1) Solve the following system by using a) GEM method b) A=LU, where U (U=A^T) is the upper triangular matrix obtained by using Gaussian elimination, L is the lower triangular matrix with l_{ii}=1. Determine other elements of L by using A=LU. Then, solve Lz=b for z and solve for Ux=z for x. You should get the same result.

\[
\begin{bmatrix}
4 & 2 & -3 \\
8 & 2 & -9 \\
-8 & -8 & 6
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= 
\begin{bmatrix}
3 \\
1 \\
-10
\end{bmatrix}
\]

2) Consider the transformation in which

- first, x'-frame is obtained from x-frame by rotating it by 90° about x_3-axis and reflecting x_2 with respect to x_1x_3 - plane (see Fig. 1),
- then, x''-frame is obtained from x'-frame by rotating it by 30° about x'_1-axis (see Fig. 2)

a) Find the transformation matrix retaining the base vectors of x''-frame to those of x-frame.

b) Check if the transformation matrix found in part (a) is orthogonal.

c) For the force vector \( \mathbf{F} = (-2,3,1) \) given in x-frame (local coordinate system), obtain the force vector in x''-frame (in global coordinate system).

3) i) Find the characteristic (eigen) values and the corresponding characteristic (eigen) vectors of the following matrices:

a) \( A = \begin{bmatrix}
2 & -1 & 0 \\
-1 & 2 & -1 \\
0 & -1 & 2
\end{bmatrix} \), (Ax=\lambda x)

b) \( A = \begin{bmatrix}
2 & -1 & 0 \\
-1 & 2 & -1 \\
0 & -1 & 2
\end{bmatrix} \) and \( B = \begin{bmatrix}
3 & 0 & 0 \\
0 & 4 & 0 \\
0 & 0 & 3
\end{bmatrix} \), (Ax=\lambda Bx).

ii) In each case, verify all orthogonality relations. Express the vector \( v = (1,2,3)^T \) as a linear combination of the eigenvectors.

iii) Obtain the diagonal form of the matrix A in part (a).

4) Find the value of \( A^{\frac{1}{6}} \) and \( A^{41} \) for the matrix \( A = \begin{bmatrix}
2 & -1 & 0 \\
-1 & 2 & 0 \\
0 & 0 & 2
\end{bmatrix} \).