediator
Strong	10	Chromodynamics	Gluon
Electromagnetic	10^{-2}	Electrodynamics	Photon
Weak	10^{-13}	Flavordynamics	W and Z
Gravitational	10^{-42}	Geometrodynamics	Graviton

GR: gravity is simply too weak to play a significant role in elementary particle physics.
QED: The quantum theory of electrodynamics was perfected in the 1940s.
Weak: Account for nuclear beta decay, decay of pions, muons and many of strange particles.
Strong: Reponsible for the interactions among quarks and and hadrons.

A Feynman diagram is a pictorial representation of the mathematical expressions describing the behavior and interaction of subatomic particles. Consider the scattering process:

$$m_1 + m_2 \rightarrow m_1 + m_2$$

It can be shown in lab and CM frames as follows:



Lab Frame

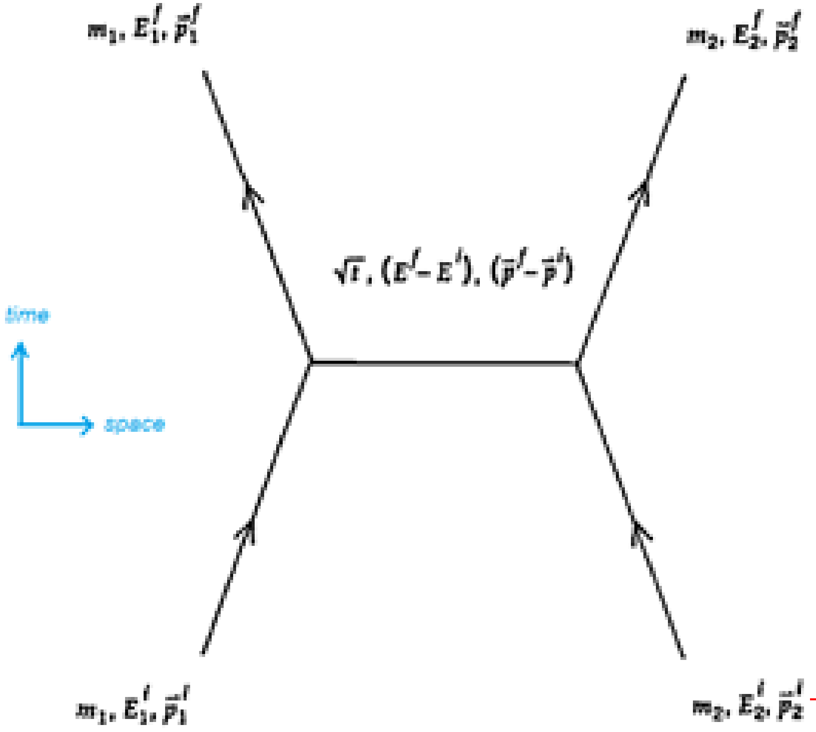
CM Frame

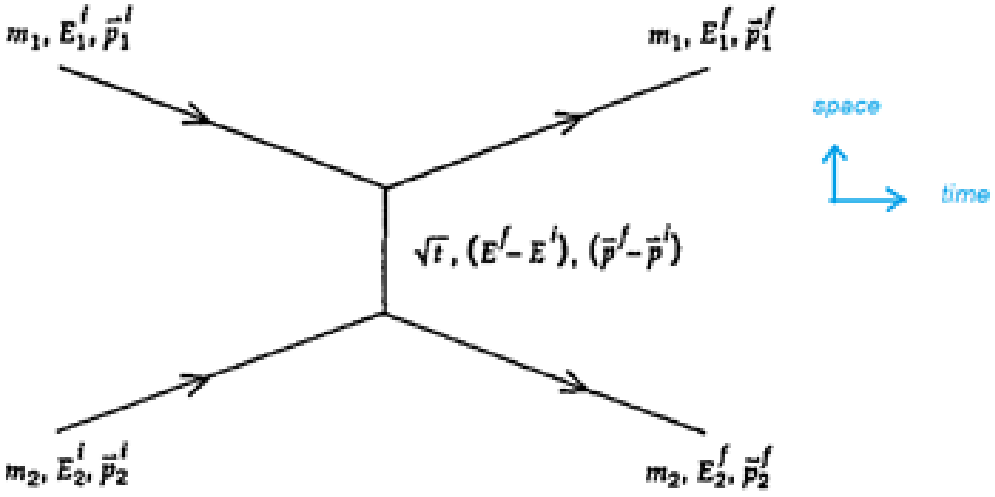
In discussing scattering, it is often convenient to define an invariant called *t*, the square of the **four-momentum transfer** in a collision:

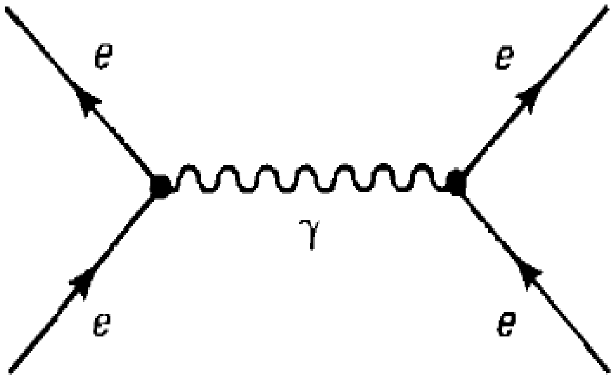
$$t = \left(p_{1\mu}^{f} - p_{1\mu}^{i}\right)^2 = \left(E_1^f - E_1^i\right)^2 - \left(\mathbf{p}_1^f - \mathbf{p}_1^i\right)^2$$

$$t = \left(p_{2\mu}^{f} - p_{2\mu}^{i}\right)^2 = \left(E_2^f - E_2^i\right)^2 - \left(\mathbf{p}_2^f - \mathbf{p}_2^i\right)^2$$

One can think of *t* as the square of the mass of an exchanged particle that mediates the scattering. Consequently, we must conclude that if such an exchange process can be used to describe scattering, then the object being exchanged cannot be physical since it has an imaginary rest mass. This means that although this "virtual" object cannot be detected, if the picture is correct, its consequences can be calculated and observed. Diagrams of the two kinds shown below.



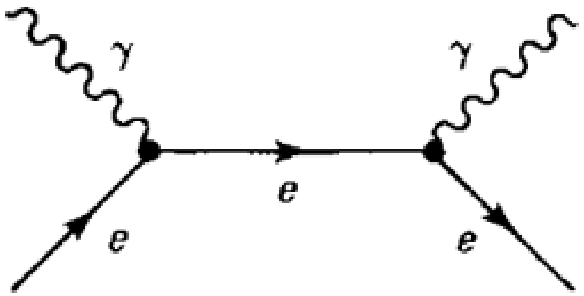




$$e^- + e^- \rightarrow e^- + e^-$$

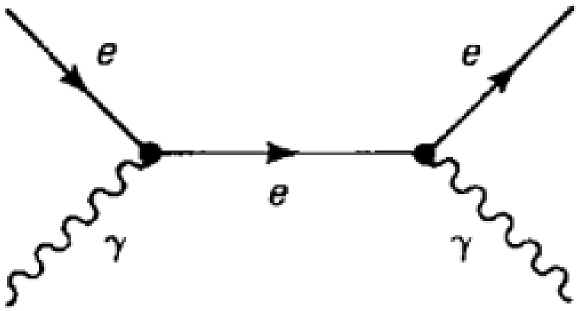
Pair annihilation

$e^- + e^+ \rightarrow \gamma + \gamma$



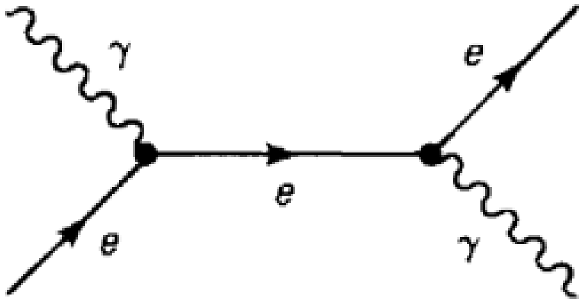
Pair production

$\gamma + \gamma \rightarrow e^- + e^+$

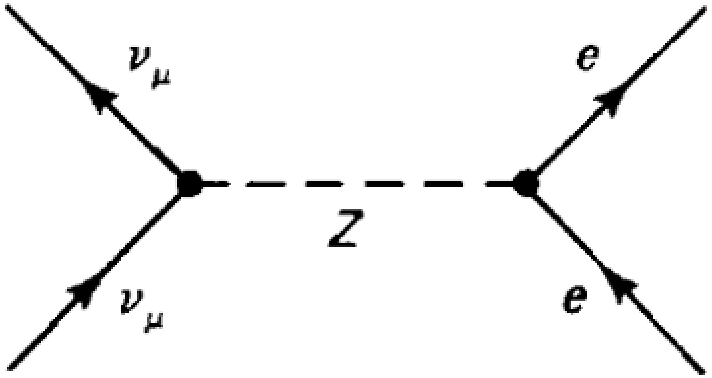


Compton scattering

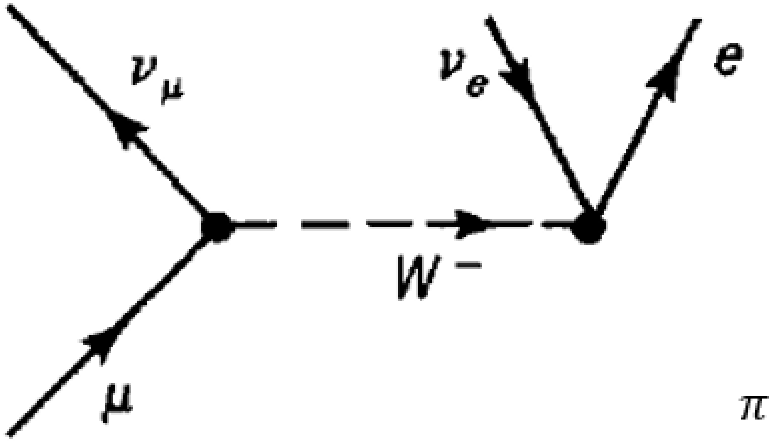
$e^- + \gamma \rightarrow e^- + \gamma$



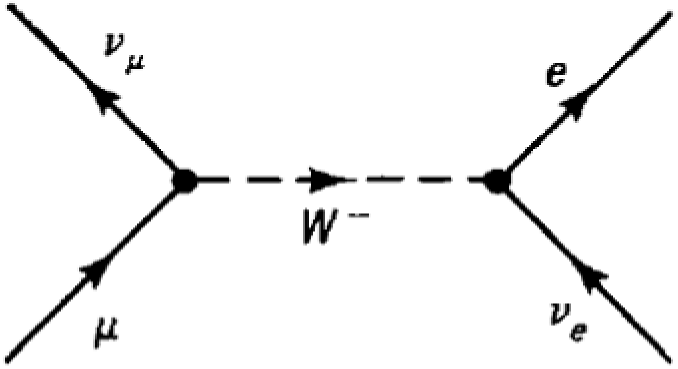
$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$



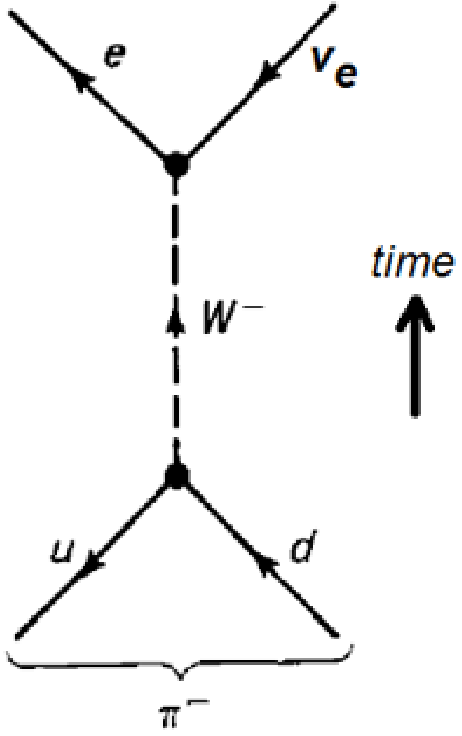
$\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$

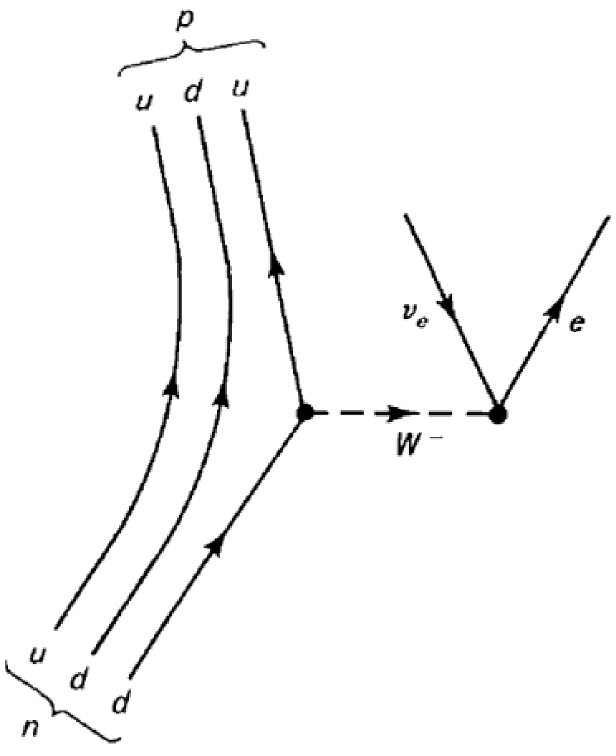


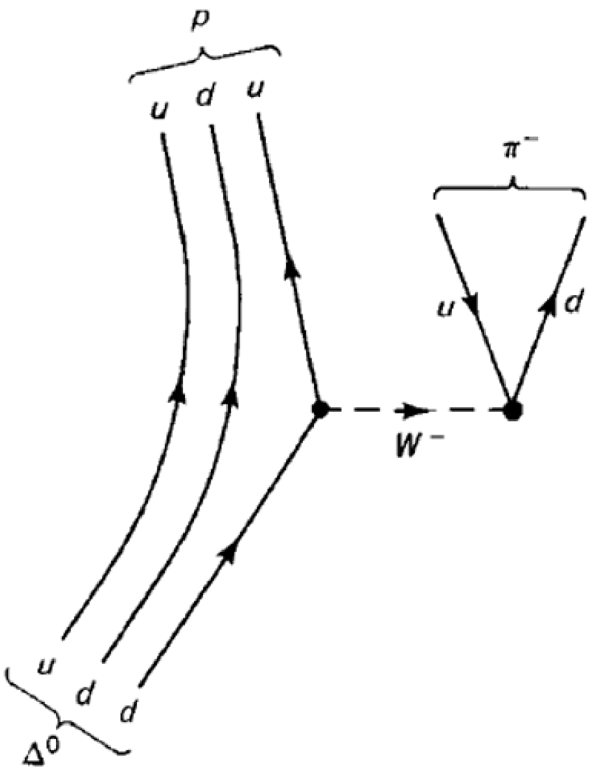
$\mu^- + \nu_e \rightarrow e^- + \nu_\mu$

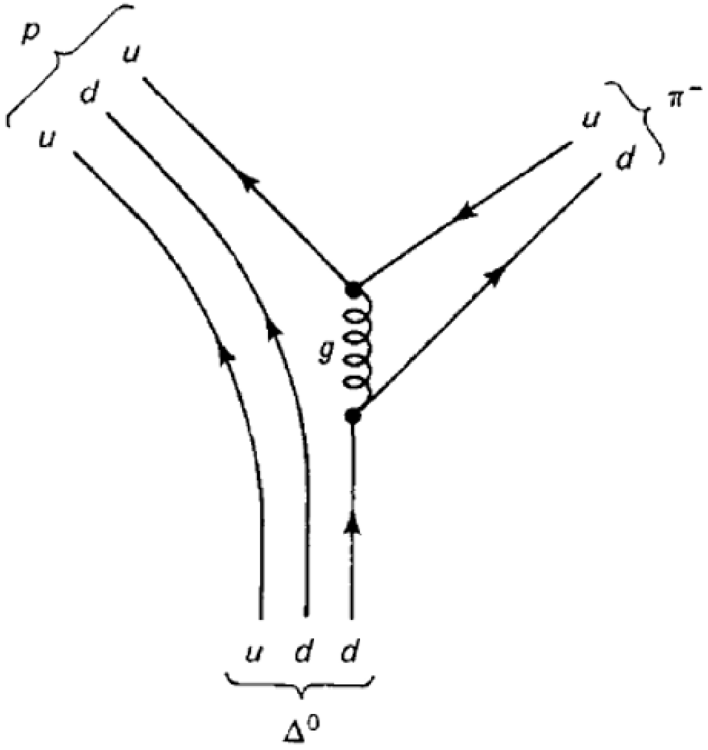


$\pi^- \rightarrow e^- + \nu_e$




p
u d u
ν_e
e
W^-
u
d
d
n


p
u d u
π^-
u
d
W^-
u
d
d
Δ^0


p
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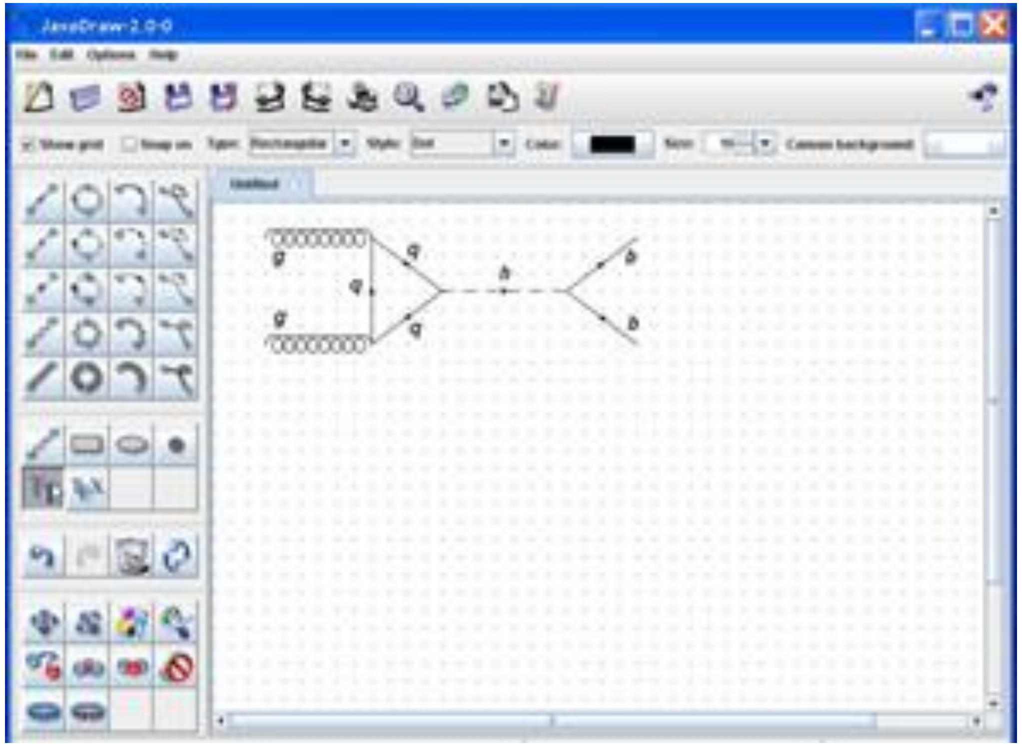
d
u
π^-
u
d
g
u d d
Δ^0

JaxoDraw

In drawing Feynman Diagrams, one can use the program JaxoDraw. It has a complete graphical user interface that allows to carry out all actions in a mouse click-and-drag fashion.

https://jaxodraw.sourceforge.io



Problems

1. Kinetic energy of an electron is 10 MeV. Calculate

(a) total energy of the electron in MeV

(b) the momentum of the electron in MeV

(c) the speed of the electron in m/s

2. Find the threshold energy of the antineutrino in the following reaction:

$$\overline{\nu_e} + p \rightarrow n + e^{+}$$

Additionally, determine what the maximum energy of the positron will be if a solar neutrino with an energy of 2 MeV initiates this reaction.

3. Consider a neutrino originating from the decay $\pi^+ \rightarrow \mu^+ + \nu_\mu$.

(a) What is the speed of the muon if pion is at rest?

(b) What are the minimum and maximum speeds of the muon if pion has momentum of 10 GeV/c?

4. Consider π^0 is flying in x-direction in the decay $\pi^0 \to \gamma + \gamma$ decay.

What is the angle between photons if photon energies are measured as E_1 = 2 GeV and E_2 = 6 GeV?

5. Suppose the production of Z boson at LEP. $e^- + e^+ \rightarrow Z$ where m(Z) = 91.2 GeV.
Find the required total beam energy to generate a Z boson at E_{cm} = 91.2 GeV

(a) in a fixed target experiment [Ans: E = 8 PeV]

(b) in a colliding beam experiment [Ans: E = 45.6 GeV]

6. Consider a reaction where two protons undergo a head-on collision to produce a proton-antiproton pair:

$$p + p \rightarrow p + p + p + \bar{p}$$

Find the threshold kinetic energy of one of the incident protons required to start this reaction.

7. Consider the reaction where m_2 is at rest:

$$m_1 + m_2 \to M_1 + M_2 + \cdots + M_n$$

Find the threshold *kinetic energy* of particle m_1.

8. Consider the decay $M \rightarrow m_1 + m_2$. Assume that (p_T, η, ϕ) values associated to decay products are given. Show that at ultra relativistic limit the invariant mass of the parent particle can be found from:

$$M = \sqrt{2p_{T1}p_{T2}[\cosh(\eta_1 - \eta_2\ ) - \cos(\phi_1 - \phi_2)]}$$

9. Consider the decay $X \rightarrow \mu^+ + \mu^-$. $(p_T, \eta, \phi, \text{charge})$ values associated to 5 muons in an event are given as follows. Determine which (μ^+, μ^-) combination(s) may belong to a $J/\psi \rightarrow \mu^+ + \mu^-$ decay.

Muon	pT (GeV)	eta	phi	ch
1	6.09024	0.618300	0.83360	-1
2	11.57960	0.274272	-2.26349	+1
3	14.65500	0.279672	-2.02579	-1
4	10.01640	-1.027030	1.94910	+1
5	7.43239	0.943696	0.51260	+1

10. Draw lowest order Feynman Diagrams for the following reactions or decays.

$$e^- + \mu^- \rightarrow e^- + \mu^-$$

$$\pi^- \rightarrow \mu^- + \overline{\nu_\mu}$$

$$\mu^- \rightarrow e^- + \overline{\nu_e} + \nu_e$$

$$\Sigma^0 \rightarrow \Delta^0 + \gamma$$

$$\overline{\nu_e} + p \rightarrow n + e^+$$