PHYS 111 5(4-2)
PHYSICS I (MECHANICS)

Chapter 6

Gravitation

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METU, Nov 2024

Content

Tycho Brahe (1546 – 1601)

- Danish Astronomer who made observations on Planets and Stars.
- He studied law for a while until 1565.

- In 1571 he discovered a bright star in the constellation Cassiopeia and made precise measurements of its position. This star gradually faded away after weeks and gradually disappeared from the sky (is this a supernova?)

- The Danish king gave him a fully equipped observatory to continue his work. Location: Ven island in the Baltic sea.

- On the island, using a Sextant (protractor), he measured the positions of 777 stars and planets and prepared a catalogue. There is no error of more than 1-2 minutes in its measurements.

- Johannes Kepler served as Brahe's assistant for a long time before his death. When Brahe died, he left his entire inheritance to him.


Ven
Adası
Frederikshavn
Vendsyssel-Thy
Brønderslev
Aalborg
Thisted
Hobro
Skive
Viborg
Randers
Grenå
Holstebro
Herning
Silkeborg
Aarhus
Horsens
Grindsted
Vejle
Danmark
(Denmark)
Kolding
Odense
Fyn
Haderslev
Svendborg
Åbenrå
Sylt
Kalundborg
Roskilde
Sjælland
Slagelse
Køge
Næstved
Hillerød
Helsingborg
København
Landskrona
Lund
Malmö
Trelleborg
Ystad
Halmstad
Ängelholm
Sweden
Varberg
Falkenberg
Värnamo
Sävsjö
Ljungby
Växjö
Kristianstad
Karlshamn
Ronneby
Rønne
E6
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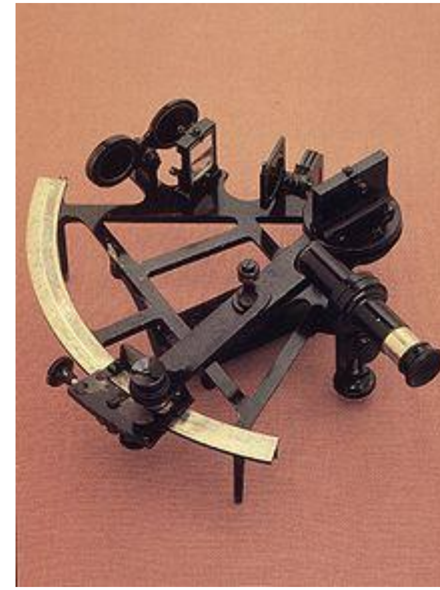
E45
E4
E22
E55



TYCHONIS BRAHE DANI
SCRIPTA ASTRONOMICA

EDIDIT

I. L. E. DREYER

AUXILIO IOANNIS RÆDER

SUMPTUS FECIT G. A. HAGEMANN

TOMUS II

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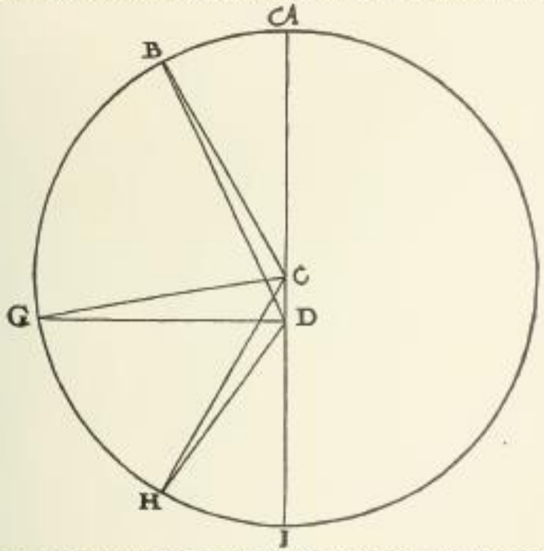
ſecundis, ſed hic paulo limitatiorem, & in pauculis ſcrupulis ſecundis ſubtiliorem exactioremque Poli Altitudinem applicui. Sic etiam Declinatione maxima tum contentus eram P. 23. M. 31 præciſe, quæ nunc exactius aßumitur, dimidio videlicet ſcrupulo maior. Atque ex his con- † currentibus cauſis per ſe quidem non adeo euidentibus, ea tantillula diuerſitas in Apogæo & Eccentricitate ad Annum 1583, tacite ſeſe inſinuabat, quæ tamen inſenſibiliter Solis motum alterare potuit.

Atque iſta hoc loco indicanda duxi, vt illi, qui Ephemerides noſtras ☉ iam antea obtinuerant, quas e conſtitutione Apogæi & Eccentricitatis anno 1583 inuenta ſupputauimus, rationes etiam cur hæc vltima reſtitutio in ſcrupulis ſecundis motum Solis non vbique ſimilem priori exhibeat, cognitas haberent: vtque vna conſtare poßet, vnde Libro ſequente Eccentricitatem ipſius in Hypotheſi Cometæ exponenda Capite Octauo aßumſerimus part. 360, qualium ſemidiameter Eccentrici 10000, ſiquidem is numerus quam proxime accedit ad inuentionem modo indicatam, quemadmodum Apogæum etiam ibidem paulo vlterius promotum inuenitur, quod etiam hiſce alijs Obſeruationibus quamproxime fundatur, anticipatione tamen aliquantula conſulto facta. Atque hæc de his monuiße ſufficiat. ‖

EX hac adinuenta Eccentricitatis quantitate Proſthaphæreſes motus 29
Solis ad ſingulos gradus Eccentrici deduxi, vt conſtare poßet, quantum addendum demendumue ſit medio & æquali motui per totum Orbem curriculi Solaris, vtque eius locus verus e Terra, Centro vniuerſi, prodiens, innoteſceret. Ratio autem qua Proſthaphæreſes illæ conquiſitæ ſunt, in hunc modum ſe habet.

Intelligatur in appoſito Schemate Eccentricus Solis per Circulum AGI, cuius Centrum ſit C, Centrum vero Mundi ſiue Terra D, per quæ duo loca ducatur linea, vſque dum contingat vtrinque Circuli circumferentiam, quæ ſit ACDI, manifeſtum eſt quod CD ſit Eccentricitas, quæ noſtro æuo (vt ‖ oſtendimus) reperta eſt partium 3584, qualium AC Semidiameter aßumitur 100000: locus Solis intelligatur vel in B, vel G aut H, nam cum eſt in A aut I, Apogæo videlicet & Perigæo, nullam prorſus admittit Proſthaphæreſin, coincidente tunc loco ſimplici cum vero. Ducantur autem ex his tribus intermedijs locis binæ lineæ a B & G & H, ſingulæ ad C, centrum Eccentrici & D centrum Terræ; Jamque per hæc tria loca Proſthaphæreſin ſcrutari lubeat.

Sit primum Sol in B, diſtans ab Apogæo part. 30, quam intercapedinem metitur Angulus BCA, erit itaque in Triangulo BCD Angulus, qui ad C notus, eſt enim prioris complementum ad duos rectos, vt conſtat e tyrocinijs Geometricis, eſtque hoc loco 150: bina vero latera ipſum Angulum continentia etiam nota ſunt, BC Semidiameter Eccentrici quæ perpetuo aßumitur 10000000, & DC, Eccentricitas prius inquiſita 358400 (tantam enim vbique retinere placuit). Ergo per data duo latera Angulum notum ambientia, non latebit Scientiæ Triangulorum planorum gnaro Angulus DBC, qui menſurat ipſam Proſthaphæreſin, differentiam videlicet qua locus Solis aliter videtur ex C quam D. Inueni autem hanc peracta operatione part. 0. min. 59. ſec. 44, quæ oſtendit quantum demendum ſit æquali motui, vt verus Solis locus pateat.



Pari modo Sole iuxta H conſtituto in diſtantia ab Apogæo 150 partium, erit in Triangulo CHD, Angulus HCD tanquam complementum antedicti, part. 30. Latera vero ambientia vbique manent eadem, vt prius HC 10000000, & CD eorundem 358400, ergo dabitur Angulus
31 CHD, part. 1. /. 3. //. 33, qui deſignat Proſthaphæreſin ad diſtanti-‖am ab Apogæo 150 partium, ſiue (quod idem eſt) 5 Signorum 0 grad. quæ adhuc ablatiua eſt. Nam in toto priori Semicirculo ab A, per G in I ſit ſubtractio Proſthaphæreſis a ſimplici motu, eo quod illic locus Solis verus antecedat medium, in reliquo vero requiritur additio ob contrariam rationem.

Maxima autem Proſthaphæreſis quæ contingere poteſt, eſt iuxta G, in eo videlicet Eccentrici loco, vbi linea a D, Centro Mundi producta in

ÆQVALIS MOTVS LONGITVDINIS SOLIS

IN MENSIBVS				
ANNI COMMVNIS	S.	G.	′	″
Ianuarius	1	0	33	18
Februarius	1	28	9	11
Martius	2	28	42	30
Aprilis	3	28	16	39
Maius	4	28	49	58
Iunius	5	28	24	7
Iulius	6	28	57	26
Auguſtus	7	29	30	44
September	8	29	4	54
October	9	29	38	12
Nouember	10	29	12	22
December	11	29	45	40

IN MENSIBVS				
ANNI BISEXTILIS	S.	G.	′	″
Ianuarius	1	0	33	18
Februarius	1	29	8	20
Martius	2	29	41	38
Aprilis	3	29	15	48
Maius	4	29	49	6
Iunius	5	29	23	16
Iulius	6	29	56	34
Auguſtus	8	0	29	53
September	9	0	4	3
October	10	0	37	21
Nouember	11	0	11	31
December	0	0	44	49

IN DIEBVS				IN HORIS			IN MINVTIS					
D.	G.	M.	S.	H.	M.	S.	M.	M.	S.	M.	M.	S.
1	0	59	8	1	2	28	0	0	0	30	1	14
2	1	58	17	2	4	56	1	0	2	31	1	16
3	2	57	25	3	7	24	2	0	5	32	1	19
4	3	56	33	4	9	51	3	0	7	33	1	21
5	4	55	42	5	12	19	4	0	10	34	1	24
6	5	54	50	6	14	47	5	0	12	35	1	26
7	6	53	58	7	17	15	6	0	15	36	1	29
8	7	53	7	8	19	34	7	0	17	37	1	31
9	8	52	15	9	22	11	8	0	20	38	1	34
10	9	51	23	10	24	38	9	0	22	39	1	36
11	10	50	32	11	27	6	10	0	25	40	1	39
12	11	49	40	12	29	34	11	0	27	41	1	41
13	12	48	48	13	32	2	12	0	30	42	1	43
14	13	47	57	14	34	30	13	0	32	43	1	46
15	14	47	5	15	36	58	14	0	34	44	1	48
16	15	46	13	16	39	26	15	0	37	45	1	51
17	16	45	21	17	41	53	16	0	39	46	1	53
18	17	44	30	18	44	21	17	0	42	47	1	56
19	18	43	38	19	46	49	18	0	44	48	1	58
20	19	42	47	20	49	17	19	0	47	49	2	1
21	20	41	55	21	51	45	20	0	49	50	2	3
22	21	41	3	22	54	13	21	0	52	51	2	6
23	22	40	12	23	56	40	22	0	54	52	2	8
24	23	39	20	24	59	8	23	0	57	53	2	11
25	24	38	28				24	0	59	54	2	13
26	25	37	37				25	1	2	55	2	15
27	26	36	45				26	1	4	56	2	18
28	27	35	53				27	1	7	57	2	20
29	28	35	1				28	1	9	58	2	23
30	29	34	10				29	1	11	59	2	25
31	30	33	18				30	1	14	60	2	28
IN DIEBVS				IN HORIS			IN MINVTIS					

TABVLA

TABVLA PROSTHAPHÆRESIVM SOLARIVM
NOSTRIS TEMPORIBVS ACCOMMODATA

	0 Subt.			Diff. A.		1 Subt.			Diff. A.		2 Subt.			Diff. A.		
G.	P.	′	″	′	″	P.	′	″			P.	′	″			G.
0	0	0	0	2	5	0	59	44	1	49	1	44	47	1	6	30
1	0	2	5	2	5	1	1	33	1	48	1	45	53	1	4	29
2	0	4	10	2	4	1	3	21	1	47	1	46	57	1	2	28
3	0	6	14	2	4	1	5	8	1	46	1	47	59	1	0	27
4	0	8	18	2	4	1	6	54	1	44	1	48	59	0	58	26
5	0	10	22	2	4	1	8	38	1	43	1	49	57	0	56	25
6	0	12	26	2	4	1	10	21	1	42	1	50	53	0	54	24
7	0	14	30	2	3	1	12	3	1	41	1	51	47	0	52	23
8	0	16	33	2	3	1	13	44	1	40	1	52	39	0	51	22
9	0	18	36	2	3	1	15	24	1	39	1	53	30	0	49	21
10	0	20	39	2	2	1	17	3	1	37	1	54	19	0	48	20
11	0	22	41	2	2	1	18	40	1	36	1	55	7	0	45	19
12	0	24	43	2	2	1	20	16	1	35	1	55	52	0	42	18
13	0	26	45	2	2	1	21	51	1	33	1	56	34	0	40	17
14	0	28	47	2	1	1	23	24	1	32	1	57	14	0	38	16
15	0	30	48	2	1	1	24	56	1	31	1	57	52	0	36	15
16	0	32	49	2	0	1	26	27	1	29	1	58	28	0	34	14
17	0	34	49	1	59	1	27	56	1	27	1	59	2	0	32	13
18	0	36	48	1	59	1	29	23	1	26	1	59	34	0	30	12
19	0	38	47	1	58	1	30	49	1	24	2	0	4	0	28	11
20	0	40	45	1	58	1	32	13	1	23	2	0	32	0	26	10
21	0	42	43	1	57	1	33	36	1	22	2	0	58	0	24	9
22	0	44	40	1	56	1	34	58	1	20	2	1	22	0	21	8
23	0	46	36	1	55	1	36	18	1	18	2	1	43	0	19	7
24	0	48	31	1	54	1	37	36	1	16	2	2	2	0	17	6
25	0	50	25	1	53	1	38	52	1	14	2	2	19	0	14	5
26	0	52	18	1	53	1	40	6	1	12	2	2	33	0	12	4
27	0	54	11	1	52	1	41	18	1	11	2	2	45	0	10	3
28	0	56	3	1	51	1	42	29	1	9	2	2	55	0	8	2
29	0	57	54	1	50	1	43	38	1	8	2	3	3	0	6	1
30	0	59	44			1	44	46			2	3	9			0
G.	11 Adde			Diff. S.		10 Adde			Diff. S.		9 Adde			Diff. S.		G.

7 PRÆSVPPONENS

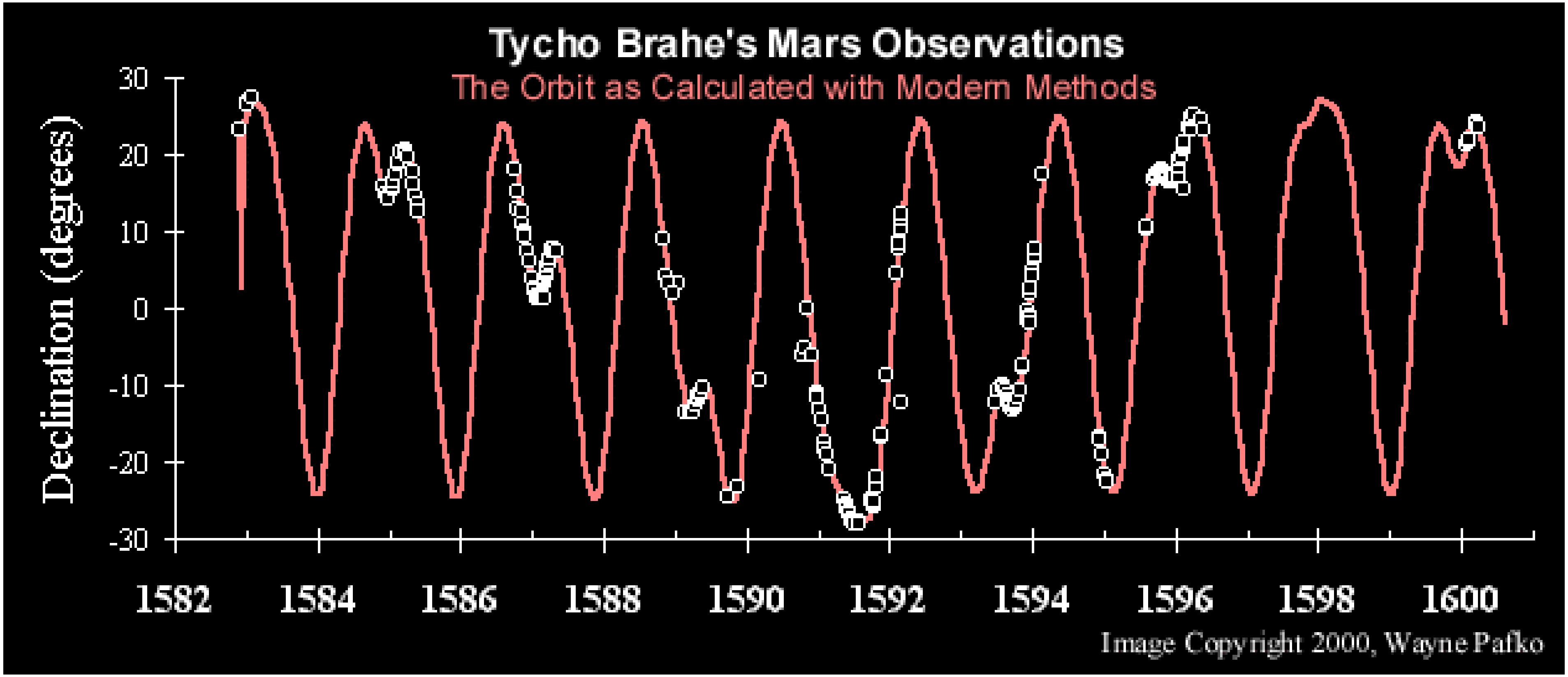
Tycho Brahe's Mars Observations

Source: Tychonis Brahe Dani Opera Omnia

Input by: Wayne Pafko (March 24, 2000)

[MS] = Mars Symbol (you know...the "male" sign)

Year	Month	Day	*Day (adj)*	Hour	Min	*Days since 1 AD*	*Date*		Dec (deg)	Dec (min)	Dec (sec)	Declination
1582	11	12	*22*			*578150.50*	*1582.89*	1.00	23	7		23.12
1582	12	30	*40*			*578198.50*	*1583.02*	1.00	26	56		26.93
1582	12	27	*37*			*578195.50*	*1583.01*	1.00	26	22	20	26.37
1583	1	18	*28*			*578217.50*	*1583.07*	1.00	27	18		27.30
1584	11	13	*23*	13	26	*578883.06*	*1584.90*	1.00	15	54		15.90
1584	11	27	*37*	2	15	*578896.59*	*1584.93*	1.00	14	42		14.70
1584	12	20	*30*			*578919.50*	*1585.00*	1.00	14	24		14.40
1584	12	21	*31*			*578920.50*	*1585.00*	1.00	14	21	30	14.36
1584	12	21	*31*			*578920.50*	*1585.00*	1.00	14	21	15	14.35
1585	1	7	*17*			*578937.50*	*1585.04*	1.00	15	35		15.58
1585	1	9	*19*			*578939.50*	*1585.05*	1.00	15	50		15.83
1585	1	14	*24*	16	40	*578945.19*	*1585.07*	1.00	16	27		16.45
1585	1	22	*32*	14	55	*578953.12*	*1585.09*	1.00	17	31		17.52
1585	1	31	*41*			*578961.50*	*1585.11*	1.00	18	43		18.72
1585	2	3	*13*	9	43	*578964.90*	*1585.12*	1.00	19	1	10	19.02
1585	2	3	*13*	9	39	*578964.90*	*1585.12*	1.00	19	3		19.05
1585	2	3	*13*	6	15	*578964.76*	*1585.12*	1.00	19	2		19.03
1585	2	4	*14*	9	14	*578965.88*	*1585.12*	1.00	19	9	45	19.16
1585	2	4	*14*	8	16	*578965.84*	*1585.12*	1.00	19	8		19.13
1585	2	4	*14*	6	40	*578965.78*	*1585.12*	1.00	19	9	45	19.16
1585	2	17	*27*	9	45	*578978.91*	*1585.16*	1.00	20	21	45	20.36
1585	2	17	*27*	9	30	*578978.90*	*1585.16*	1.00	20	21	30	20.36
1585	2	17	*27*	9	50	*578978.91*	*1585.16*	1.00	20	21	30	20.36
1585	3	12	*22*	9	20	*579001.89*	*1585.22*	1.00	20	32	45	20.55
1585	3	16	*26*	7	50	*579005.83*	*1585.23*	1.00	20	23		20.38
1585	3	19	*29*	8	15	*579008.84*	*1585.24*	1.00	20	5	30	20.09
1585	3	26	*36*	8	20	*579015.85*	*1585.26*	1.00	19	44	0	19.73
1585	4	15	*25*	9	48	*579035.91*	*1585.31*	1.00	17	38	40	17.64
1585	4	15	*25*	9	50	*579035.91*	*1585.31*	1.00	17	38	45	17.65
1585	4	26	*36*	9	50	*579046.91*	*1585.34*	1.00	16	8	30	16.14
1585	5	7	*17*	11	24.5	*579057.98*	*1585.37*	1.00	14	22	30	14.38
1585	5	7	*17*	9	20	*579057.89*	*1585.37*	1.00	14	22		14.37
1585	5	7	*17*	11	15	*579057.97*	*1585.37*	1.00	14	22	30	14.38
1585	5	12	*22*			*579062.50*	*1585.39*	1.00	13	30	15	13.50
1585	5	17	*27*	11	30	*579067.98*	*1585.40*	1.00	12	38	30	12.64
1585	5	18	*28*	10	40	*579068.94*	*1585.40*	1.00	12	27		12.45



http://www.pafko.com/tycho

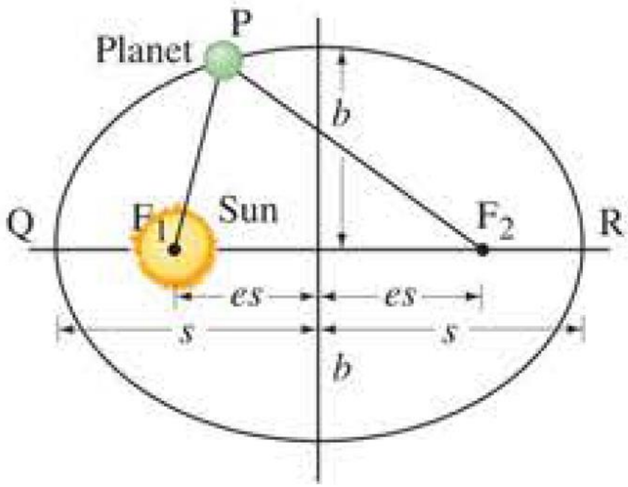
Johannes Kepler (1571 - 1630)

- German astronomer, physicist, and mathematician.
- He was a professor of mathematics in Graz.
- He joined the famous astronomers with the planetary system he explained in his work Mysterium Cosmographicum (Mysteries of the Universe, 1596)
- He started working with Tycho Brahe in 1598.
- He made studies on the **movements of the planets** from astronomical observations compiled by Tycho Brahe.
- He changed the face of astronomy with his great contributions.

Kepler’s Laws of Planetary Motion

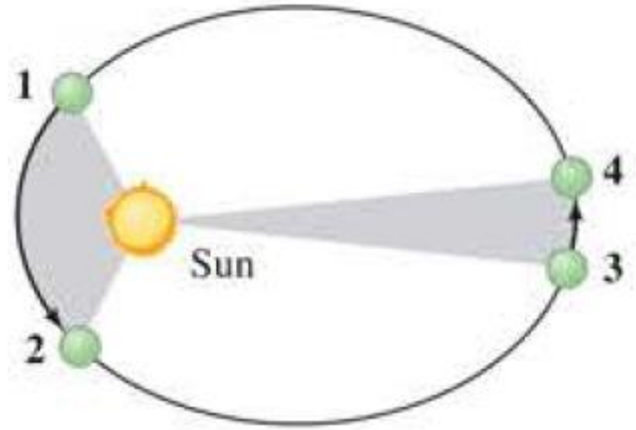
Kepler's first law:

The path of each planet about the Sun is an ellipse with the Sun at one focus.



Kepler's second law:

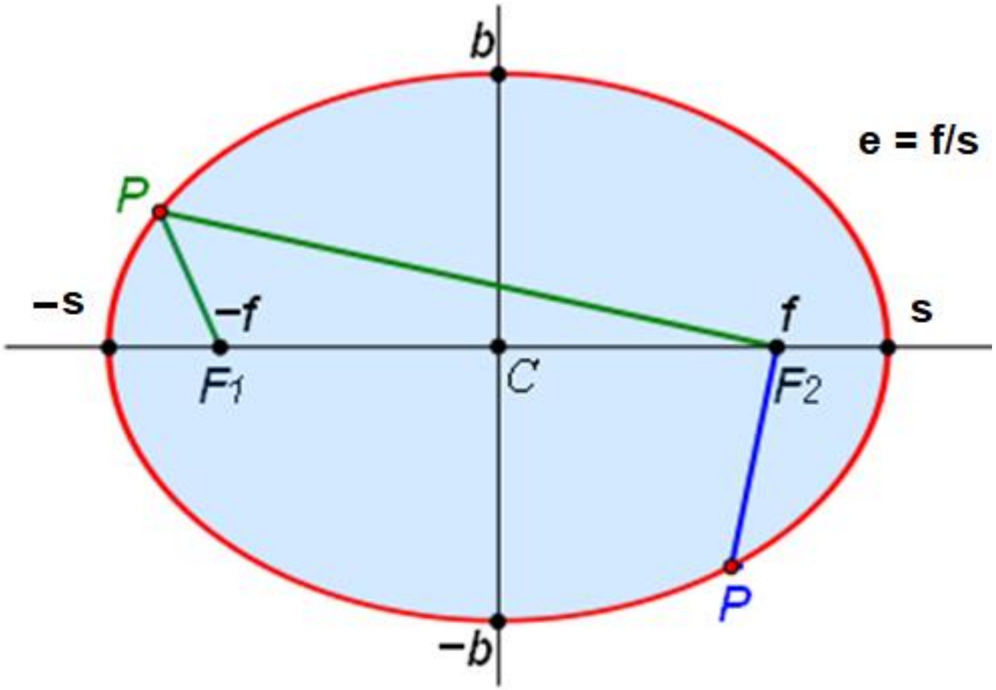
Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal periods of time.



Kepler's third law:

The ratio of the squares of the period a planet to cube of its semimajor axis is a constant (same for all planets)

$$\frac{T^2}{s^3} \equiv constant$$



T = period of planet

s = semimajor axes

f = one of the focus of ellipse

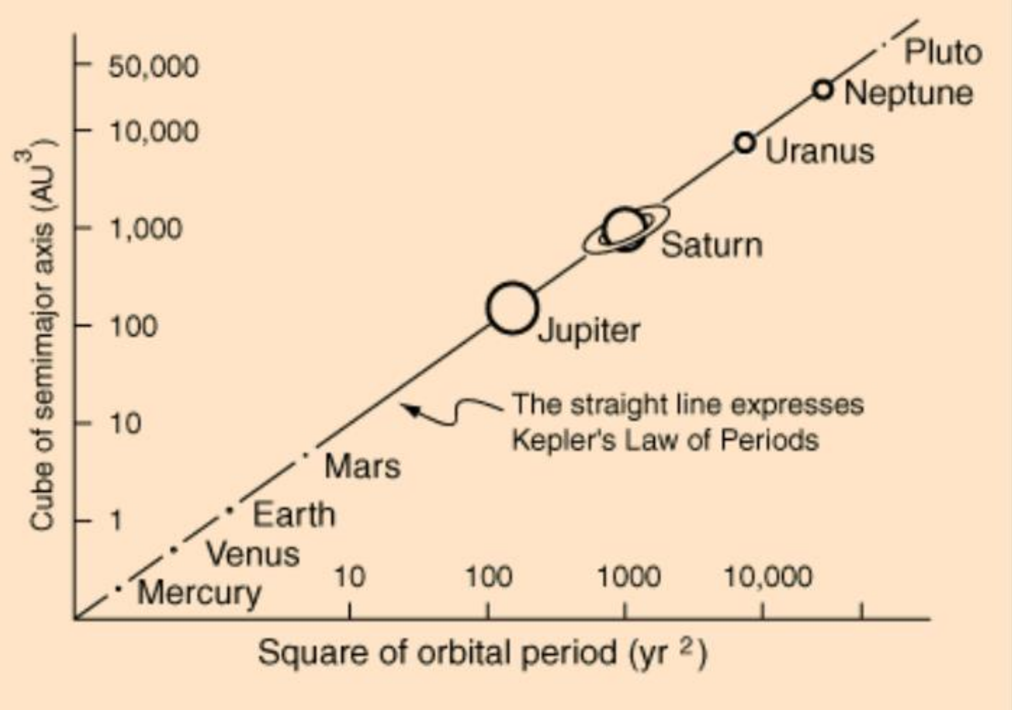
$e = f / s$ = eccentricity

(e = 0 for circle and e = 0.0167 for Earth)

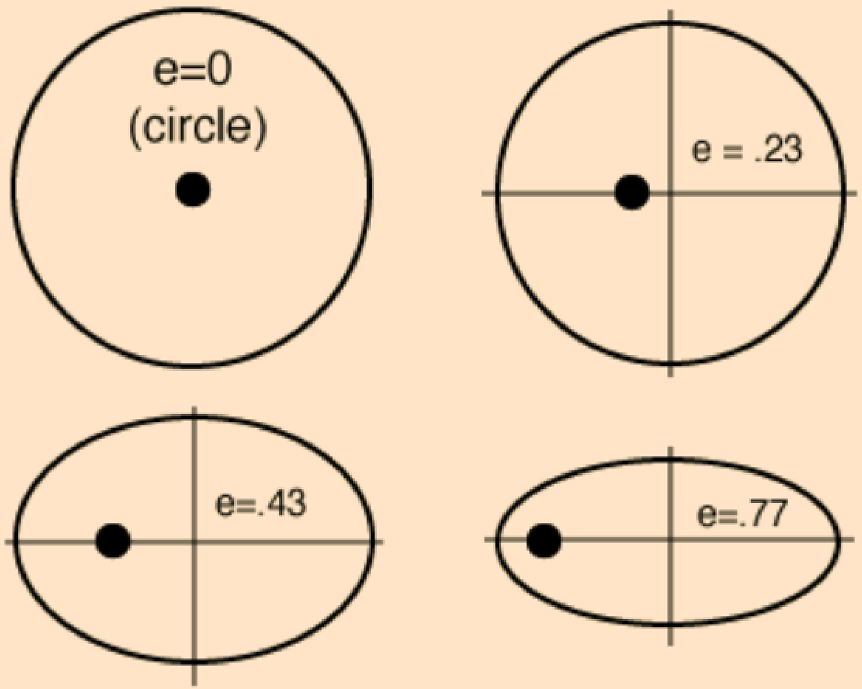
Planet	Semimajor axis (10^{10}m)	Period T (y)	T^2/a^3 ($10^{-34}y^2/m^3$)
Mercury	5.79	0.241	2.99
Venus	10.8	0.615	3.00
Earth	15.0	1	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84	2.98
Neptune	450	165	2.99
Pluto	590	248	2.99

Halliday, Resnick, Walker, Fundamentals of Physics 4th I

$$\frac{T^2}{s^3} \equiv constant$$



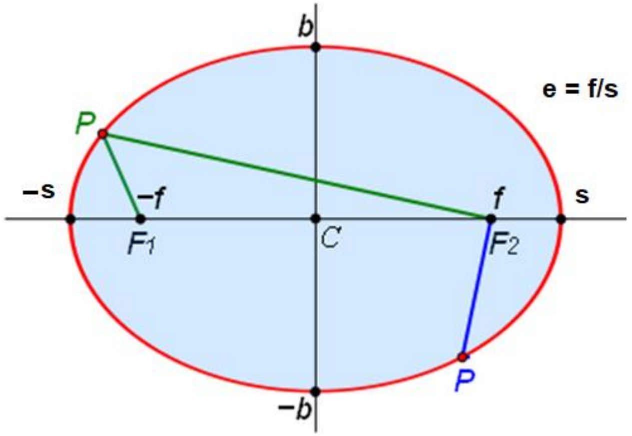
Examples of Ellipse Eccentricity



Planetary orbit eccentricities

Mercury	0.206
Venus	0.0068
Earth	0.0167
Mars	0.0934
Jupiter	0.0485
Saturn	0.0556
Uranus	0.0472
Neptune	0.0086
Pluto	0.25

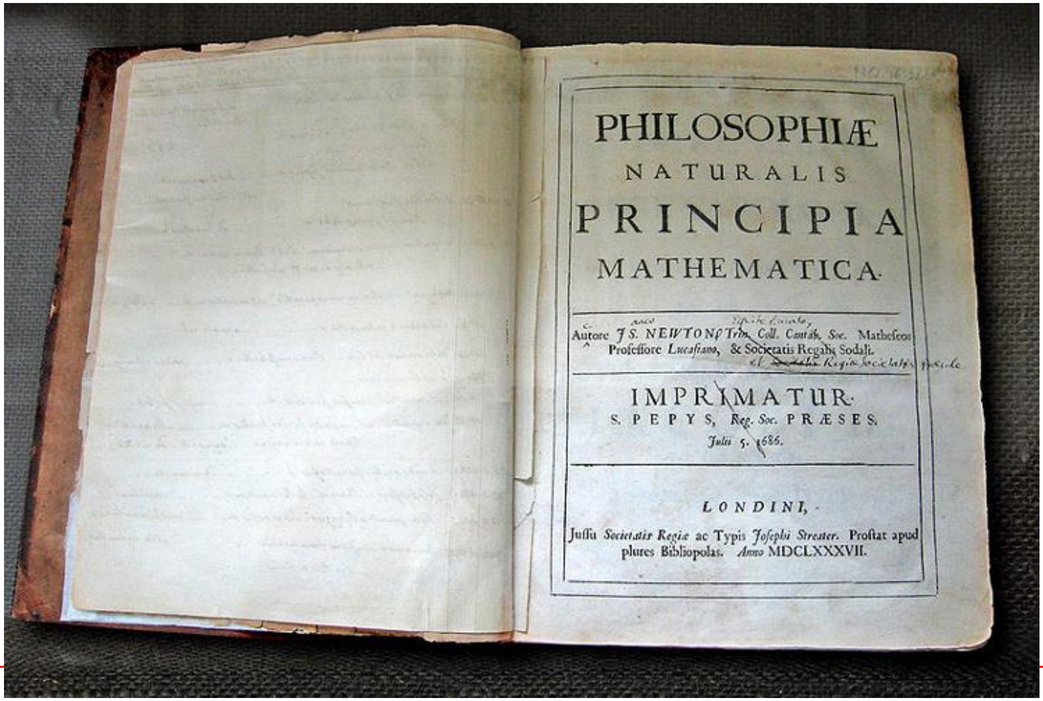
$$e = \frac{f}{s}$$



Newton's Law of Universal Gravitation

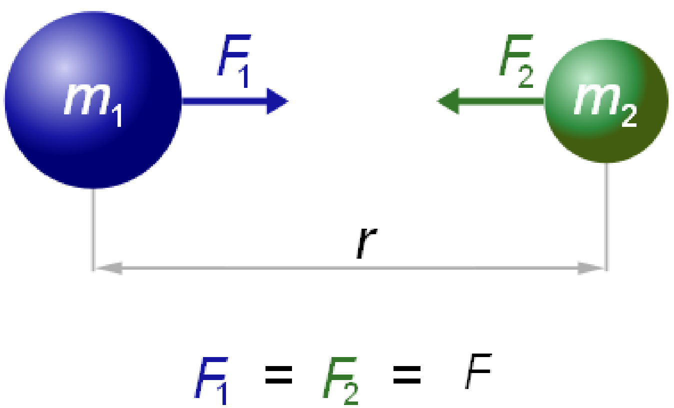
- In 1687, Newton published his famous work Mathematical Principles of Natural Philosophy.
- In this book, he mentioned:
  Laws of Motion and **Law of Universal Gravitation**

- You can find the book at:
  https://archive.org/details/philosophiaenatu00newt/page/n5/mode/2up



Law of universal gravitation:

Every particle in the universe attracts every other particle with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them. This force acts along the line joining the two particles.



$$F = G\frac{m_1 m_2}{r^2}$$

m_1, m_2 are masses of particles

r is distance between them

G is Universal Gravitational Constant

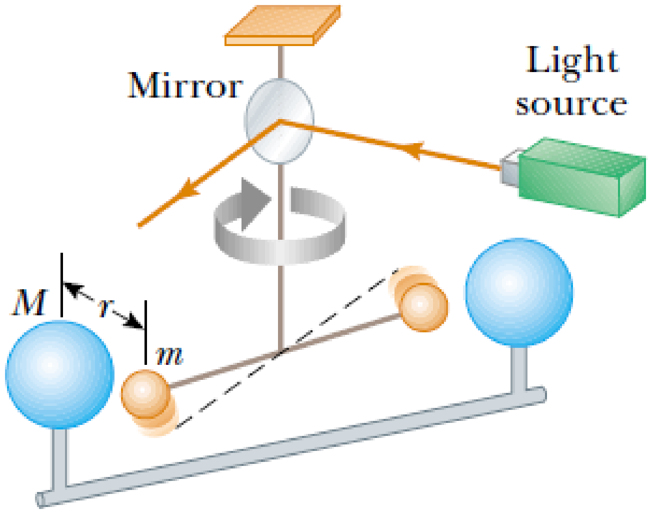
$G = 6.673 \times 10^{-11}\ \mathrm{N.m^2/kg^2}$

Measurement of Constant *G*

The Universal Gravitational Constant was first measured by H. Cavendish in 1798.

In the experiment, two small and two large masses are used.

G value is evaluated by measuring the deviation angle between incoming and reflected light rays as shown in Figure.



See:
http://en.wikipedia.org/wiki/Cavendish_experiment

Example:

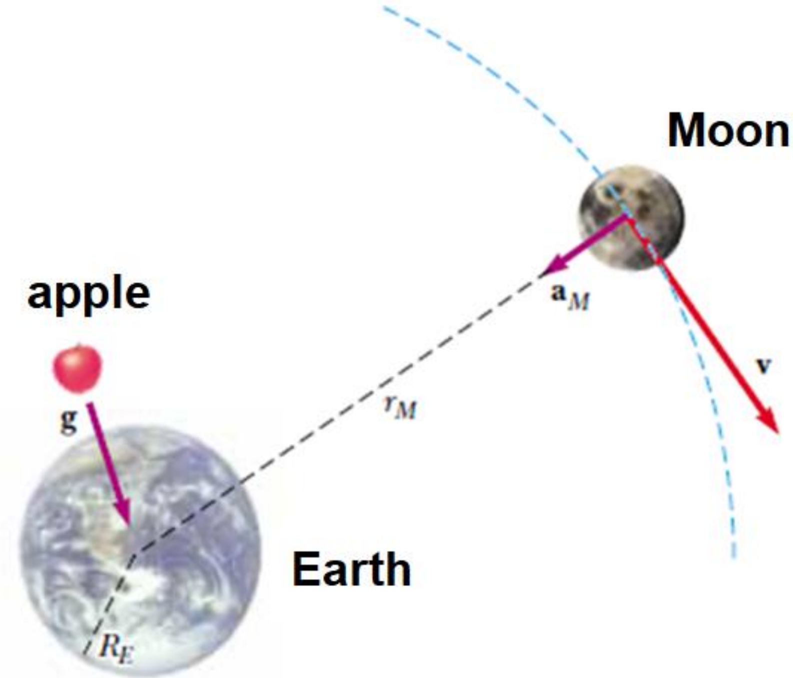
What is the gravitational force between two people each weighing 75 kg if the distance between them is half a meter?

$$F = (6.673 \times 10^{-11}\mathrm{N.m^2/kg^2})\frac{(75\ \mathrm{kg})(75\ \mathrm{kg})}{(0.5\ \mathrm{m})^2} = 1.5 \times 10^{-6}N$$

How about moon?

- Newton compared the acceleration of the Moon and that of a freely falling body on the surface of the earth.

- If the source of both accelerations is the same (*we do'nt know mass of earth*)



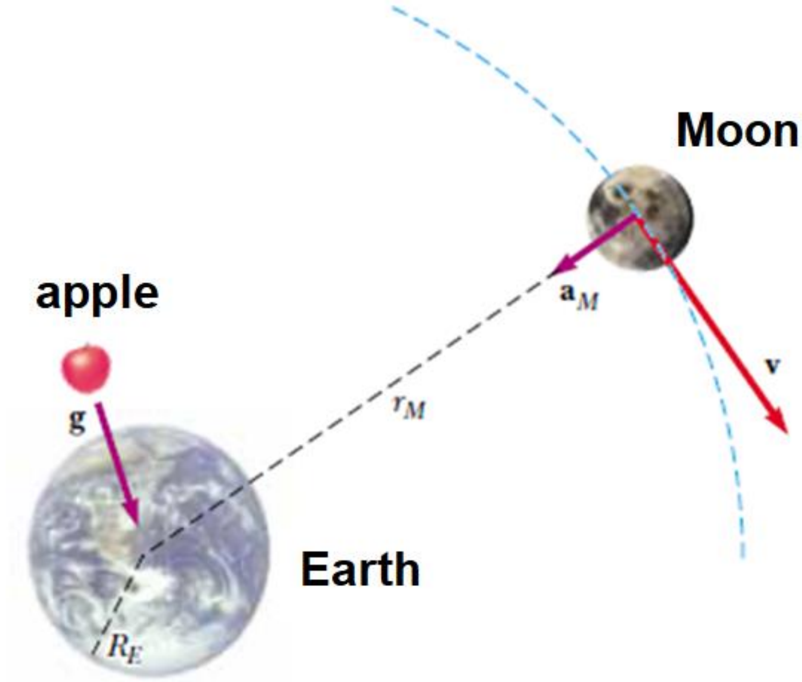
$$\frac{\text{Acceleration of moon}}{\text{Acceleration of apple}} = \frac{a_M}{g} = \frac{GM_E/r_M^2}{GM_E/R_E^2} = \left(\frac{R_E}{r_M}\right)^2 = \left(\frac{6.37 \times 10^6 m}{3.84 \times 10^8 m}\right)^2 = 2.75 \times 10^{-4}$$

- Hence, acceleration of the moon

$$a_M = (2.75 \times 10^{-4})(9.8\ \mathrm{m/s^2}) = 2.70 \times 10^{-3}\ \mathrm{m/s^2}$$

- On the other hand

$$a_M = \frac{v^2}{r_M} = 2.72 \times 10^{-3}\ \mathrm{m/s^2}$$



- Percentage Error:

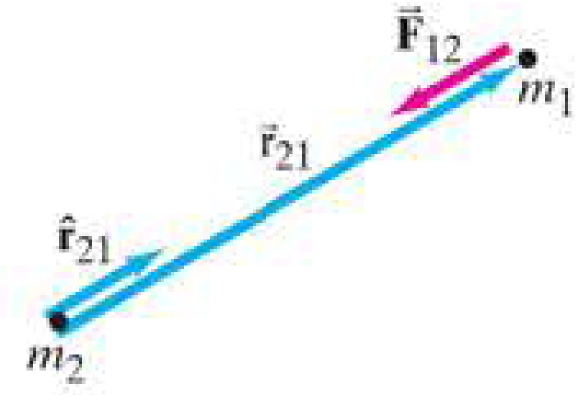
$$P.E. = \frac{2.72 \times 10^{-3} - 2.70 \times 10^{-3}}{2.70 \times 10^{-3}} \times 100 = \%\, 0.7$$

- This result strongly confirms the inverse-square law.

Vector Form of Newton's Law of Gravitation

We can write Newton's law of universal gravitation in vector form as

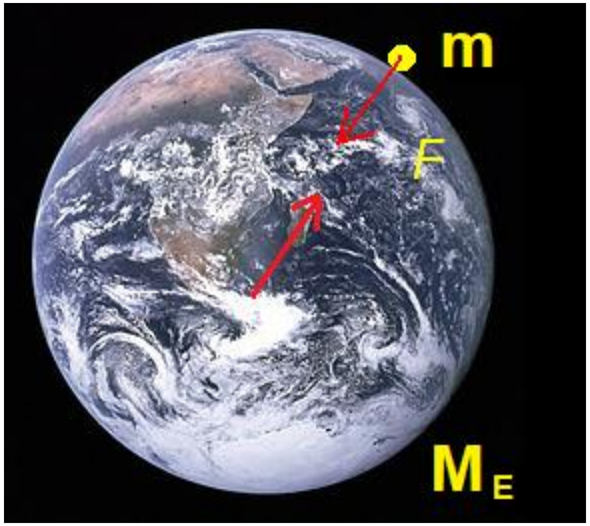
$$\mathbf{F}_{12} = -G\frac{m_1 m_2}{r_{21}^2}\hat{\mathbf{r}}_{21}$$



When many particles interact, the total gravitational force on a given particle is the vector sum of the forces exerted by others.

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \cdots + \mathbf{F}_{1n}$$

Gravity Near the Earth's Surface



Formula can be applied to obbjects having spherical symmetry.

So, gravitational force and acceleration of an object on Earth surface is:

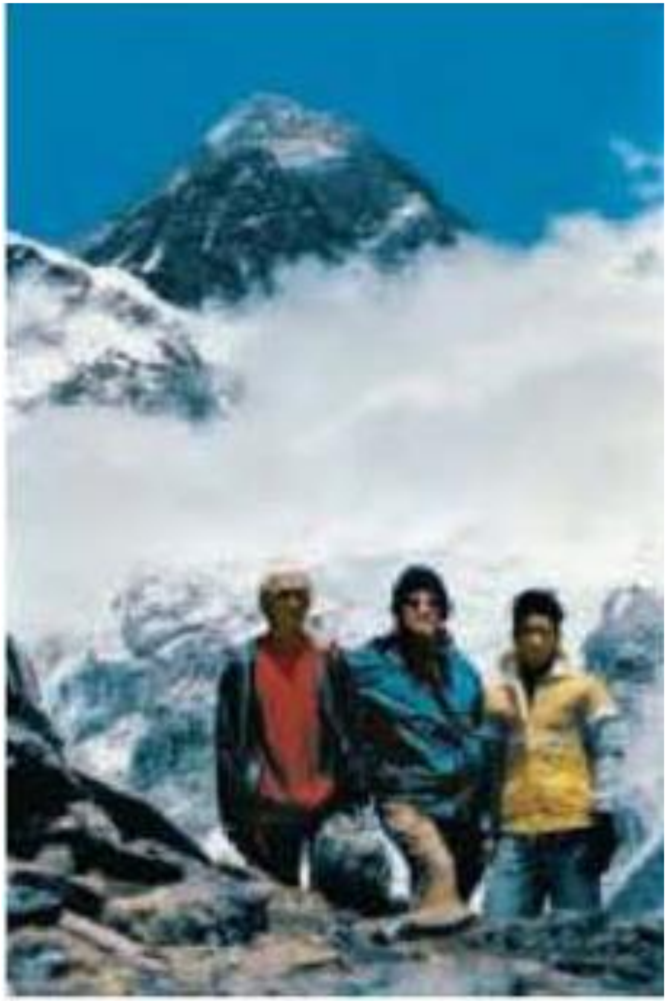
$$F = G\frac{M_E m}{{R_E}^2} = mg \quad \Rightarrow \quad g = G\frac{M_E}{{R_E}^2}$$

Solving for **Mass of Earth**:

$$M_E = \frac{g{R_E}^2}{G} = \frac{(9.8\,\mathrm{kg.m/s^2})(6.38 \times 10^6 m)^2}{6.673 \times 10^{-11}\,\mathrm{N.m^2/kg^2}} = 6 \times 10^{24}\mathrm{kg}$$

Example

Estimate the effective value of *g* on the top of Mt. Everest, 8850 m above sea level .



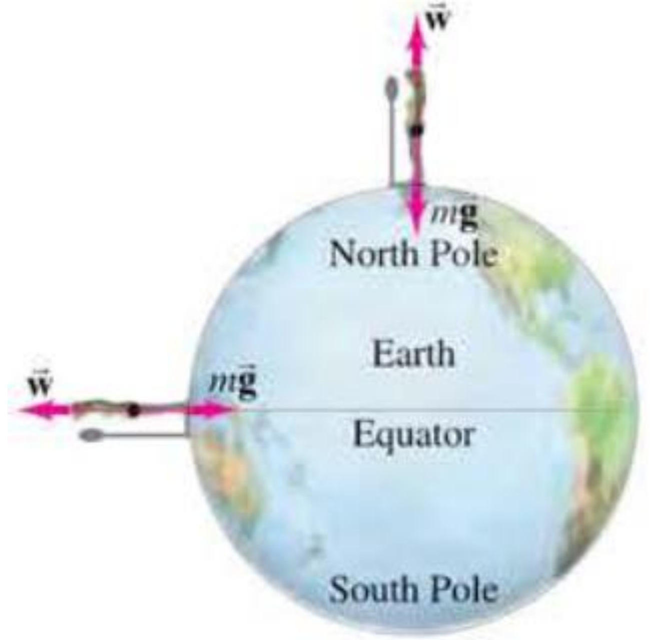
Solution

r = 6380 km + 8.85 km = 6388.9 km

$$g = G\frac{M_E}{r^2} = 9.77 \text{ m/s}^2$$

Example: Effect of Earth's rotation on g.

Assuming the Earth is a perfect sphere, determine how the Earth's rotation affects the value of g at the equator compared to its value at the poles.



Solution

North pole: Normal force – gravitaonal force = 0 (w – mg = 0)

Equator:

Speed of an object at rest on equator: $v = \dfrac{2\pi R_E}{1\ \text{day}} = 4.64 \times 10^2\ \text{m/s}$

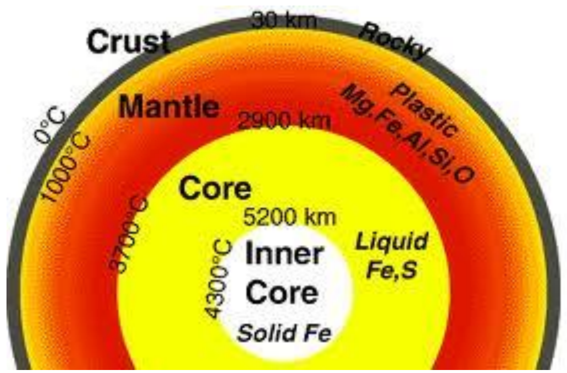
Consider centripedal force: $w - mg' = \dfrac{mv^2}{R_E}$

Or $g' = \dfrac{w}{m} - \dfrac{mv^2}{mR_E} = \dfrac{mg}{m} - \dfrac{mv^2}{mR_E} = g - \dfrac{v^2}{R_E} \Rightarrow \Delta g = \dfrac{v^2}{R_E} = 0.0337\ \text{m/s}^2$

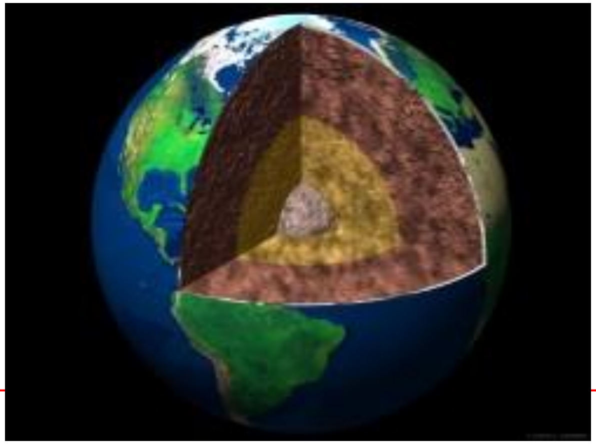
Example: Density of Earth

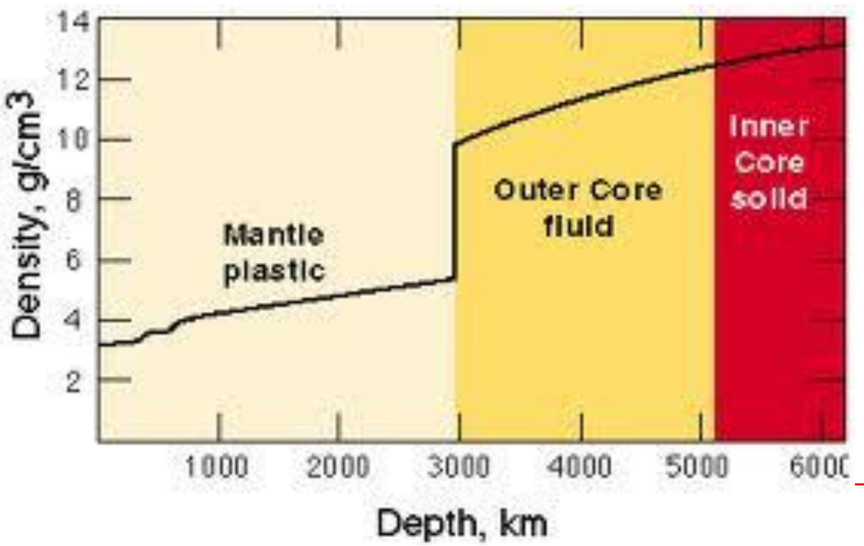
$$\text{Average Density} = \frac{\text{mass}}{\text{volume}} = \frac{6 \times 10^{24}\ \text{kg}}{\frac{4}{3}\pi(6.4 \times 10^{6}\ \text{m})^{3}} = \mathbf{5.5\ g/cm^{3}}$$

Density of objects (rock, soil, etc) on Earth surface is about **3 g/cm³.**



Therefore the density of the earth's interior is much higher!





Satellite Motion

What keeps a satellite up? => its speed and gravity

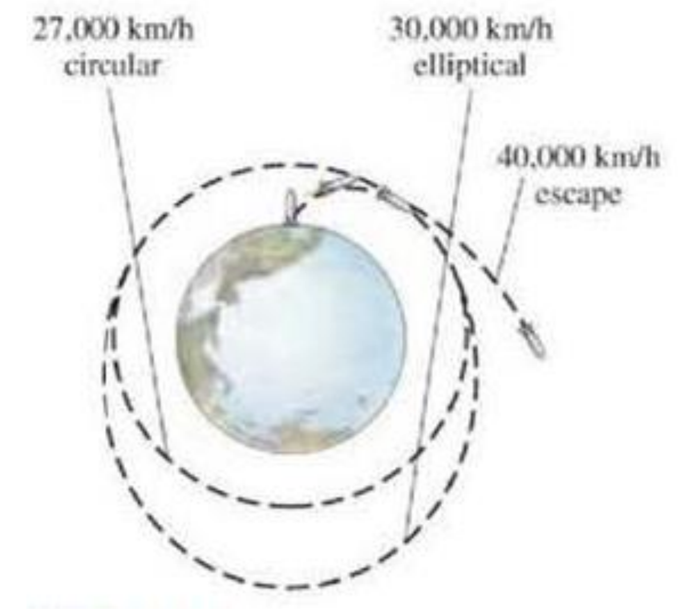
Satellite Motion

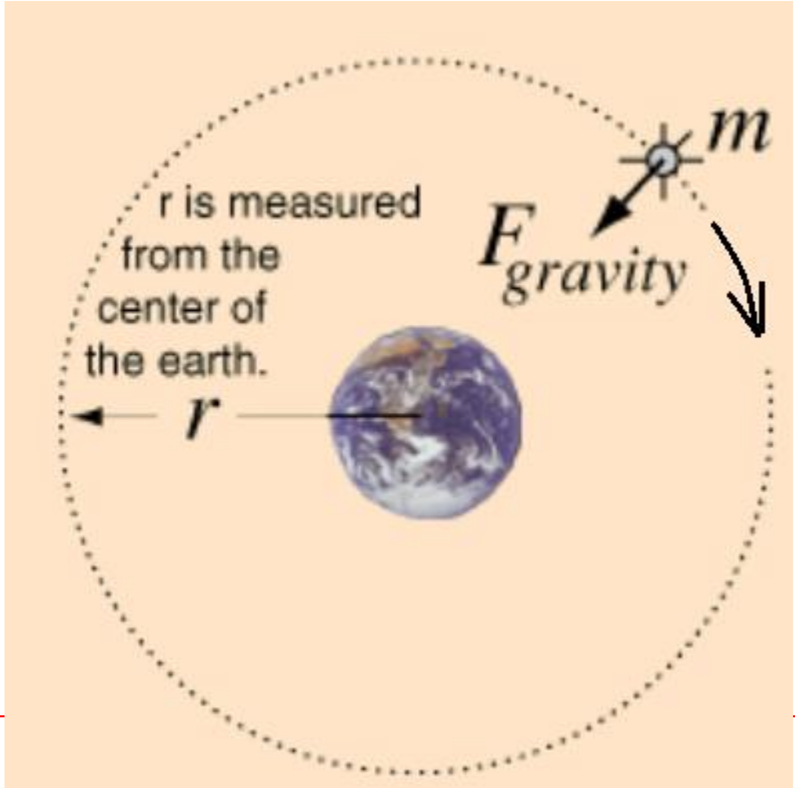
Force on satellite of mass *m* in circular orbit:

$$F = G\frac{mM_E}{r^2} = \frac{mv^2}{r}$$

Solving for velocity:

$$v = \sqrt{\frac{GM_E}{r}}$$





Example:

Find the height above the earth surface and velocity of a satellite whose period is 1 day.

Solution

Speed of the satellite: $v = \dfrac{2\pi r}{T}$ and $v = \sqrt{\dfrac{GM_E}{r}}$

Equating them:

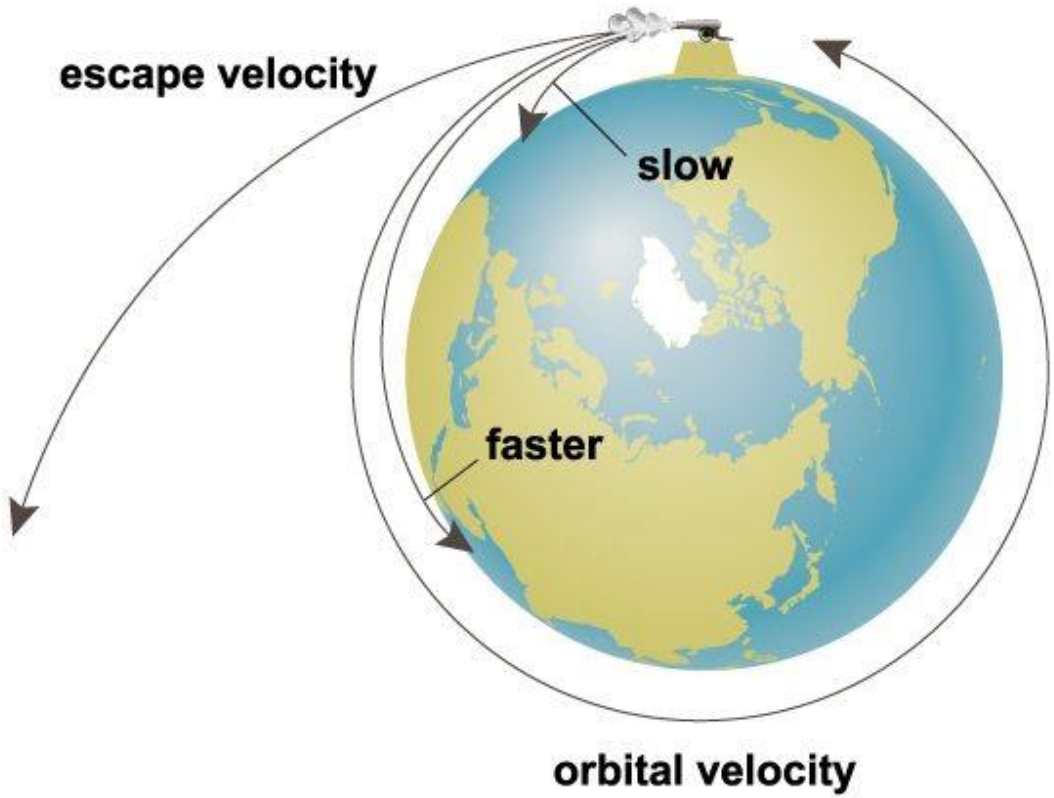
$$\frac{2\pi r}{T} = \sqrt{\frac{GM_E}{r}}$$

Solving for radius: *r* = 42000 km from center earth.

Solving for the speed: *v* = 3070 m / s

Example:

Calculate the critical speed on the Earth surface for a circular orbit.



$$v = \sqrt{\frac{GM_E}{R_E}} \approx 7900 \text{ m/s}$$

Weightlessness

In elevator, apply Newton's 2nd law:

in general

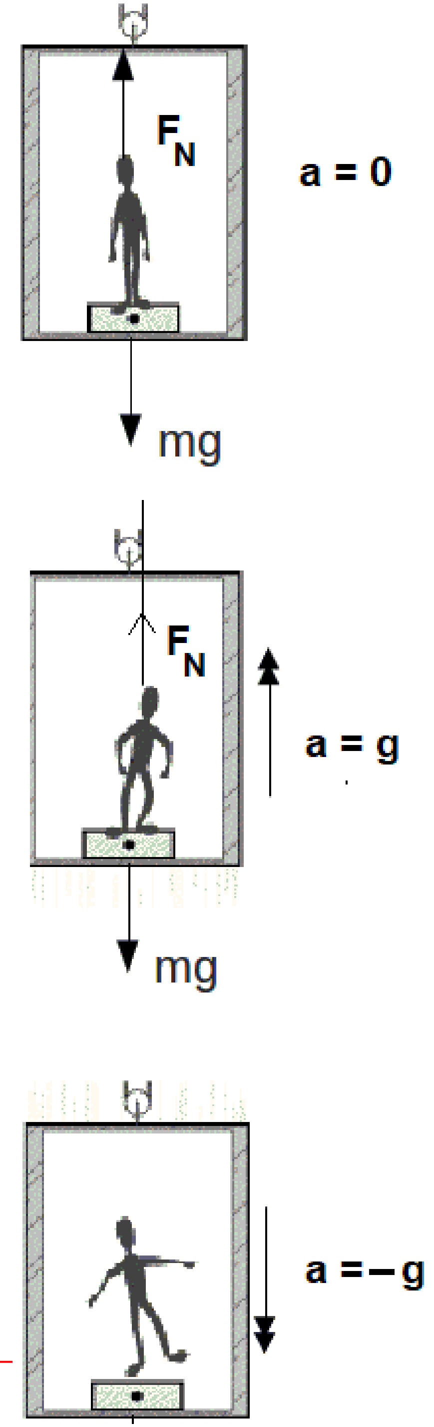
$$F_{\mathrm{N}} - mg = ma$$

$$a = 0 \quad \Rightarrow \quad F_N - mg = 0 \quad \Rightarrow F_N = mg$$

$$a = g \quad \Rightarrow \quad F_N - mg = mg \quad \Rightarrow \quad F_N = 2mg$$

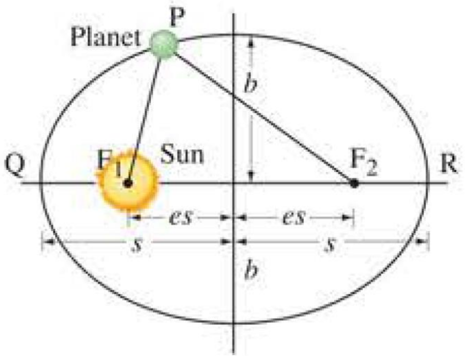
$$a = -g \quad \Rightarrow \quad F_N - mg = -mg \Rightarrow \quad F_N = 0$$

In last condition, you'll feel **weightless**!



The "weightlessness" experienced by people in a satellite orbit close to the Earth is the same apparent weightlessness experienced in a freely falling elevator.

Kepler's Laws and Newton's Synthesis



Kepler's 1st and 2nd laws requrie more analysis.

Kepler's 3rd law:

$$\frac{T^2}{s^3} \equiv const$$

Assuming, nearly circular orbit and put average distance ***s* = *r***,

$$F = G\frac{M_E M_S}{r^2} = \frac{M_E v^2}{r} \quad \text{and} \quad v = \frac{2\pi r}{T} \quad \Rightarrow \frac{T^2}{r^3} = \frac{4\pi^2}{G M_S}$$

Using this relation,

we can evaluate the mass of the sun!

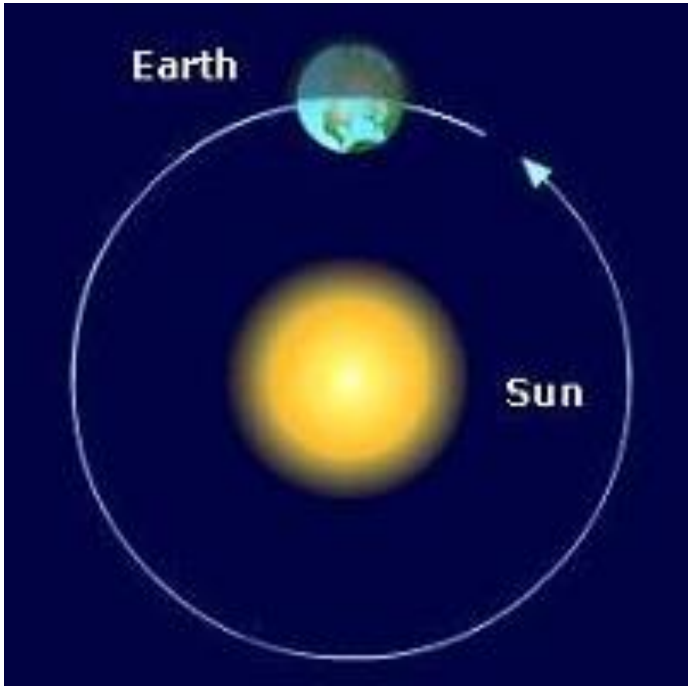
How?

Example:

Period of Earth: T = 1 year,

Earth-Sun distance: $r = 1.5 \times 10^{11}$ m,

Evaluate mass of Sun.



Solution:

From last relation:

$$M_S = \frac{4\pi^2 r^3}{GT^2} = 2 \times 10^{30}\ \mathrm{kg}$$

Q: Can you find mass of Earth using Moon's data?

A: Yes. It gives $M_E = 6 \times 10^{24}$ *kg.*

For any two planets:

$$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3} = const = \frac{4\pi^2}{GM_S}$$

Example:

Determine the mean distance of Mars from the Sun using the Earth as a reference. (T_{Mars} = 687 days)

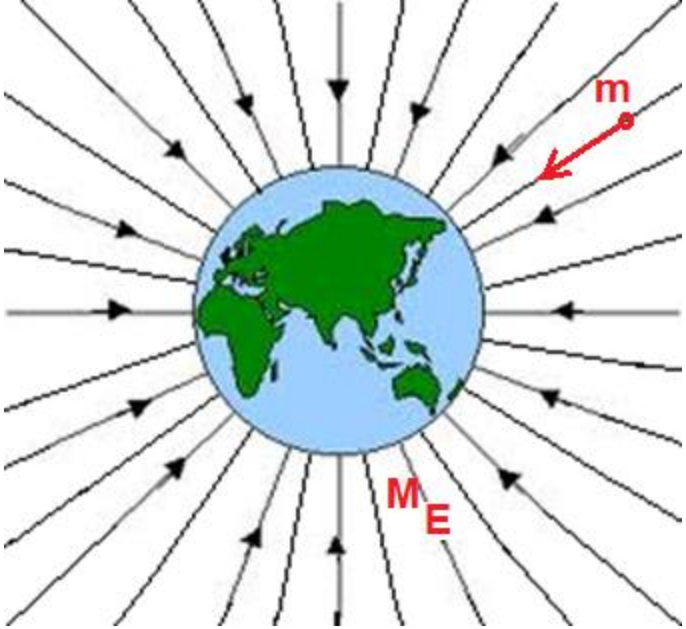
$$\frac{(365)^2}{1^3} = \frac{(687)^2}{r_2^3} \quad \Rightarrow \quad r_2 = 1.52$$

So, Mars is 1.52 times the Earth's distance from the Sun, or 2.28 X 10^{11} m.

Gravitational Field

Most of the forces we meet in everyday life are contact forces: **you push or pull something**.

The gravitational force acts over a distance.
Earth exerts a force on a falling apple.



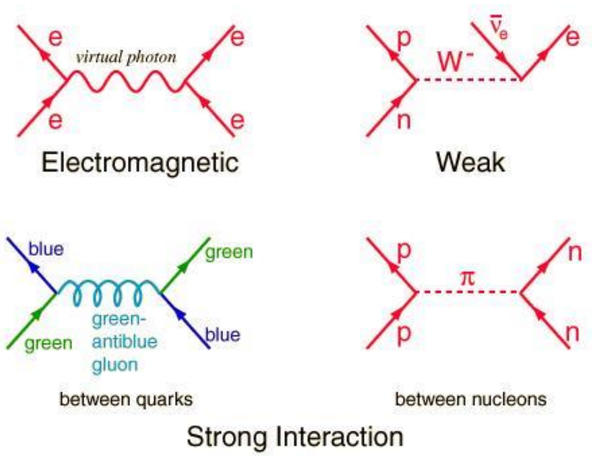
We can define the gravitational field as
the gravitational force per unit mass at any point in space.
Near Earth surface at a distance from center, we can evaluate field as:

$$\vec{\mathbf{g}} = \frac{\vec{\mathbf{F}}}{m} = -\frac{GmM_E}{mr^2}\hat{\mathbf{r}} = -\frac{GM_E}{r^2}\hat{\mathbf{r}}$$

$\hat{\mathbf{r}}$ = the unit vector to the direction of center (shown as red in figüre).

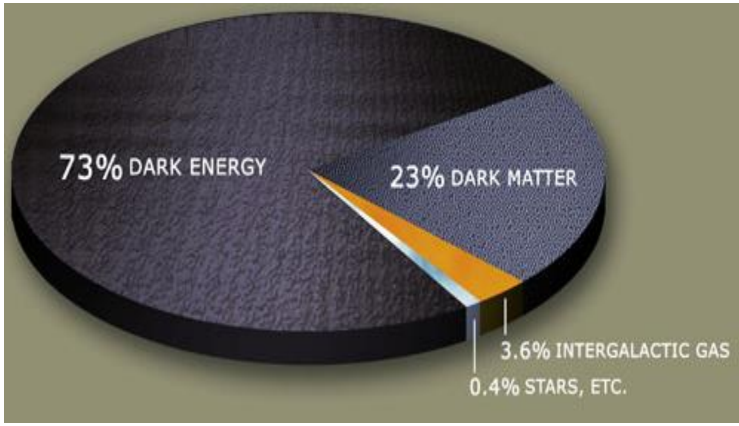
Four Fundamental Forces (Interactions)

Force	Source	Range (m)	Intensity	Theory	Particle
Gravity	Mass	∞	1	General Relativity	Graviton ?
Weak	Weak charge	10^{-17}	10^{25}	Electroweak	W ve Z
Electro-magnetic	Electric charge	∞	10^{36}	QED	Photon
Strong	Color charge	10^{-15}	10^{38}	QCD	Gluon



Dark Matter

- It is not radiate or reflect light (invisible).
- It interacts with visible matter.
- It surrounds the whole Galaxy!

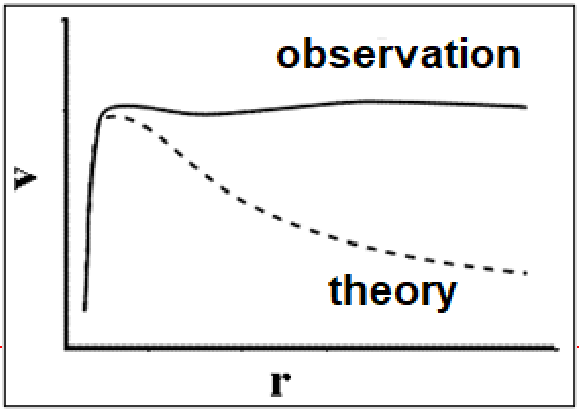


Evidence of dark matter:

$$\frac{mv^2}{r} = G\frac{mM}{r^2} \rightarrow v = (GM/r)^{1/2}$$

Center of galaxy: $M \propto r^3 \rightarrow v \propto r$

Outer part: $M \cong \textbf{constant} \rightarrow v \propto r^{-1/2}$



Turkish Appendix

Kütleçekimin Doğası

- Kütleçekim yasası çok sade!
- Kütleçekimin arkasındaki işleyiş (mekanizma), Fizik ve Hikmet nedir?
- Neden kütleler birbirini çekiyor?
- Newton bu mekanizma için bir şey söylememiş.
- Fiziğin birçok yasası matematiksel kurallardan oluşur ve mekanizma konusunda bir fikir vermez.
- Neden doğa olaylarını açıklamak için matematik kullanıyoruz ama arkadaki mekanizma ile uğraşmıyoruz?
  Kimse Bilmiyor.
- Ancak buna devam ediyoruz çünkü işe yarıyor.

Kütleçekimin Doğası

Kütleçekim mekanizması için bir çok fikir öne sürülmüş.

Küçük parçacıklar hipotezi (1750)

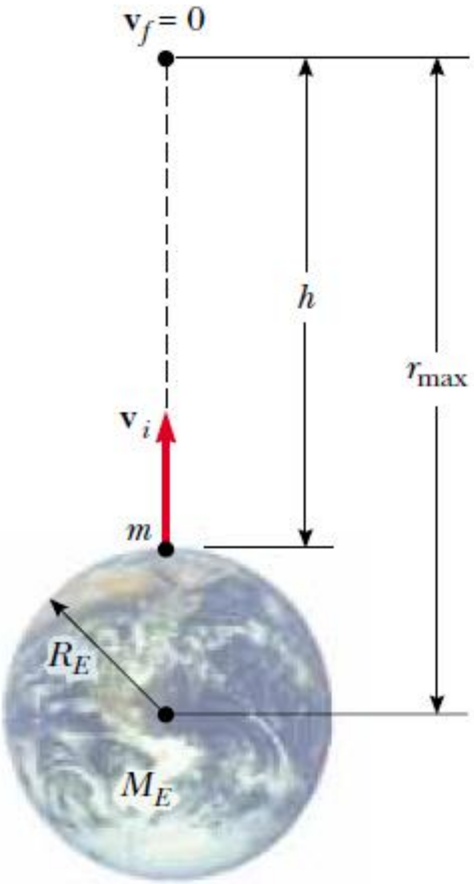
Uzayın her yöne çok hızlı hareket eden küçük parçacıklarla dolu olduğunu varsayalım. Bu parçacıklar Dünya ile çarpıştığında ona bir itme kazandırır. Güneş doğrultusunda gelen parçacıklar kısmen daha azdır.
Çünkü bu parçacıkların bir kısmını Güneş soğurur. Bu sayede Dünya, diğer yönlerden gelen itmelerle Güneşe doğru sürüklenir.

Bu mekanizma çöker!

Çünkü (yağmurda koşan adam misali) Dünya Güneş etrafında dönerken ön yüzey daha çok itmeye maruz kalır. Bu, dünyanın yavaşlamasına sebep olur.

Kurtulma Hızı

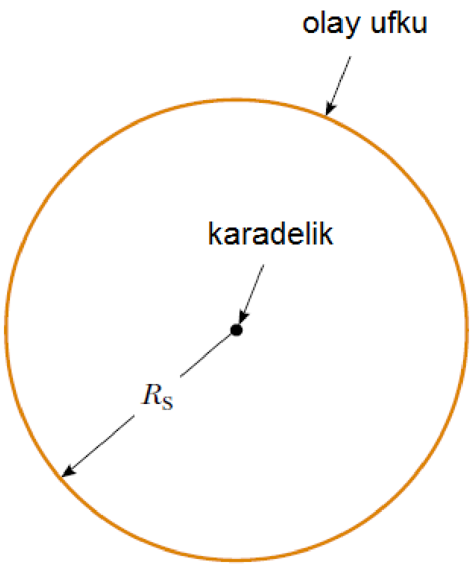
Dünya yüzeyinden v hızıyla bir cisim fırlatılsın. Bu cismin dünyanın çekim etkisinden kurtulabilmesi için gereken en küçük hız:



$$v_{kurtulma} = \sqrt{\frac{2GM}{R}}$$

Astronomik nesne	$v_{kurtulma}$
Ay	2.3 km/s
Dünya	11.2 km/s
Jupiter	60.0 km/s
Güneş	618.0 km/s

Karadelikler

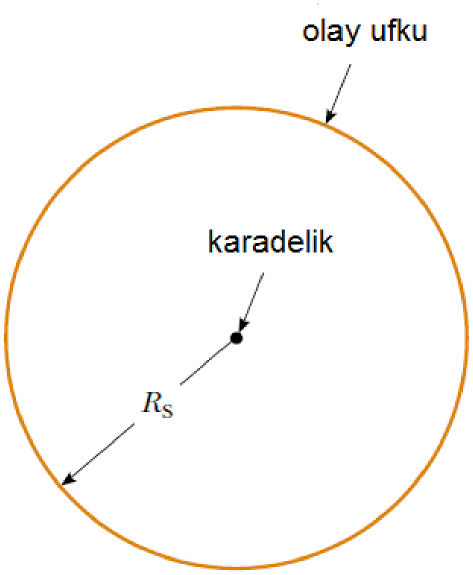


Yıldızlar iç yakıtlarını tüketince,
kütleçekimi yıldızın içine çökmesine
sebep olur.

M = 1.4 Güneş kütlesi üzerinde ise,
çökme devam eder => Nötron yıldızı oluşur.

M = 3 Güneş kütlesi üzerinde ise,
çökme devam eder => Karadelik oluşabilir.

Karadelikler



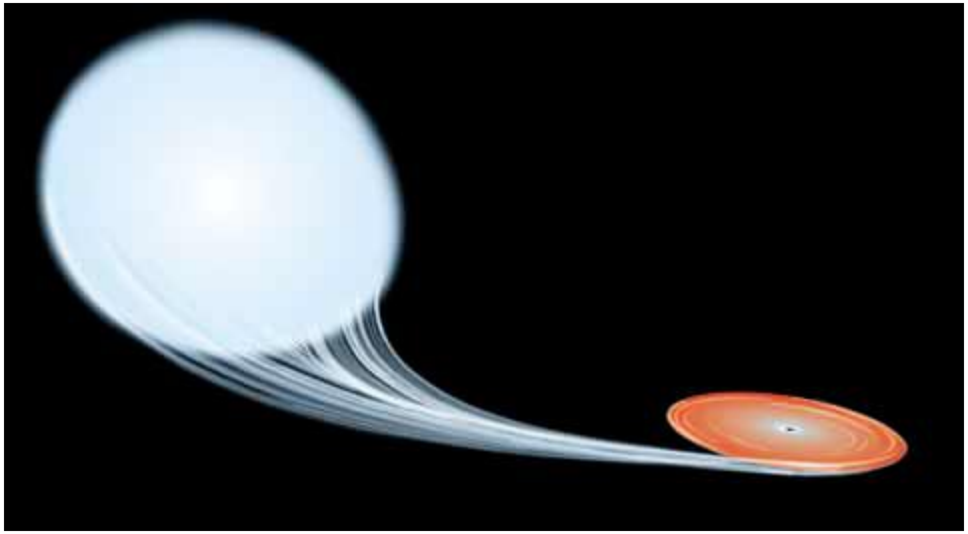
Karadelikler için kurtulma hızı çok büyüktür (Neden?)

$\mathbf{v_{kurtulma} = c}$ olduğu duruma karşılık gelen yarıçapa Schwarzschild yarıçapı (Rs), karadeliği çevreleyen Rs yarıçaplı hayali küreye olay ufku denir.

Olay ufku civarında (dışında) hayatta kalmak olasıdır.

Karadelikler

Çift yıldızlardan birinin karadelik olması durumuna ait bir benzetim (simülasyon).



Karadelik etrafındaki madde, karakteristik x-ışını yaymaya başlar. Neden?

Genel Görelilik ve Kütleçekim

Newton: “kütleçekim sonsuz hızda iletilir”.
Einstein: “hiçbirşey ışıktan hızlı gidemez”.

Einstein, Görelilik İlkesini dikkate alarak, Kütleçekim yasasında bir düzeltme yapıyor.

Yeni yasa:

Genel Görelilik Yasası

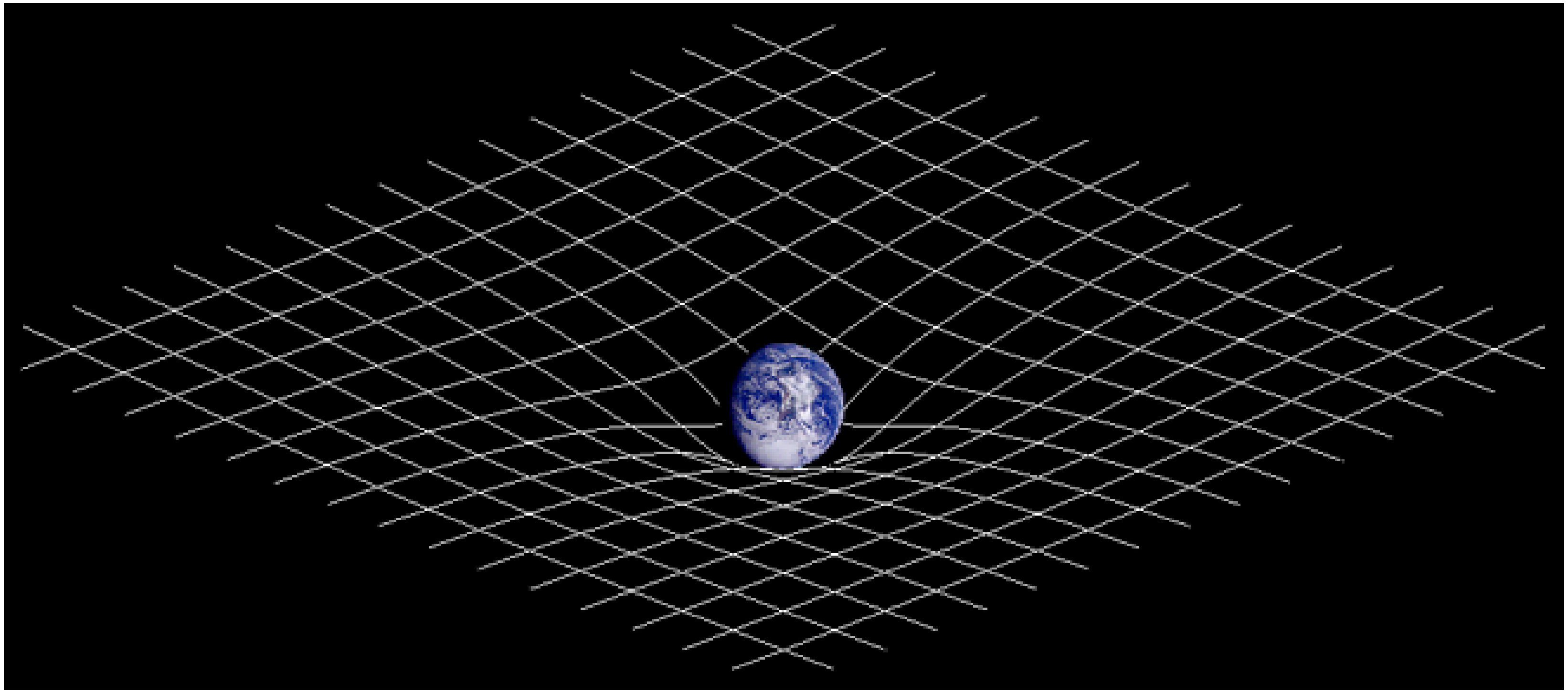
Genel Görelilik ve Kütleçekim

Dahası ($E = mc^2$ den), kütle ve enerji eşdeğerdir ve her ikisi de kütleçekiminden etkilenir.

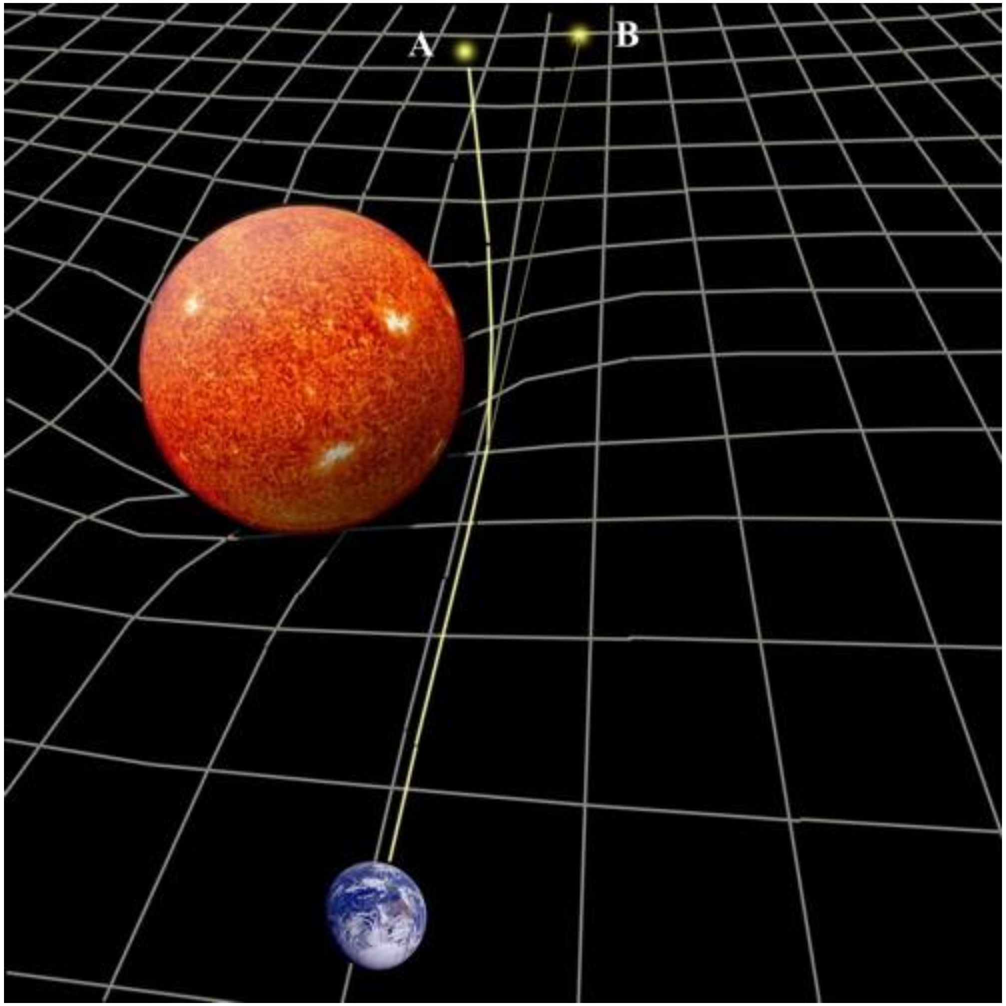
Buna göre, ışık (külliyen enerji) uzayda düz dizgi boyunca değil eğri-büğrü bir yol izler.

1. Kanıt: Güneş tutlulması esnasında bir yıldızın konumu
2. Kanıt: Mekür gezegeninin güneş çevresindeki hareketi
3. Kanıt: Kütleçekimsel Mercek etkisi

Genel Görelilik ve Kütleçekim



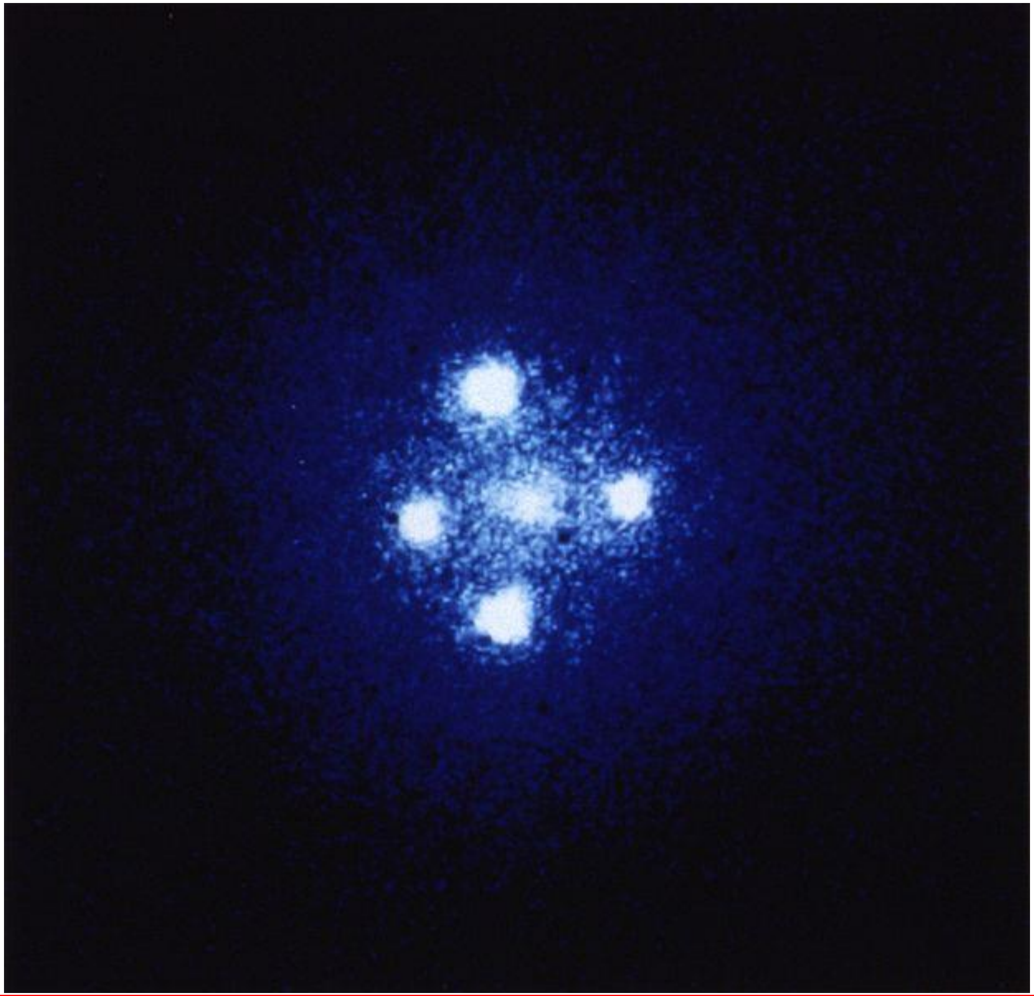
Genel Görelilik ve Kütleçekim



Genel Görelilik ve Kütleçekim

Aynı astronomik nesnenin

kütleçekimsel mercek etkisi ile dört görüntüsü

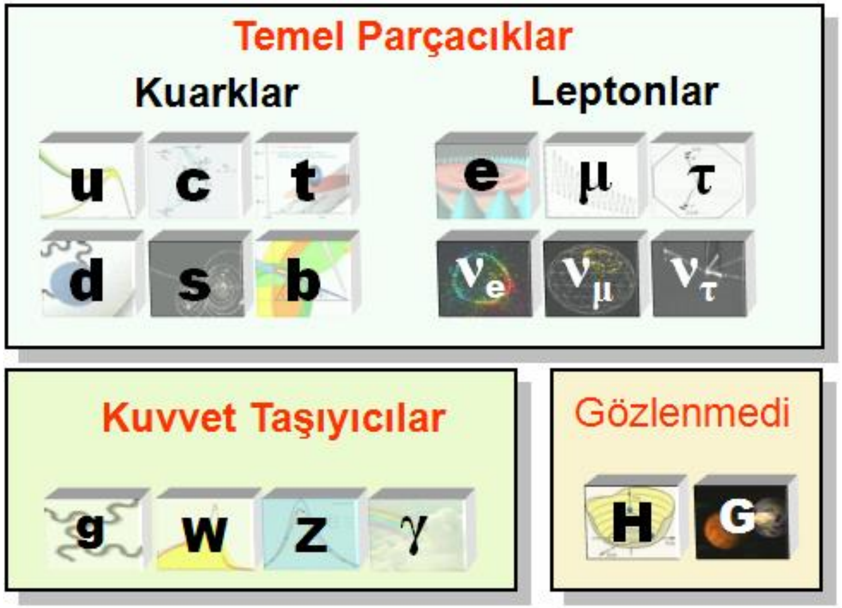


Kuantum Fiziği ve Kütleçekim

Bütün madde temel parçacıklardan oluşmuştur.

Ölçek küçüldükçe,
Kuantum etkileri ortaya çıkar.

Çok zayıf olan Kütleçekim,
henüz Kuantum doğası
kazanmamıştır.



Fizik yasalarının matematiksel olarak tutarlı olmalı.
Kütleçekimin de kuantum yapısı olmalıdır.

Beklenti:

Newton yasası -> Einstein yasası

Einstein yasası -> Kuantum kütleçekimi (belirsizlik ilkesi)

Sicim Kuramı (Herşeyin Kuramı)

String Theory (Theory of Everyting)

Amaç:

- Bütün parçacıkları ve dört temel kuvveti bir çatı altında toplamak.

Fikir:

- Temel parçacıklar nokta şeklinde değil titreşen sicim ilmikleridir

- Kuram ek boyutların varlığını öngörüyor:
  4+6 = 10 boyut veya 4+22 = 26 boyut !