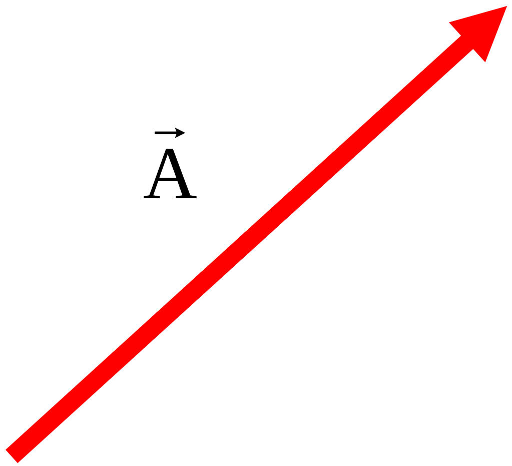
# PHYS111 MECHNICS

## Vectors




**METU**

**METU, OCT 2025**

# Scalars and Vectors

A scalar is a quantiy whose value is independent of the coordinate system you have selected.

- Single real number is a scalar.
- Mass, time, length, speed, energy, power, pressure, temperature are scalars.

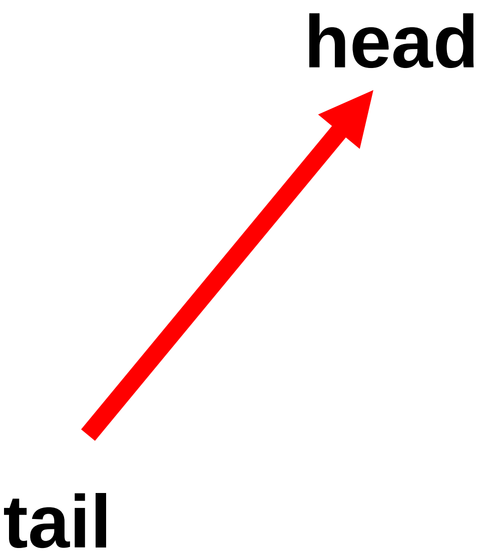
A quantity which has direction as well as magnitude is a vector quantity. One value is not enogh to describe a vector.

- We need at least two real numbers to describe a vector
- Displacement, velocity, acceleration, force, momentum are vectors

***** Speed is a scalar however velocity is a vector *****

# Vector Represantaion

A vector is represented by an arrow.




A vector is denoted by

$$\vec{\mathrm{A}} \quad \text{or} \quad \mathbf{A}$$

The magnitude (length) of a vector is represented by:

$$|\vec{\mathrm{A}}| \quad \text{or} \; A$$

# Product of a Scalar and a Vector

if *k* is a scalar and **A** is a vector, then *k***A** is also a vector.

$\vec{A}$  $k = +1$

$-\vec{A}$  $k = -1$

$\mathbf{0}$ $k = 0$ *(Zero or null vector)*

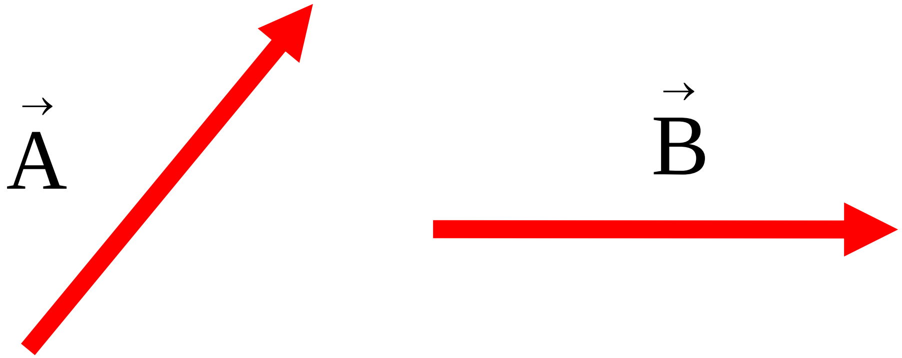
$k\vec{A}$  $k > 1$

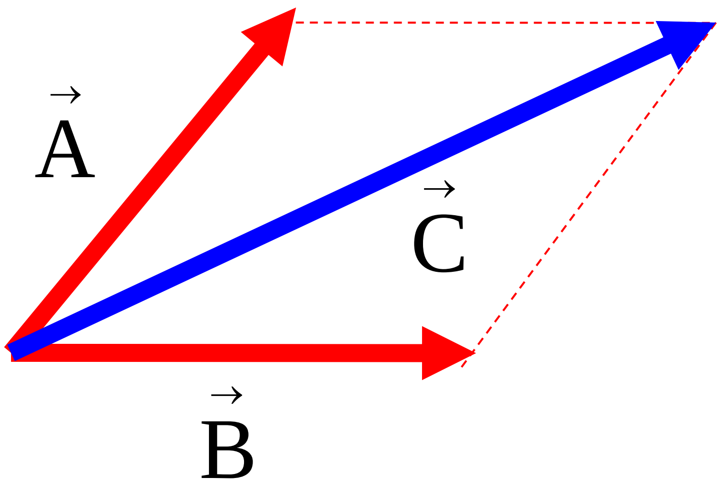
$k\vec{A}$  $0 < k < 1$

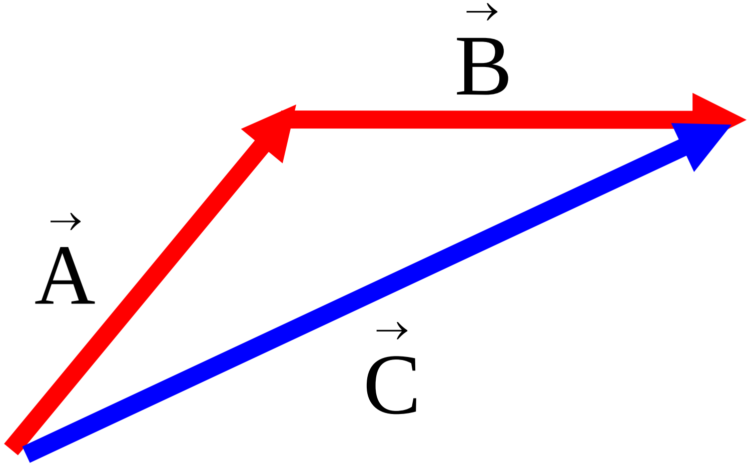
# Addition of Vectors

Consider two vectors

We want: $\vec{C} = \vec{A} + \vec{B}$



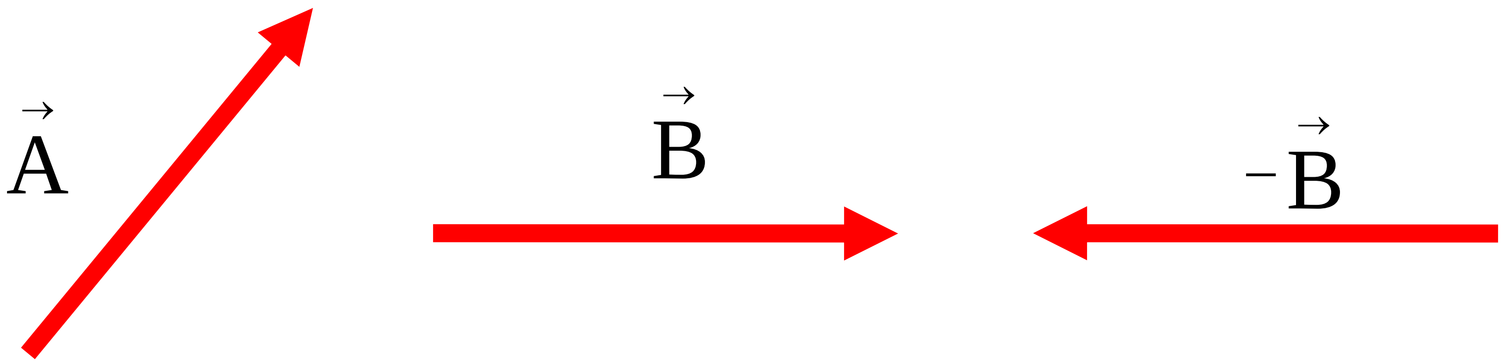

$\vec{C} = \vec{A} + \vec{B}$

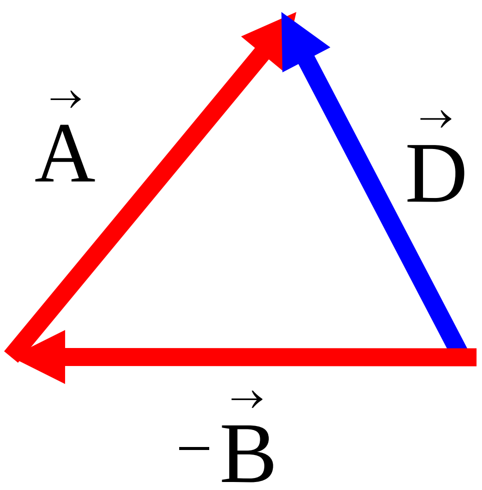
$\vec{C} = \vec{A} + \vec{B}$

# Subtraction of Vectors

Consider two vectors

We want: $\vec{D} = \vec{A} - \vec{B}$

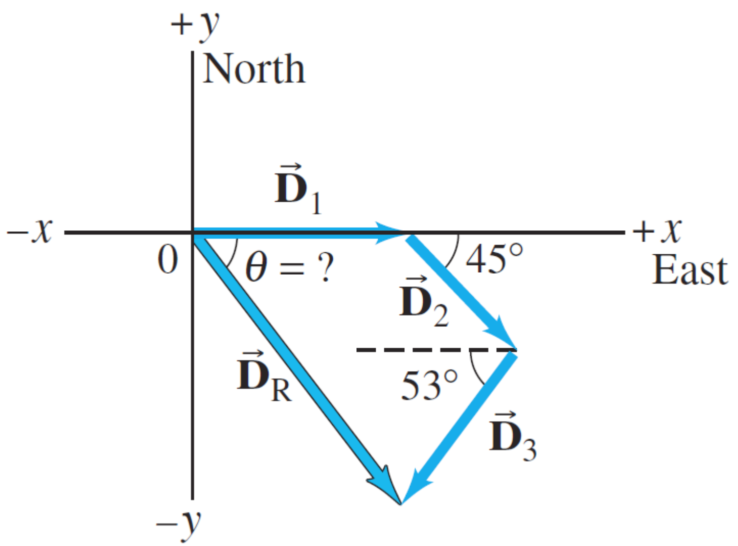



$$\vec{D} = \vec{A} + (-\vec{B}) = \vec{A} - \vec{B}$$

# Example

An airplane trip involves three legs, with two stopovers, as shown in Fig. The first leg is due east for 620 km; the second leg is southeast (45°) for 440 km; and the third leg is at 53° south of west, for 550 km, as shown. What is the plane's total displacement?




**Ans:**

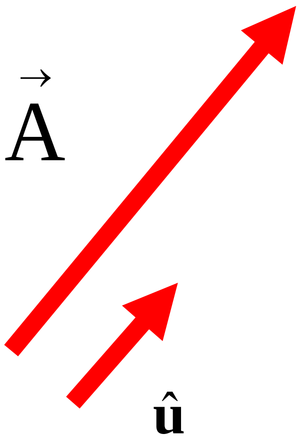
| Vector | Components x (km) | y (km) |
|---|---|---|
| $\vec{D}_1$ | 620 | 0 |
| $\vec{D}_2$ | 311 | −311 |
| $\vec{D}_3$ | −331 | −439 |
| $\vec{D}_R$ | 600 | −750 |

$D_R$ = 960 km

$\theta$ = 51°.

# Unit Vector

Unit vector is a vector in the direction of any vector whose magnitude is one. Consider a vector **A**.

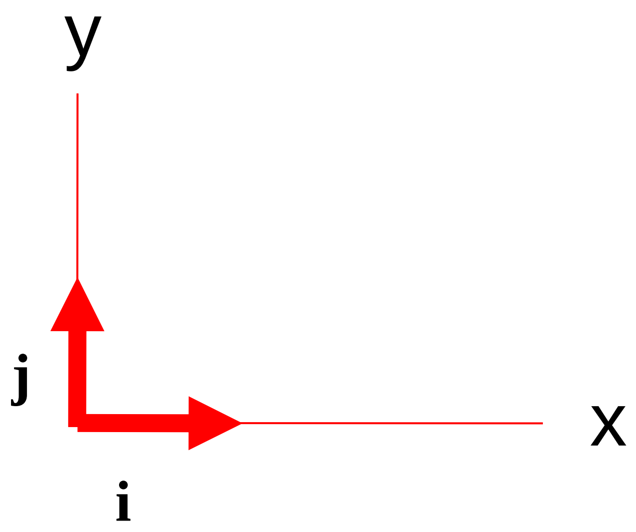


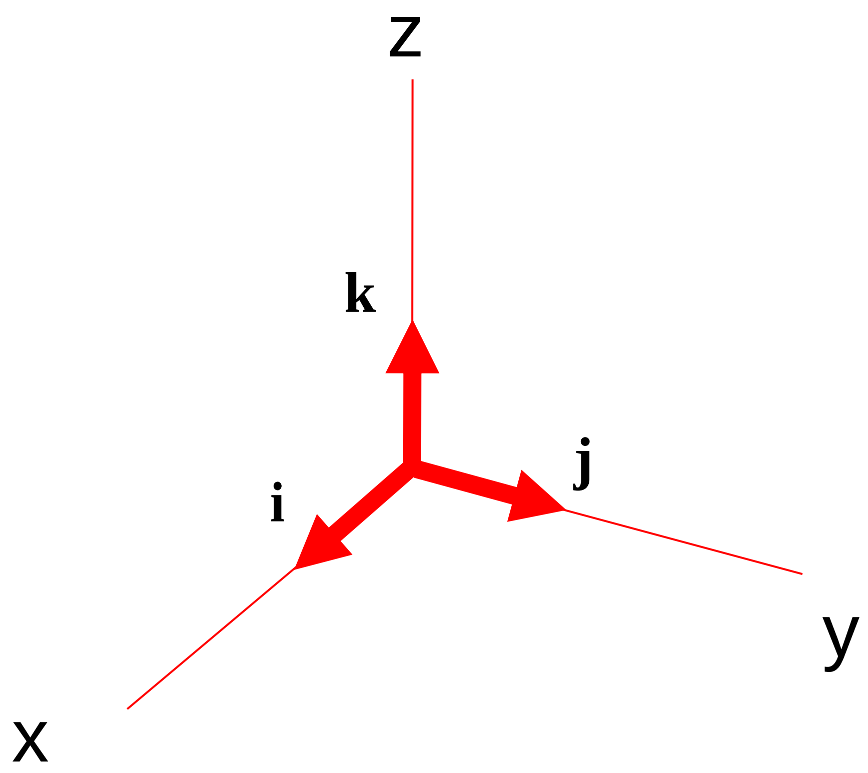

The unit vector in the direction of **A** is defined as:

$$\hat{u} = \frac{\vec{A}}{|\vec{A}|}$$

# Rectangular Unit Vectors

In Cartesian coordinate system, we define three basis (unit) vectors in the direction of positive x, y and z axes as follows:

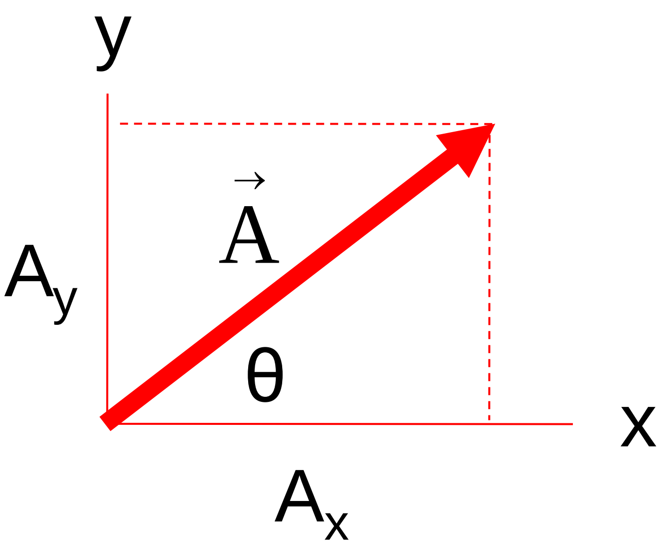


# Components of a Vector in 2D

The projection of a vector on axes are called its components.

The following realtions hold in 2D:




$$\vec{A} = A_x\mathbf{i} + A_y\mathbf{j}$$

$$A_x = |\vec{A}|\cos\theta$$

$$A_y = |\vec{A}|\sin\theta$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

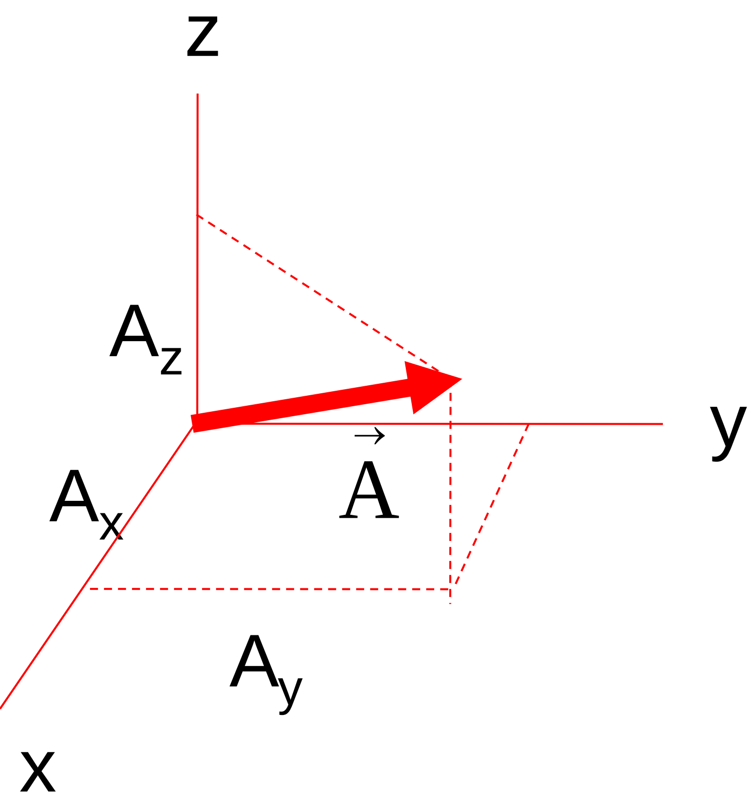
$$\tan\theta = \frac{A_y}{A_x}$$

# Components of a Vector in 3D

The following relations hold in 3D:

$$\vec{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$



# Example

Two vectors are given: **A** = 3**i** + 4**j** and **B** = –**i** + 2**j**.

(a) Find |**A**| [Ans: 5]

(b) Find |**B**| [Ans: 2.23]

(c) Find |**A+B**| [Ans: 6.32]

(d) Find |**A** – **B**| [Ans: 4.47]

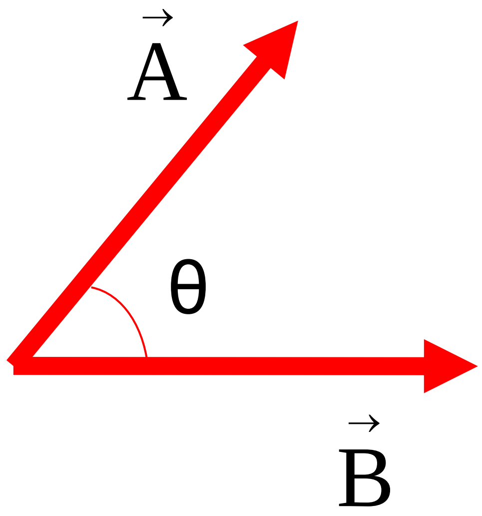
(e) Find the unit vector in the direction of vector **A** [Ans: **u**=0.6**i**+0.8**j**]

# Dot Product of Two Vectors

The dot (or scalar) product of two vectors is defined as:

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

Where 0<=θ<=180 is the angle between two vectors.



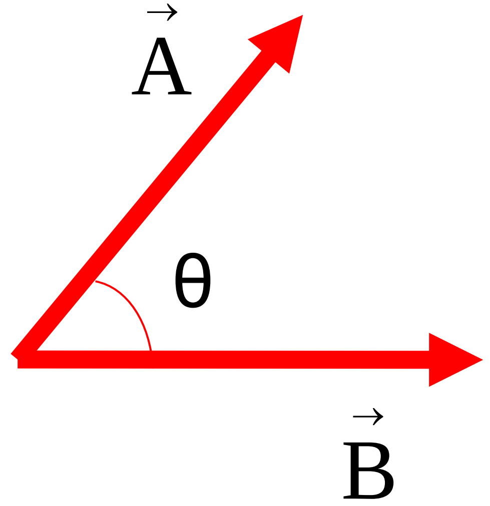

**GEOMETRICAL MEANING**

$A\cos\theta$ is the projection of vector **A** on vector **B**. Therefore:

$$\mathbf{A}.\mathbf{B} = (A\cos\theta)\,B$$

is the projection of **A** on **B** times magnitude of **B**.

# Some Properties of Dot Product




$$\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{A}$$

$$(\vec{A}+\vec{B})\cdot\vec{C}=\vec{A}\cdot\vec{C}+\vec{B}\cdot\vec{C}$$

**i** . **i** = **j** . **j** = **k** . **k** = 1

**i** . **j** = **i** . **k** = **j** . **k** = **0**

If $\vec{A}=A_x\mathbf{i}+A_y\mathbf{j}+A_z\mathbf{k}$ and $\vec{B}=B_x\mathbf{i}+B_y\mathbf{j}+B_z\mathbf{k}$

then

$$\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z$$

$$\vec{A}\cdot\vec{A}=A_x^2+A_y^2+A_z^2 \quad\Rightarrow\quad |\vec{A}|=\sqrt{\vec{A}\cdot\vec{A}}=\sqrt{A_x^2+A_y^2+A_z^2}$$

If **A** and **B** are non-zero vectors and **A**.**B** = 0, then $\theta = 90^o$.

So, **A** and **B** are <u>perpendicular</u> to each other if **A**.**B** = 0

# Example

Two vectors are given: **A** = 3**i** + 4**j** + 5**k** and **B** = **i** – 2**j**.

(a) Find |**A**| [Ans: 7.07]

(b) Find |**B**| [Ans: 2.23]

(c) Find **A.B** [Ans: –5]

(d) Find the angle between vectors [Ans:108.5$^o$]

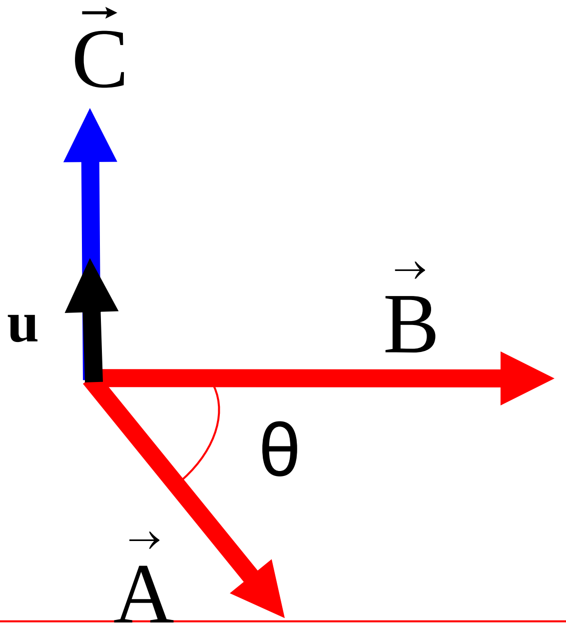
# Cross Product of Two Vectors

The cross (or vector) product of two vectors is defined as:

$$\vec{A} \times \vec{B} = AB\sin\theta\,\mathbf{u}$$

where 0<=θ<=180 is the angle between two vectors.

**u** is the unit vector in the direction of **A** x **B**.



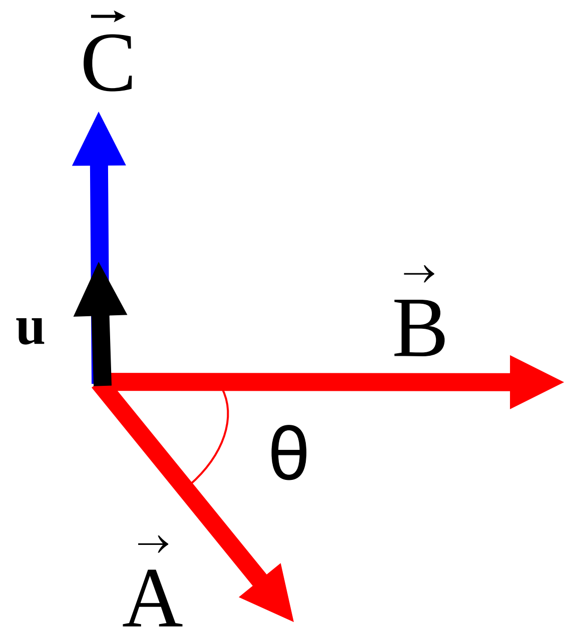

**GEOMETRICAL MEANING**

$A\sin\theta$ is the perpendicular component of vector **A** with respect to vector **B**. Therefore:

$$|\mathbf{A}\times\mathbf{B}| = (A\sin\theta)\,B$$

is the area of a paralelogram bounded by vectors **A** and **B**.

# Some Properties of Cross Product




$$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$$

$$(\vec{A}+\vec{B})\times\vec{C} = \vec{A}\times\vec{C}+\vec{B}\times\vec{C}$$

$$\mathbf{u} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$$

**i** x **i** = **j** x **j** = **k** x **k** = 0

**i** x **j** = **k**, **j** x **k** = **i**, **k** x **i** = **j**

If $\vec{A} = A_x\mathbf{i}+A_y\mathbf{j}+A_z\mathbf{k}$ and $\vec{B} = B_x\mathbf{i}+B_y\mathbf{j}+B_z\mathbf{k}$

then

$$\vec{A}\times\vec{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

If **A** and **B** are non-zero vectors and **A**x**B** = 0, then $\theta = 0^o$.

So, **A** and **B** are <u>parallel</u> to each other if **A** x **B** = 0.

# Example

Two vectors are given: **A** = 3**i** + 5**k** and **B** = **i** – 2**j** + 3**k**.

(a) Find |**A**| [Ans: 5.83]

(b) Find |**B**| [Ans: 3.74]

(c) Find **A**x**B** [Ans: (10, –4, –6)]

(d) Find the angle between vectors [Ans: 46.5$^{o}$]

(e) Find the unit vector in the direction of **A** x **B**.

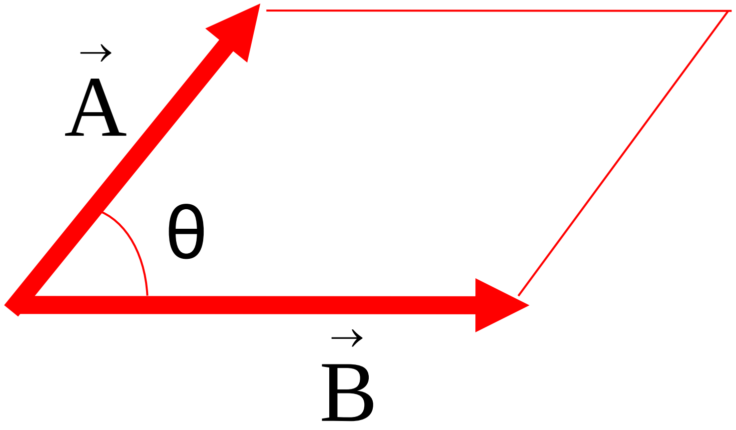
[Ans: **u** = (0.81111, -0.32444 ,-0.48666)]

# Example

Magnitude of vector **A** = 10 cm

Magnitude of vector **B** = 15 cm

Angle between vectors = 45$^o$




Find the area of the paralelogram.

**Ans:**

Area = |**A**x**B**| = |$AB$ sin$\theta$| = |(10)(15)(sin45)| = 106 cm$^2$.

# Exercise

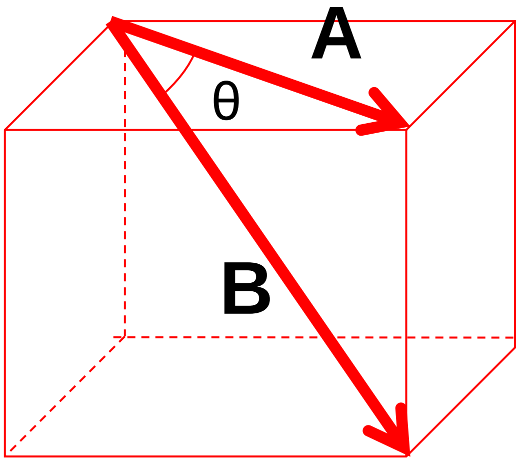
Consider two vectors (**A** and **B**) having the same length (magnitude), namely $A = B$. If $|\mathbf{A}+\mathbf{B}| = 10\,|\mathbf{A}-\mathbf{B}|$, calculate the angle between vectors.

**Ans:** $\cos\theta = 9/11$ or $\theta = 35.1^{o}$

# Exercise

Consider a unit cube.

Find the angle between vectors **A** and **B**.




**Ans:** $\theta = 65.9^o$