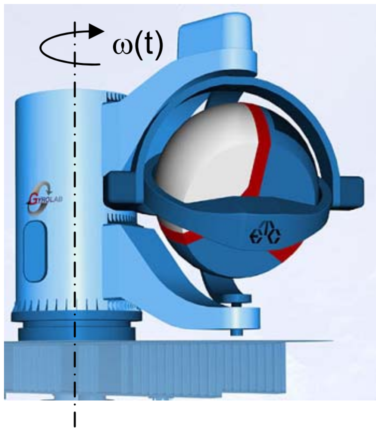
I/ Introduction & Modeling

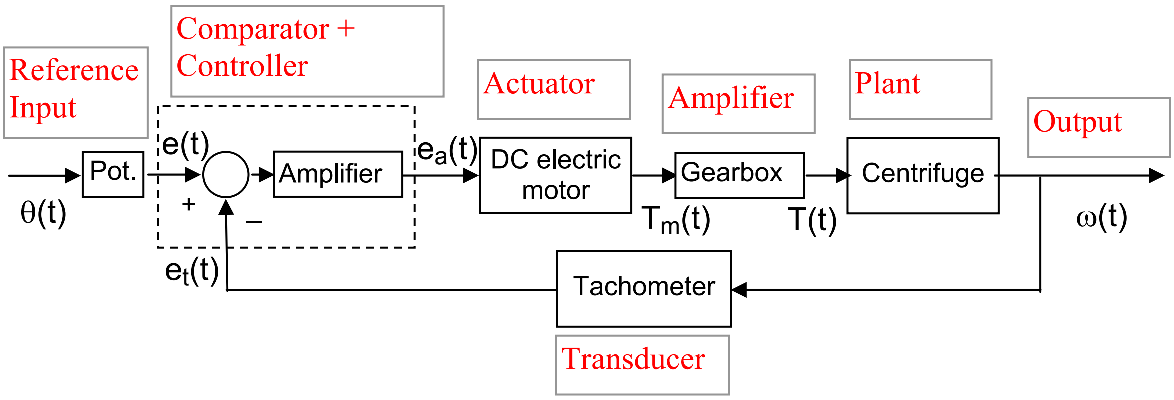
PROBLEM I-1

A centrifuge used in training pilots is shown in the picture and is illustrated with a conceptual block diagram below. The centrifuge consists of rotating arms, carrying a gondola, rotated at a controlled speed to simulate the high accelerations to which the pilots are subjected during flight.



Associate the main function of each component with one or more of the elements of the list {reference input, comparator, controller, amplifier, actuator, plant, sensor, transducer, controlled output}, and write on the rectangle just above the relevant block.

Ans. :

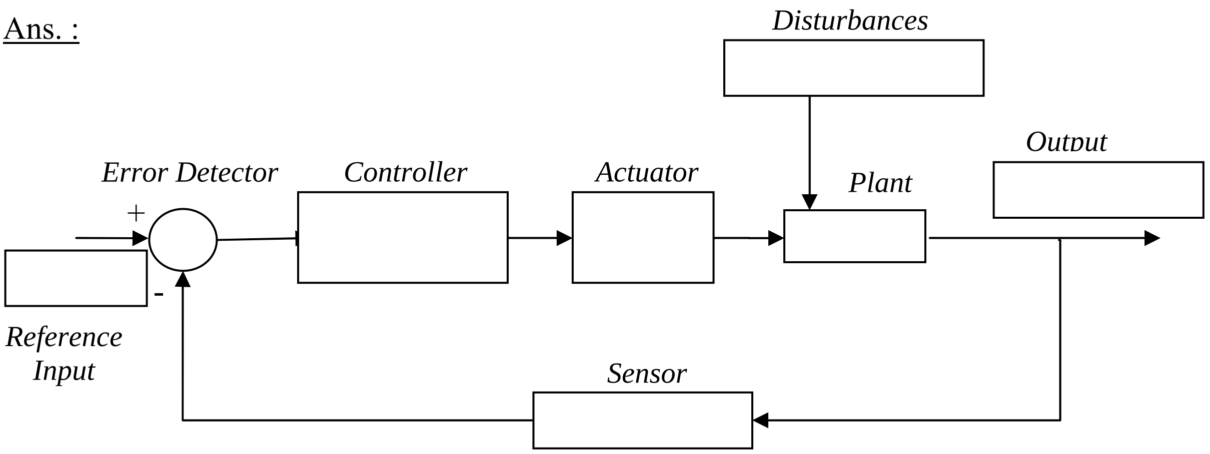


PROBLEM I-2

Automotive cruise control systems are used to hold the vehicle speed <u>steady</u> at a set value especially in long runs to prevent driver fatigue. After switched "ON" by the driver, this system takes the control of the gas throttle using an electronically controlled stepper motor, and keeps the vehicle speed constant. This system requires speed sensors to measure the controlled vehicle speed.

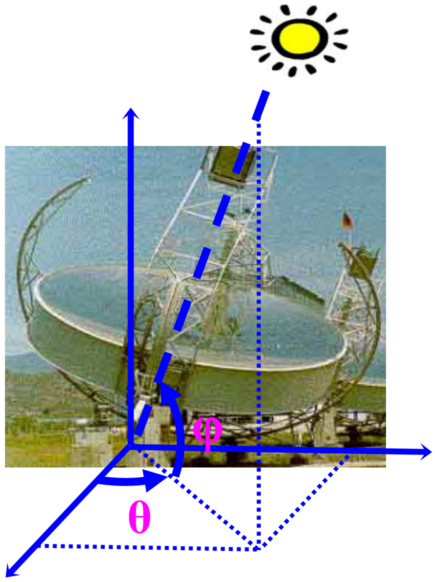
a) Identify basic elements of the cruise control system. (Reference input, output, actuator, controller, sensor, plant)
b) List any possible disturbances to the system during operation.
c) Draw a simple block diagram. Clearly identify each block as one of the basic elements.
d) Classify (also explain the reason) the system as;
   i. Open Loop or Closed Loop,
   ii. Regulator or Servomechanism.

<u>Ans. :</u>



PROBLEM I-2

Solar collectors need to be perfectly oriented towards sun in order to provide best utilization of sun light. Therefore, solar collectors must track the sun by continuously changing orientation during daytime. The orientation of a solar collector can be adjusted by changing two angles, one of which is the angle between the collector axis and the horizontal (elevation, angle φ in the figure) and the other represents the rotation of the collector about vertical (azimuth, angle θ in the figure). These two angles can be varied smoothly using two servomotors which are driven by a digital control card.



For sun tracking, two alternative methods are possible to determine the orientation (the two angles, elevation and azimuth) of the sun in the sky;

i. A sun sensor that senses the position of the sun in the sky,
ii. A software that is loaded on the digital control card which predicts the position of the sun according to date and location.

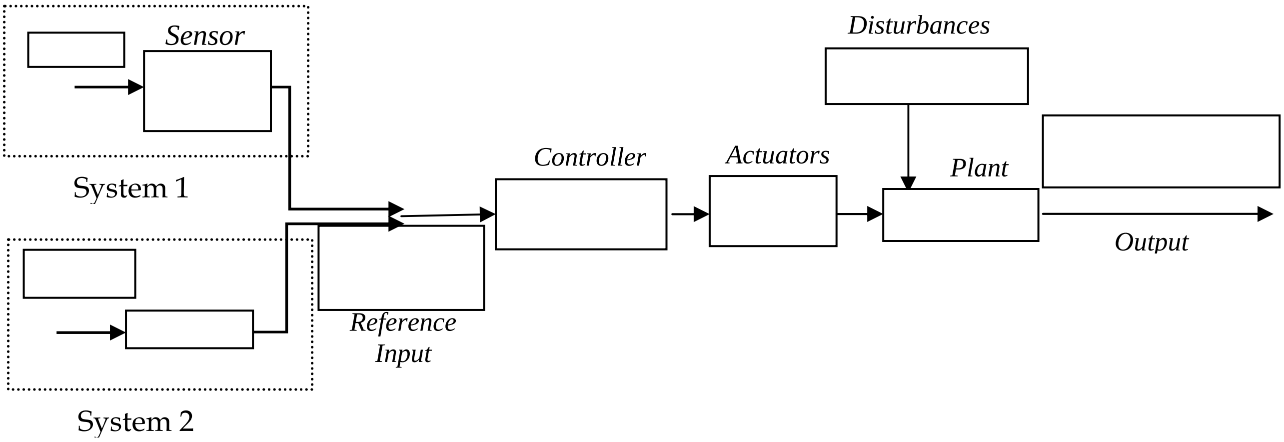
a) Identify basic elements of both sun tracking systems.

b) Classify (also explain the reason) both systems as;
   i. Open Loop or Closed Loop,
   ii. Regulator or Servomechanism.

c) List any possible disturbances to sun tracking systems and compare behavior of the two systems under the effect of disturbances.

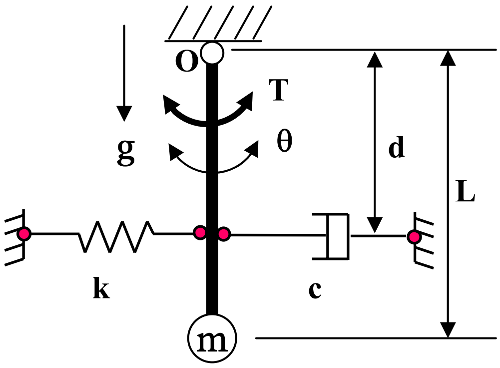
d) Draw simple block diagrams for the systems clearly identifying all elements.

Ans. :


Sensor
System 1
System 2
Reference Input
Controller
Actuators
Disturbances
Plant
Output

PROBLEM I-3:

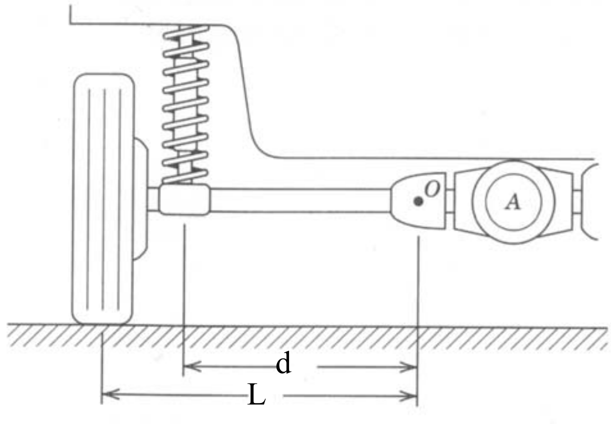
Consider the pendulum illustrated in the figure. In the equilibrium position shown the massless rigid bar connecting the mass to the pivot point **O** is vertical and the spring is undeformed. A torque **T** at the pivot point is applied to the bar, and the angular position of the bar with respect to the equilibrium position is represented by the angle **θ**.

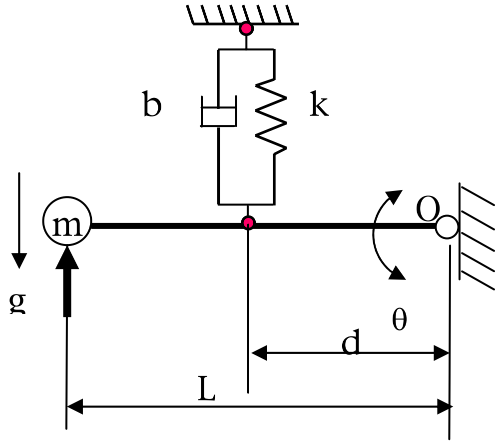


Obtain the relation between the input (torque **T**) and the output (angle **θ**) for small oscillations about the origin.

Ans.: $mL^2\ddot{\theta} + cd^2\dot{\theta} + \left(kd^2 + mgL\right)\theta = T$

PROBLEM I-4





Consider the swing axle type of independent suspension of a vehicle. The rigid half axle pivots about the fixed body point O. A strut, which consists of a spring and a damper coaxially placed, is pivoted on the half shaft at one end and on the body at the other. Neglect the masses of the strut (spring and damper) and half shaft. Wheel mass is m, the spring constant is k, and the damper damping coefficient is b. Assume the equilibrium position of the system to be the horizontal position of the half shaft. For small motions about the equilibrium position :

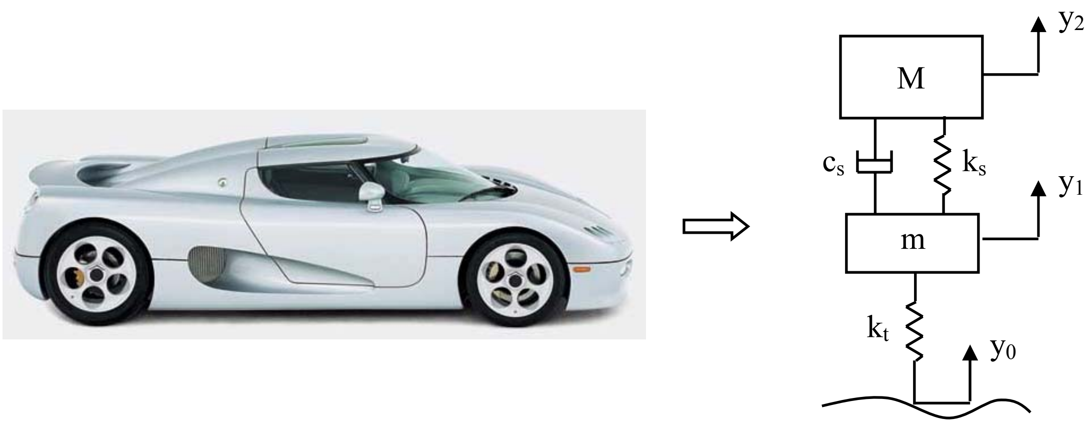
a) Identify the elements and write down the elemental equations. Hint : Give a small rotation to the half shaft first.

b) Write down the structural equation(s) and identify them as continuity or compatibility equation(s).

c) Obtain the input output relation (input : force F on the mass, output : angle θ of the half shaft)

Ans.: c) $mL\ddot{\theta}+\frac{bd^2}{L}\dot{\theta}+\frac{kd^2}{L}\theta=F$

PROBLEM I-5



Although a car is a complicated mechanical system, simple models can be used with acceptable accuracy in analyzing certain aspects of vehicle dynamics. For example, the well-known "quarter car model" is a two degree freedom model that is used in ride comfort studies. In this model one suspension of the car is modeled with its mass that is not supported by the suspension springs (unsprung mass, m) and its share of the vehicle body mass (sprung mass, M) supported by suspension springs. k_s, c_s, and k_t represent the suspension stiffness, suspension damping coefficient, and tire stiffness, respectively, for the suspension considered. For the given quarter car model:

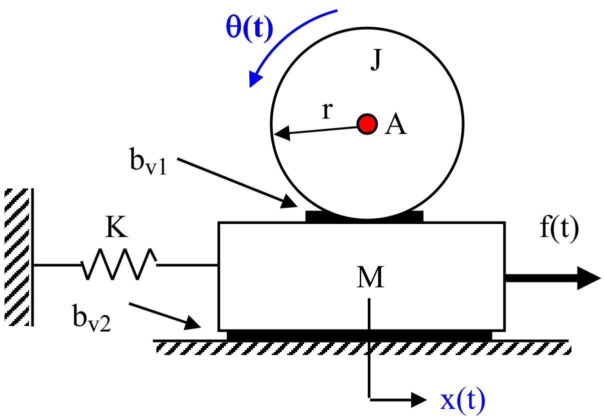
a) Identify the elements and write down the elemental equations.
b) Write down the structural equation(s) and identify them as continuity or compatibility equation(s).
c) Obtain the differential equations relating the input (road profile y_o) and the vertical motion (y_2 and y_1) of the sprung and unsprung masses.

<u>Ans. :</u> c) $M\ddot{y}_2 + c_s\dot{y}_2 + k_s y_2 = c_s\dot{y}_1 + k_s y_1$, $m\ddot{y}_1 + (k_s + k_t)y_1 + c_s\dot{y}_1 = k_s y_2 + c_s\dot{y}_2 + k_t y_o$

Note that this is a 2 dof system.

PROBLEM I-6 : (See Nise, P2.39)

A disk of inertia **J** and radius **r** is constrained to move about the stationary axis A. Viscous damping with a coefficient $\mathbf{b_{v1}}$ exists between the disk and the mass **M**. Similarly, viscous damping with a coefficient $\mathbf{b_{v2}}$ exists between the mass and the ground. The motion of the mass is restrained by an ideal spring of stiffness **K**.



Denote the angular position of the disk and the position of the mass by **θ(t)** and **x(t)**. The input to the system is the force **f(t)** applied to the mass and the outputs are **x(t)** and **θ(t)**.

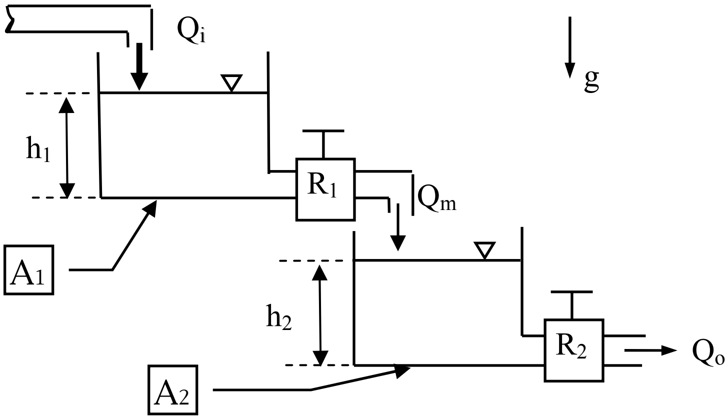
a) Identify the elements and write the elemental equations.
b) Write the structural equations.
c) Obtain the equation(s) of motion for the system.

Hint : This is a two-degree-of-freedom system and thus two equations are needed.

Ans. : c) $M\ddot{x} + (b_{v1} + b_{v2})\dot{x} + Kx - rb_{v1}\dot{\theta} = f(t)$, $J\ddot{\theta} + r^2 b_{v1}\dot{\theta} - rb_{v1}\dot{x} = 0$

PROBLEM I-7:

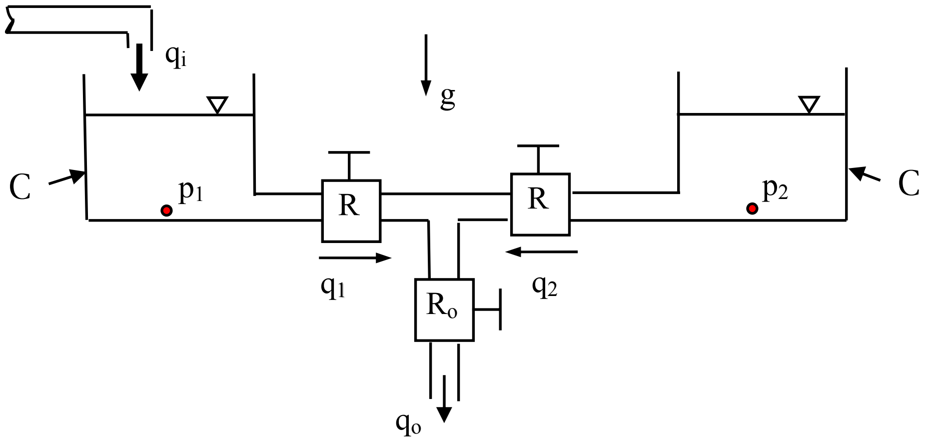
A fluid system consists of two tanks with cross-sectional areas of A_1 and A_2. Valves of resistance coefficients R_1 and R_2 are fitted to the outlet of each tank. The fluid flows from the first tank into the second tank as illustrated in the figure. Fluid flows into the first tank and out of the second tank at rates of Q_1 and Q_2. The density of fluid is denoted by ρ.



Follow the procedure outlined in the first problem and obtain the input output relation (input : Q_i, output : Q_o).

<u>Ans.:</u> $$Q_i = \frac{A_1 A_2 R_1 R_2}{\rho^2 g^2}\left(\frac{d^2 Q_o}{dt^2}\right) + \frac{A_1 R_1 + A_2 R_2}{\rho g}\left(\frac{dQ_o}{dt}\right) + Q_o$$

PROBLEM I-8



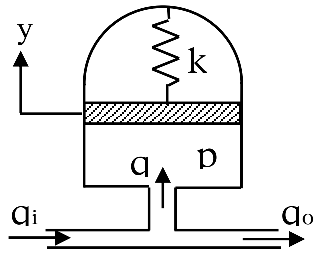
The fluid system shown above consists of two tanks each with a capacitance of C and three valves of resistance coefficients R and R_o, as illustrated. The fluid flow rates from the two tanks, and the pressures at the bottom of the tanks are denoted by q_1 and q_2, and p_1 and p_2. The density of fluid is denoted by ρ.

a) Identify the elements and write down the elemental equations.
b) Write down the structural equation(s) and identify them as continuity or compatability equation(s).
c) Derive the input (q_i)-output (q_o) relation for the system.

Ans. : c) $C(2R_o + R)\dfrac{dq_0}{dt} + q_0 = q_i$

PROBLEM I-9:

A hydraulic accumulator is illustrated in the figure. It is used to damp out pressure pulses by storing fluid during pressure peaks and releasing fluid during periods of low pressure.



Determine the governing equation for this system in terms of the variables p (pressure at the junction, q_i (incoming volume flow rate), and q_o (outgoing volume flow rate) which represent small deviations from the equilibrium conditions.

Neglect any friction as well as any pressure drop at the junction. The piston has a mass m and area A, the spring constant is k, and y is the displacement of the piston from the equilibrium position.

<u>Ans.:</u> $m(\ddot{q}_i - \ddot{q}_o) + k(q_i - q_o) = A^2\dot{p}$

PROBLEM I-10:

In order to harden a steel shaft, it is heated to a temperature of T_{os} and then it is quenched in a small can containing m_b kg of water at an initial temperature of T_{ob}. Thus while the shaft is cooling down, the bath temperature will be increased as a result of the quenching process. The convective heat transfer coefficient, and the surface area and mass of the steel shaft are denoted by h, A, and m_s, respectively. The specific heats of the steel shaft and the water are denoted by c_s and c_b.

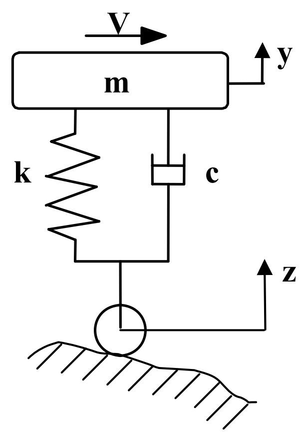
Obtain the differential equations governing the variations of the shaft temperature T_s and bath temperature T_b as functions of time.

<u>Ans.:</u> $\frac{m_b c_b m_s c_s}{hA}\frac{d^2T_s}{dt^2} + (m_b c_b + m_s c_s)\frac{dT_s}{dt} = 0$, $\frac{m_b c_b m_s c_s}{hA}\frac{d^2T_b}{dt^2} + (m_b c_b + m_s c_s)\frac{dT_b}{dt} = 0$

II/ Transfer Functions & Block Diagrams

PROBLEM II-1:

Consider a simple dynamic model of a vehicle travelling on a rough road surface. The mass represents the mass of the body and the spring and damper represent the suspension springs and dampers. The input is the road surface profile velocity, i.e. $\dot{\mathbf{z}}$. If the output is the displacement **y** of the body,

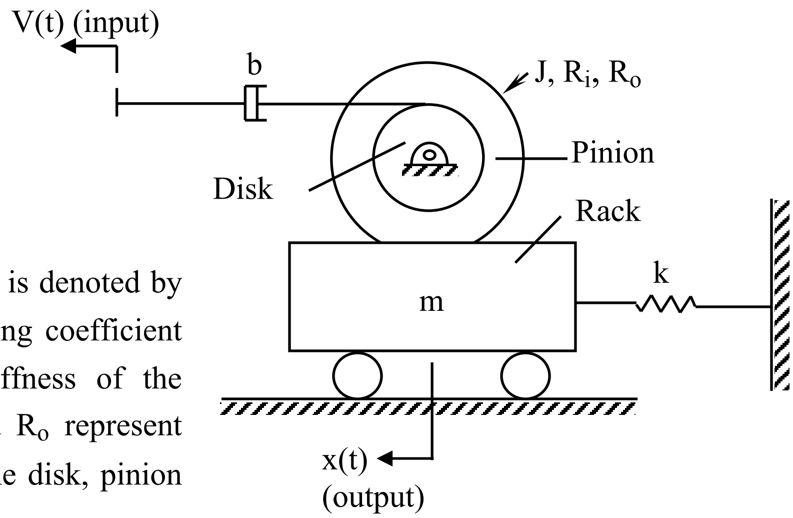


a) obtain the input-output relation,
b) write the transfer function,
c) draw the block diagram.

Ans.: a) $m\ddot{y}+c\dot{y}+ky=c\dot{z}+kz$

PROBLEM II-2:

The mechanical system shown in the figure illustrates a rack and pinion pair. The pinion has a companion disk as shown.



The mass of the translating rack is denoted by m, b denotes the viscous damping coefficient of the rope, and k is the stiffness of the restraint on the rack. J, R_i and R_o represent total inertia of the pinion and the disk, pinion and disk radii, respectively.

For this system,

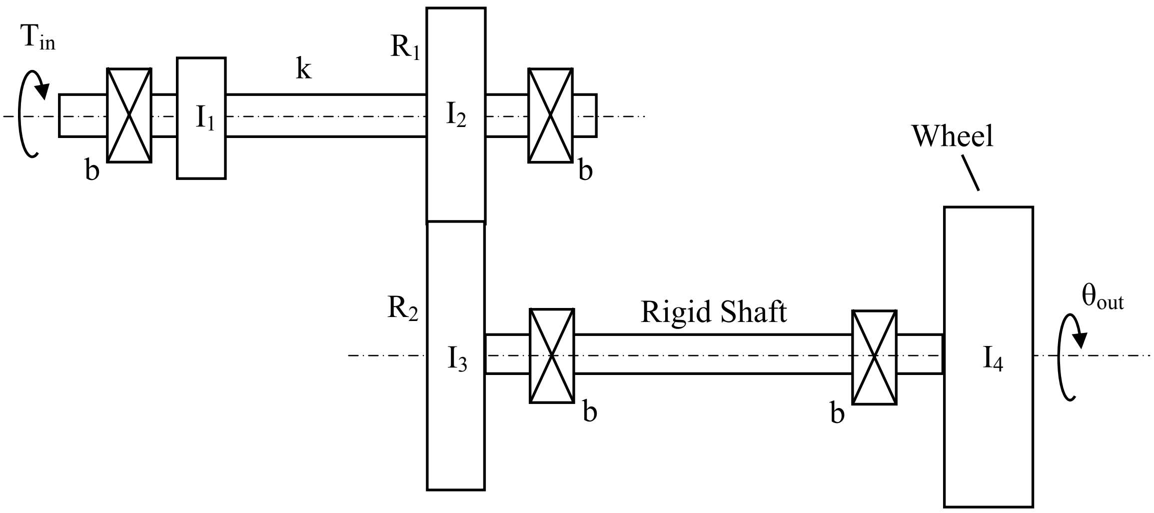
a) obtain the input-output relation,
b) write the transfer function, and
c) draw the block diagram.

Ans.: a) $(m+\frac{J}{R_o^2})\ddot{x}+\left(b\frac{R_i^2}{R_o^2}\right)\dot{x}+kx=\left(-b\frac{R_i}{R_o}\right)V$

PROBLEM II-3:

The transmission system shown below consists of 2 shafts, 2 gears and a wheel. One of the shafts is assumed to be rigid while torsional stiffness of the other is denoted by k. Inertias of the shafts, gears and the wheel are lumped and represented by I_1 to I_4. R_1 and R_2 are the radii of the pinion and the gear. Viscous friction exists in all bearings with viscous friction coefficient b. If the input of the system is the torque applied on shaft 1 and the output is the angular position of wheel,

a) obtain the input-output relation,
b) write the transfer function, and
c) draw the block diagram.



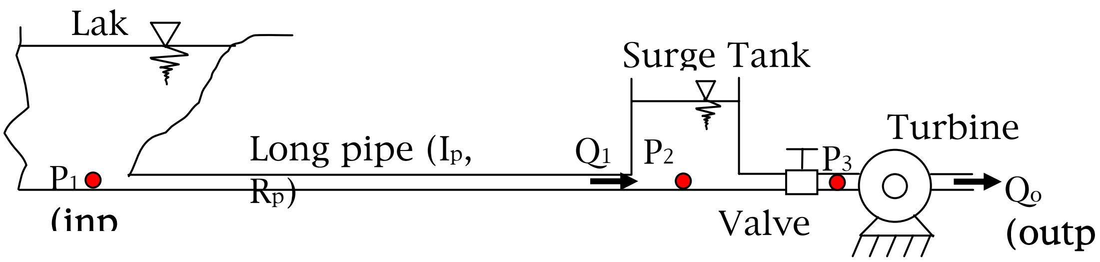
Ans. : a) $I_1\ddot{\theta}_1 + b\dot{\theta}_1 + k\theta_1 = T_{in} + kn\theta_3$ and $\left[I_2 n^2 + (I_3 + I_4)\right]\ddot{\theta}_3 + (n^2+2)b\dot{\theta}_3 + kn^2\theta_3 = kn\theta_1$

PROBLEM II-4:

For the system illustrated in the figure, model the turbine as a resistive element (load) with a resistance coefficient of R_L and

a) obtain the input-output relation,
b) write the transfer function,
c) draw the block diagram.

Note that the lake is assumed to be have infinite capacitance (show that the time rate of change of pressure becomes zero for any flow rate under this assumption !) and thus can be modeled as a constant pressure source.

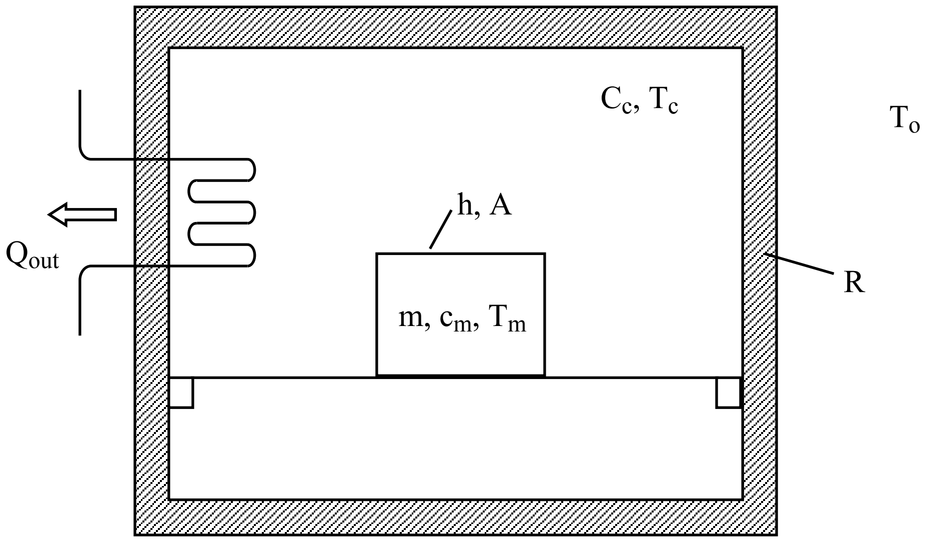


Ans.: a) $C_f I_P (R_v + R_L) \dfrac{d^2 Q_o}{dt^2} + \left[C_f (R_v + R_L) R_P + I_P\right] \dfrac{dQ_o}{dt} + (R_v + R_L + R_P) Q_o = P_1$

PROBLEM II-5:

In the cooling system shown below, heat is removed at a rate, $Q_{out,}$ from the cooling room. Although the walls of the cooler are isolated, there is still heat transfer between the cooler and the surroundings. The thermal resistance of the cooler walls is denoted by R and the ambient temperature is given as T_o. The cooler room has a thermal capacitance of C_c and the temperature inside the cooler is denoted by T_c. Consider a mass m, with a specific heat c_m, placed in the room. Taking the convective heat transfer coefficient between the mass and the cooler room as h, and the surface area of the mass as A,

a) obtain the differential equation relating T_m (temperature of the mass) to Q_{out} and T_o,
b) write the transfer functions between T_m and Q_{out}, and T_m and T_o,
c) draw the block diagram.



Ans.: a) $RC_cR_mC_m\dfrac{d^2T_m}{dt^2}+(RC_m+RC_c+R_mC_m)\dfrac{dT_m}{dt}+T_m=Rq_{out}-T_0$

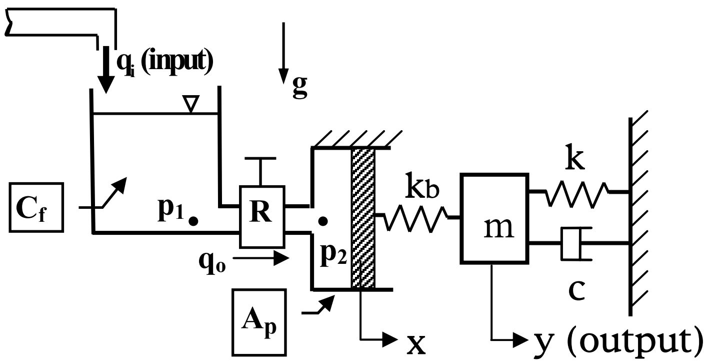
PROBLEM II-6:

Consider the hydro-mechanical system illustrated in the figure below.

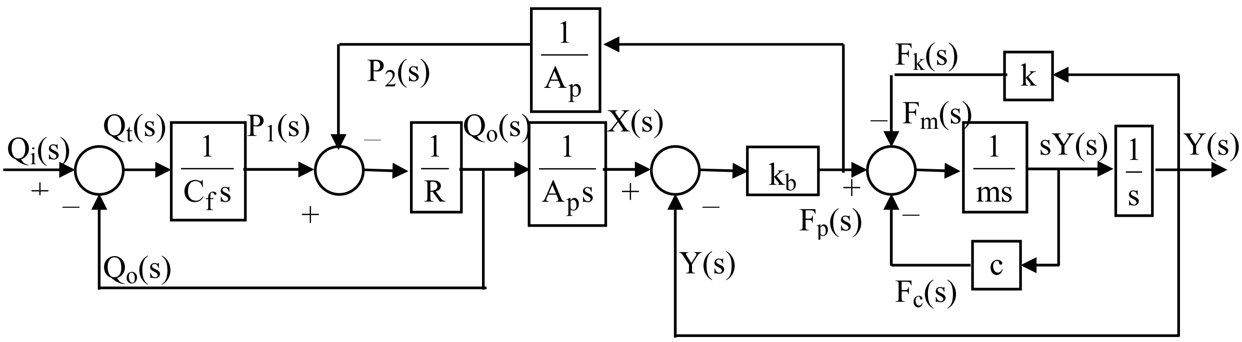
a) Identify and list all the elements in this system. Write down all the elemental equations. Note that you can write two elemental equations for the cylinder&piston as it is a transformer between hydraulic and mechanical parts of the system.

b) Write down all the structural equations; first the continuity and then the compatibility equations.

c) Draw the detailed block diagram for the system making use of the Laplace transforms of the elemental and structural equations.



Ans. : c)

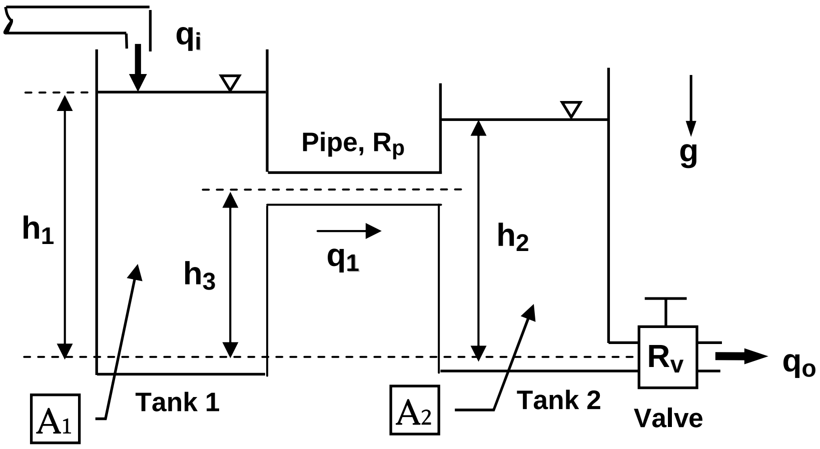


PROBLEM II-7:

A hydraulic system is illustrated in the figure below. Assume that the fluid has a density ρ and is incompressible, $\mathbf{h_2} > \mathbf{h_3}$, and the pipe is short. $\mathbf{R_p}$, $\mathbf{R_v}$, $\mathbf{A_1}$, and $\mathbf{A_2}$ denote the valve and pipe resistances and tank areas, respectively.

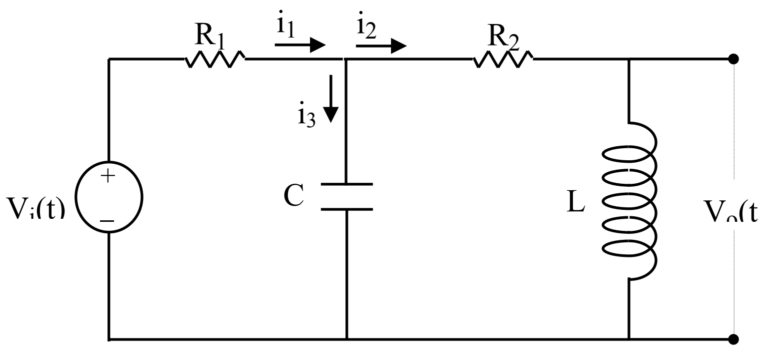
a) Identify the elements and write down the corresponding elemental equations.

b) Obtain the differential equation relating the input (fluid flow rate $\mathbf{q_i}$ into tank 1) to the output (height of the fluid $\mathbf{h_2}$ in tank 2).



Ans. : b) $\left(\frac{R_p A_1 A_2}{\rho g}\right)\frac{d^2 h_2}{dt^2} + \left[A_1\left(1+\frac{R_p}{R_v}\right) + A_2\right]\frac{dh_2}{dt} + \left(\frac{\rho g}{R_v}\right)h_2 = q_i$

PROBLEM II-8:



For the electrical circuit illustrated above, which consists of two resistances (R_1, R_2), a capacitance (C), and an inductance (L),

a) obtain the governing differential equations for the system,

b) find the transfer functions $\frac{I_2(s)}{V_i(s)}$ and $\frac{V_o(s)}{V_i(s)}$,

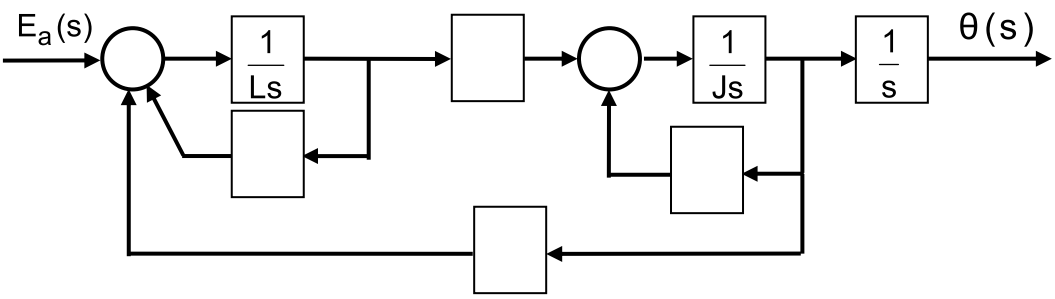
c) draw the block diagram considering $V_i(t)$ as input and $V_o(t)$ as output.

Ans. : a) $\frac{di_1}{dt}R_1 + \frac{i_1 - i_2}{C} = \frac{dV_i}{dt}$, $\frac{di_2}{dt}R_2 - \frac{i_1 - i_2}{C} + L\frac{d^2 i_2}{dt^2} = 0$

PROBLEM II-9:

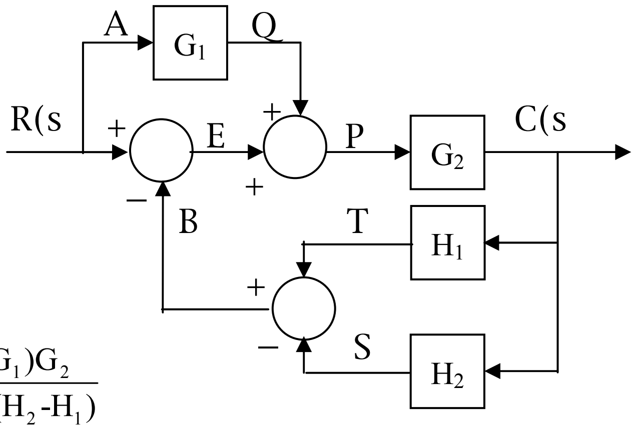
Complete the block diagram of a system represented by the equations below. Make sure that correct signs at the summers are also inserted and enter the relevant variable name on every signal.

$$e_b = K_b\omega, \quad L\frac{di}{dt} + Ri + e_b = e_a, \quad J\dot{\omega} + b\omega = T = Ki, \quad \omega = \dot{\theta}$$



PROBLEM II-10:

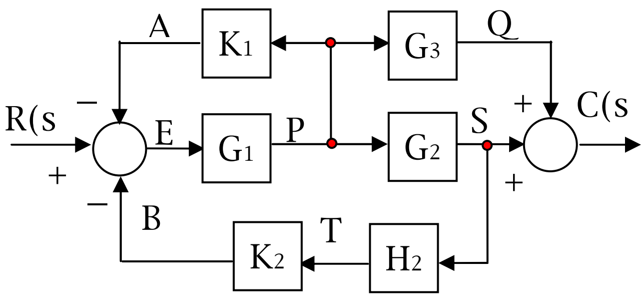
Simplify the block diagram to obtain the overall transfer function. You can choose either block diagram algebra or manipulate the equations for the blocks.



Ans. : $\frac{C(s)}{R(s)} = \frac{(1+G_1)G_2}{1+G_2(H_2-H_1)}$

PROBLEM II-11:

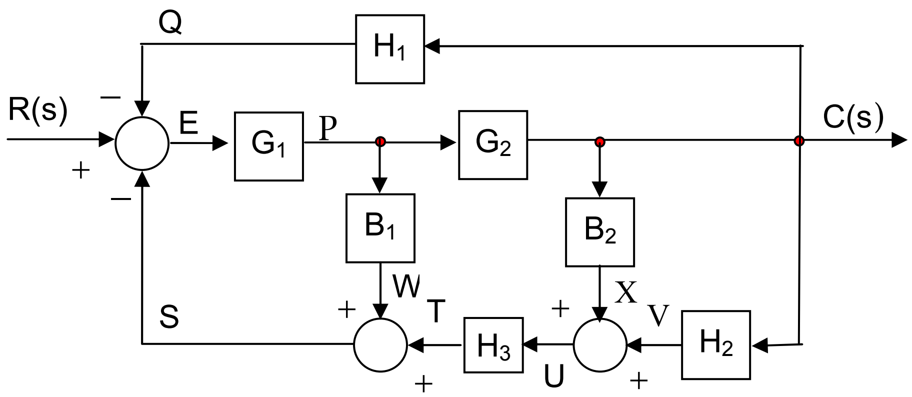
Simplify the block diagram down to a single block and obtain the overall transfer function for the system. You can choose either block diagram algebra or manipulate the equations for the blocks.



Ans. : $\frac{C(s)}{R(s)} = \frac{G_1(G_2+G_3)}{1+K_1G_1+K_2G_1G_2H_2}$

PROBLEM II-12:

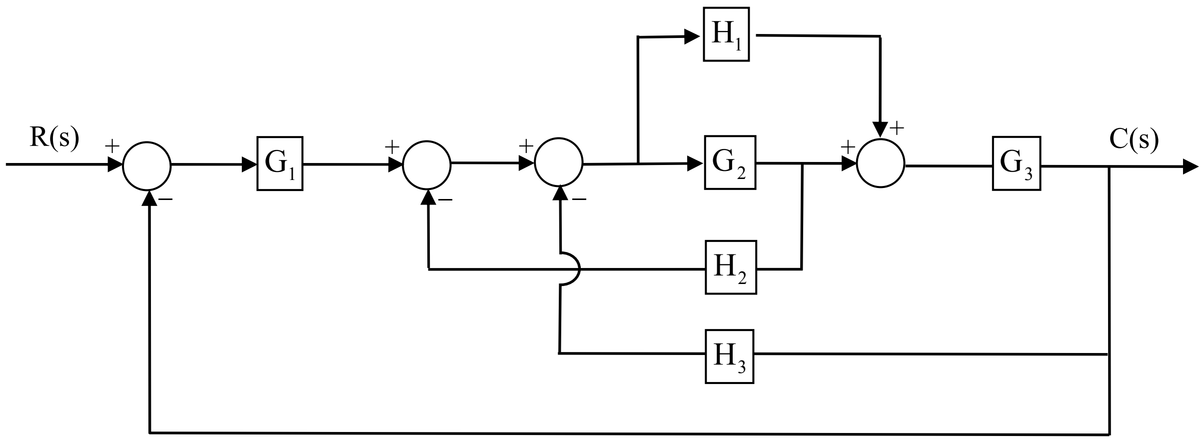
Reduce the block diagram, given below, to a single block and write down the overall transfer function for the system.



Ans. : $\frac{C}{R} = \frac{G_1G_2}{1 + G_1B_1 + G_1G_2H_1 + G_1G_2H_3B_2 + G_1G_2H_2H_3}$

PROBLEM II-13:

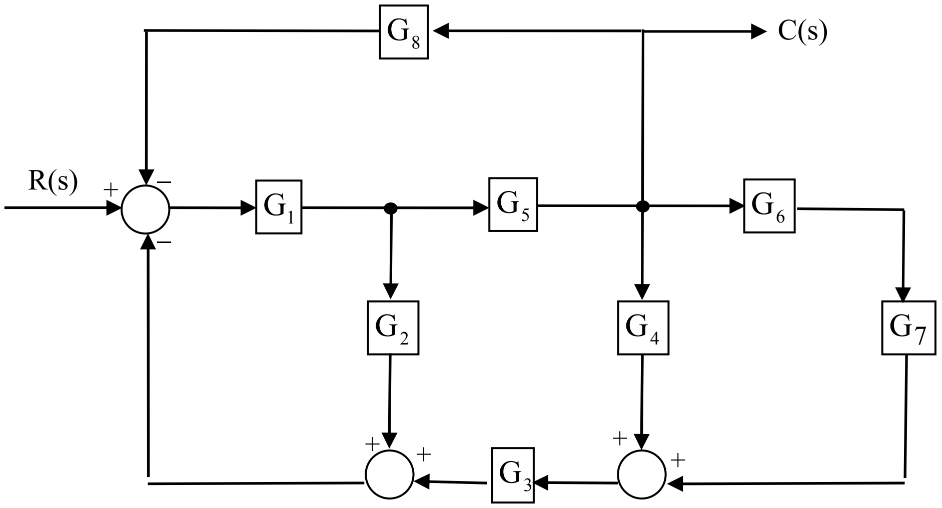
Obtain the closed loop transfer function, $T(s)=\frac{C(s)}{R(s)}$, for the block diagram given below.



Ans. : $\frac{(H_1 + G_2)G_1G_3}{1 + (H_1 + G_2)G_3H_3 + G_2H_2 + (H_1 + G_2)G_1G_3}$

PROBLEM II-14:

Obtain the closed loop transfer function, $T(s)=\dfrac{C(s)}{R(s)}$, for the block diagram given below.



$$\underline{\text{Ans. :}}\ \frac{C(s)}{R(s)}=\frac{G_1G_5}{1+G_1G_2+G_1G_5G_8+G_1G_3G_5(G_4+G_6G_7)}$$

III/ Control System Components

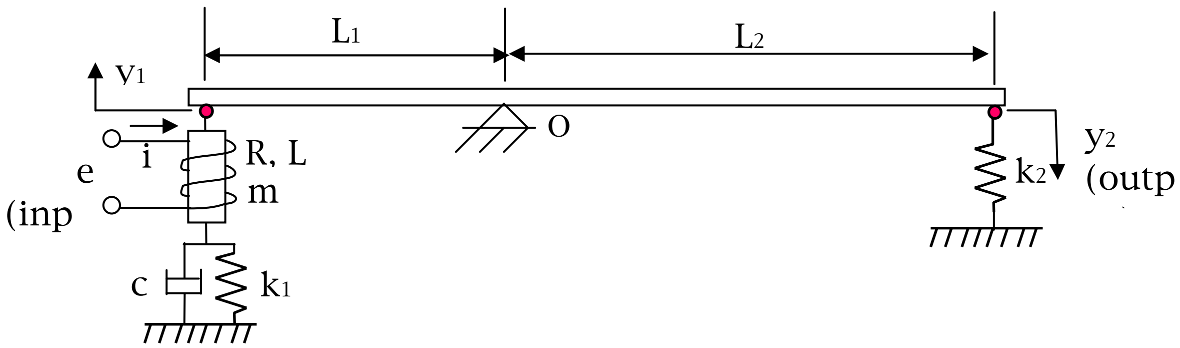
PROBLEM III-1:

A linear actuator (solenoid) consists of a mass **m,** and a parallel connection of a linear spring of stiffness $\mathbf{k_1}$ and a linear viscous damper of coefficient **c**. The resistance of the solenoid coil is **R** and its inductance is **L**. The electromagnetic coupling constant is **K.** The actuator excites one end of a massless rigid bar which is pivoted at point O as illustrated in the figure. The motion of the other end of the bar is restrained by a linear spring of stiffness $\mathbf{k_2}$.

a) Write down the elemental and structural equations for the system. Neglect gravity.

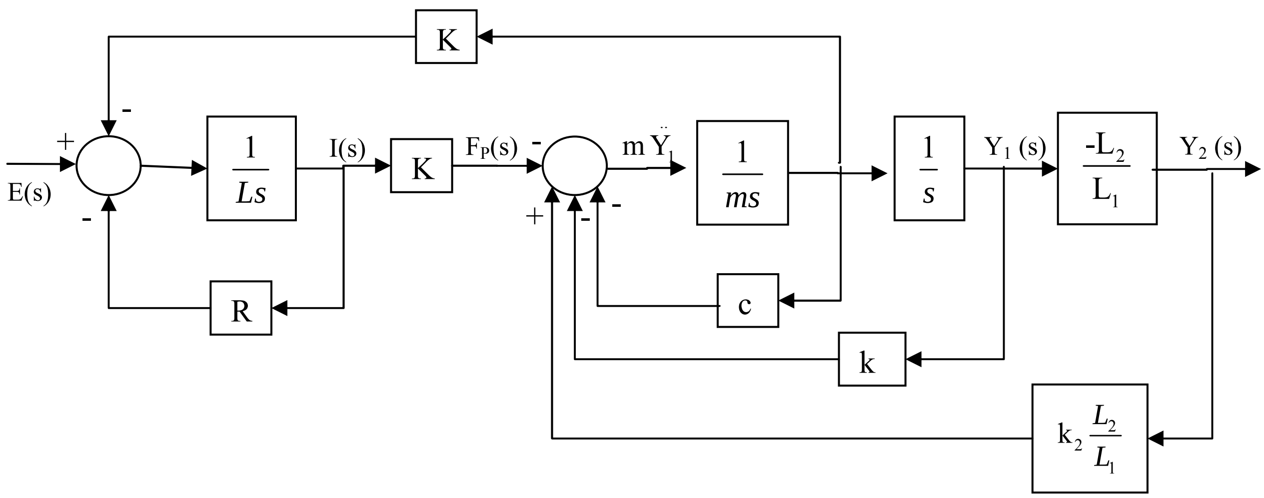
b) Draw the detailed block diagram showing each element by a block. Do not combine blocks.

c) Obtain the overall transfer function.



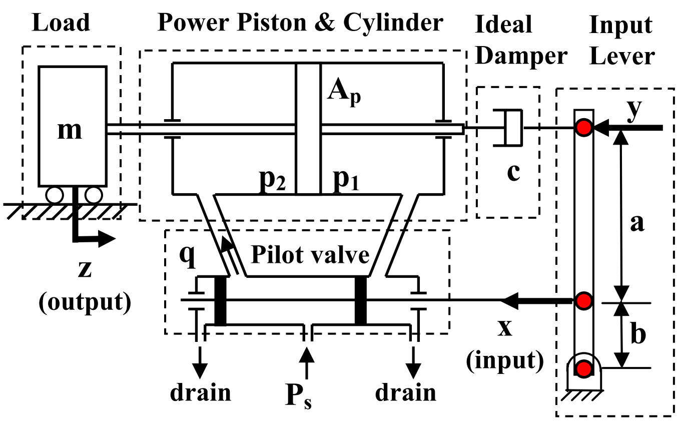
Ans.:

$$\frac{Y_2(s)}{E(s)} = \frac{K}{\left[mL\frac{L_1}{L_2}\right]s^3 + \left[(mR+cL)\frac{L_1}{L_2}\right]s^2 + \left[(cR+k_1L-K^2)\frac{L_1}{L_2} + k_2L\frac{L_2}{L_1}\right]s + \left[k_1\frac{L_1}{L_2}R + k_2\frac{L_2}{L_1}R\right]}$$



PROBLEM III-2:

A hydraulic servomotor, controlled by a rigid massless lever, drives a load as illustrated in the figure.



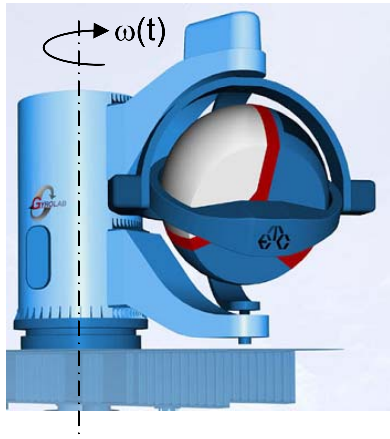
a) Write the differential equation representing the input—output relation for the system taking **x** as the input an **z** as the output.

b) Obtain the overall transfer function. Note that **m** represents the total mass of the load, piston, and piston rod.

You can use the linearized relation $q = K_1 x - K_2 \Delta p$.

Ans. : $K_2 m\ddot{z} + \left(A_p^2 + K_2 c\right)\dot{z} = -K_2 c\left(\frac{a+b}{b}\right)\dot{x} + K_1 A_p x$

PROBLEM III-3:



A centrifuge used in training pilots is shown in the picture and is illustrated with a conceptual block diagram below. The centrifuge consists of rotating arms, carrying a gondola, rotated at a controlled speed to simulate the high accelerations to which the pilots are subjected during flight.

a) Write down the elemental equations for the component models and obtain the overall transfer function relating the input **θ(t)** (angular position of the potentiometer knob) to the output **ω(t)** (angular speed of the centrifuge arms).

The main specifications of the system components are listed below.

Centrifuge : Moment of inertia **J**.

Gearbox : Reduction ratio **n,** negligible moment of inertia.

DC Servomotor : Armature resistance **R**, Armature inductance **L**, Torque constant **K**, Back emf constant $\mathbf{K_b}$, negligible moment of inertia.

DC Amplifier : Amplification factor $\mathbf{K_a}$.

Potentiometer : Constant $\mathbf{K_p}$.

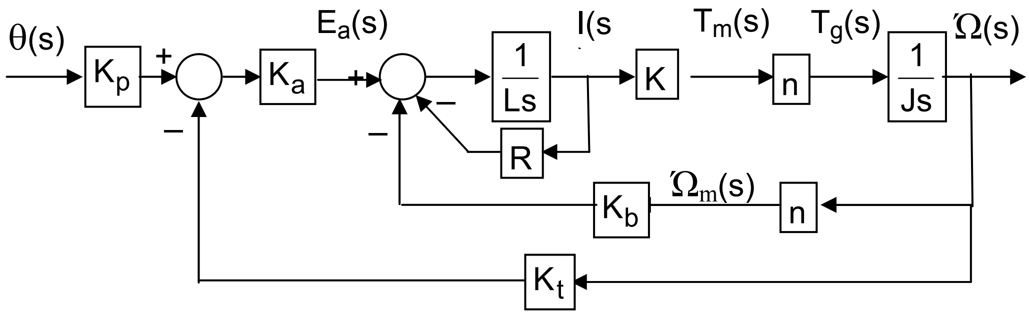
Tachometer : Constant $\mathbf{K_T}$.

b) Draw a detailed block diagram for the system.

Ans. :

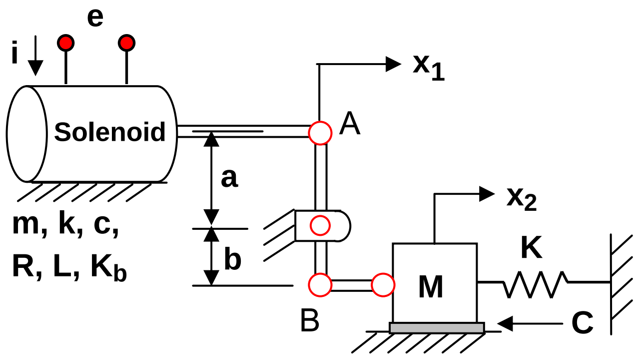
a) $$\Omega(s) = \left[\frac{nKK_aK_p}{JLs^2 + Rs + nK\left(K_aK_t + nK_b\right)}\right]\theta(s)$$

b)



PROBLEM III-4:

Consider the system illustrated in the figure. Note that the bar AB is rigid and massless (ignore the bar between point B and the mass). Consider small displacements only. The solenoid parameters are the mass **m** of the plunger, ideal spring constant **k**, viscous damping coefficient **c** and resistance **R** and the inductance **L** of the coil. The electromagnetic coupling constant, coil current, and the voltage applied to solenoid terminals are denoted by $\mathbf{K_b}$, **i**, and **e**. The load is modelled by a mass **M** and a spring of constant **K**, together with viscous friction of coefficient **C** between the mass and the ground.



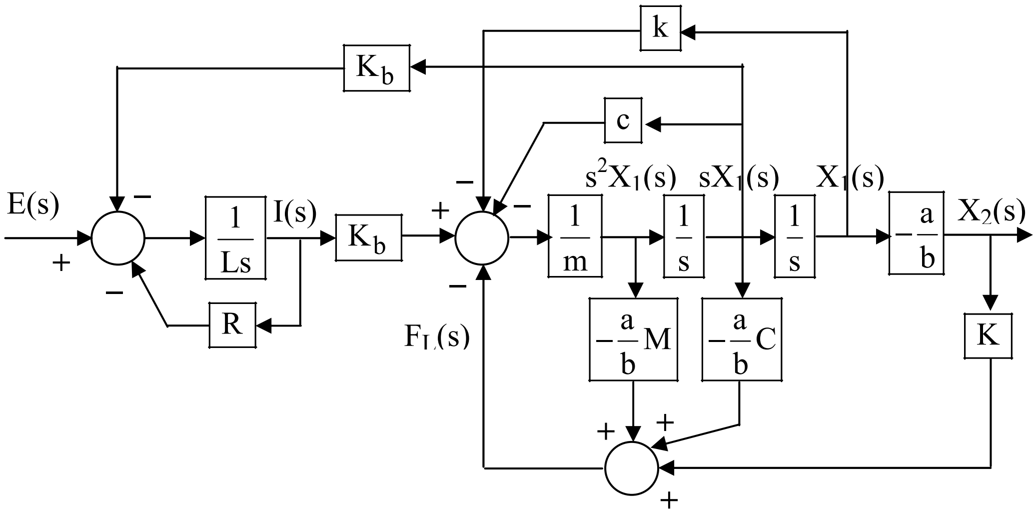
a) Represent solenoid dynamics with two differential equations; one for the mechanical and one for the electrical side. Note that a resistive force F_L acts on the mechanical side due to the load represented by the bar AB and the mass-spring-viscous friction subsystem.

b) How many degrees of freedom does the system have ? Explain.

c) Write down all the relevant elemental and structural equations for the rest of the system.

d) Draw the detailed block diagram of the system with **e** as the input and $\mathbf{x_2}$ as the output.

Ans. :

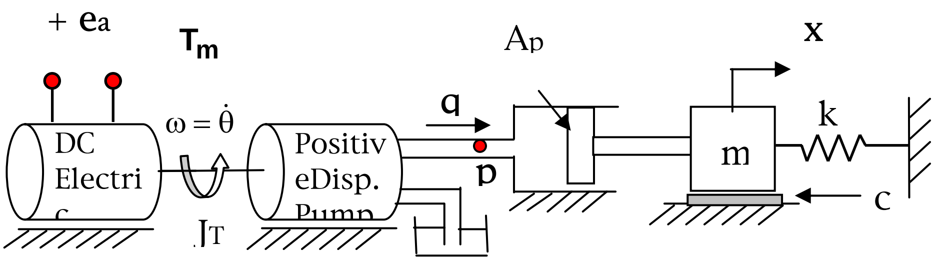

k
K_b
c
E(s)
1/Ls
I(s)
K_b
R
s²X₁(s)
sX₁(s)
X₁(s)
1/m
1/s
1/s
-a/b
X₂(s)
-a/b M
-a/b C
K
F_L(s)

PROBLEM III-5:

An electro-hydro-mechanical system is shown in the figure. A dc electric motor drives a positive displacement pump and is controlled by the armature voltage $\mathbf{e_a}$. Relevant parameters are indicated on the figure. The total inertia of the motor and the pump is denoted by $\mathbf{J_T}$. The load is represented by a mass-spring system. Viscous friction is assumed between the mass and the ground.

The pump flow rate **q** is assumed to be proportional to to shaft speed, i.e. : $q = K_f \omega$

The resistance and inertance of the pipes as well as the friction between the piston and the cylinder are negligible.



R_a = armature resistance	K_f = pump flow constant	**m** = mass
L_a = armature inductance	J_T = total inertia (motor + pump)	**k** = spring constant
K_b = back emf constant	θ = shaft angular position	**c** = coefficient of viscous friction

a) Write down a complete <u>set</u> of equations for the dc electric motor. Clearly indicate the terms coupling the electrical and mechanical parts. Draw a <u>detailed</u> block diagram of the motor (Input : armature voltage, $\mathbf{E_a}$; Output : Motor shaft angular position, θ_m).

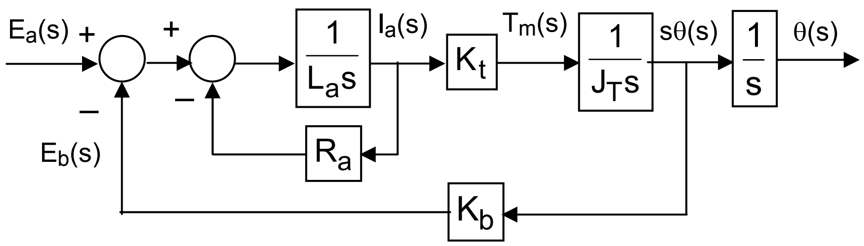
b) Draw a block diagram of the pump (Input : shaft speed, ω; Output : pump flow rate, **q**).

c) Write down a complete <u>set</u> of equations for the hydraulic actuator (cylinder+piston). Draw a block diagram of the actuator (Input : pump flow rate, **q**; Output : load displacement, **x**). Note that you need to use only one of the equations to draw the block diagram.

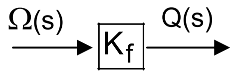
d) Now assemble all the previously drawn individual block diagrams (Input : armature voltage, $\mathbf{E_a}$; Output : load displacement, **x**). Note that the angular position of the shaft is of no interest to us and thus need not be included in the block diagram.

e) Note that the effects of the load dynamics (mass+spring+friction) do not appear in the block diagram. This is because once the flow rate is specified, the velocity of the mass is dictated. On the other hand, load dynamics determine the pressure at the outlet of the pump. Thus write a differential equation to relate pump pressure **p** to the displacement **x** of the mass. Draw a detailed block diagram of the load (Input : load displacement, **x**; Output : pressure at pump outlet, **p**). Do not worry if you have to enter "s" or its powers in the numerator !

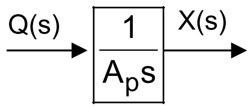
Ans. :



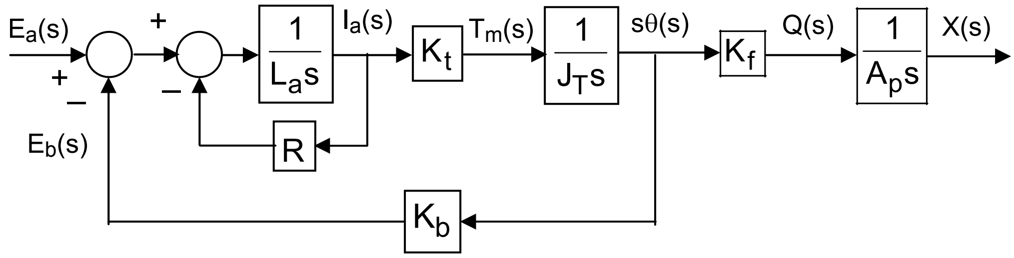
a)



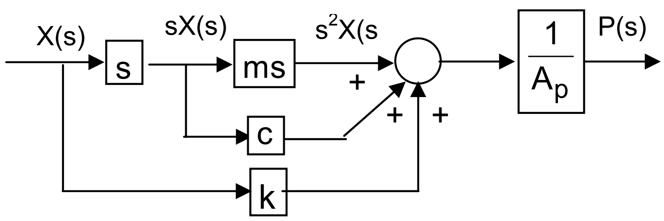
b)



c)



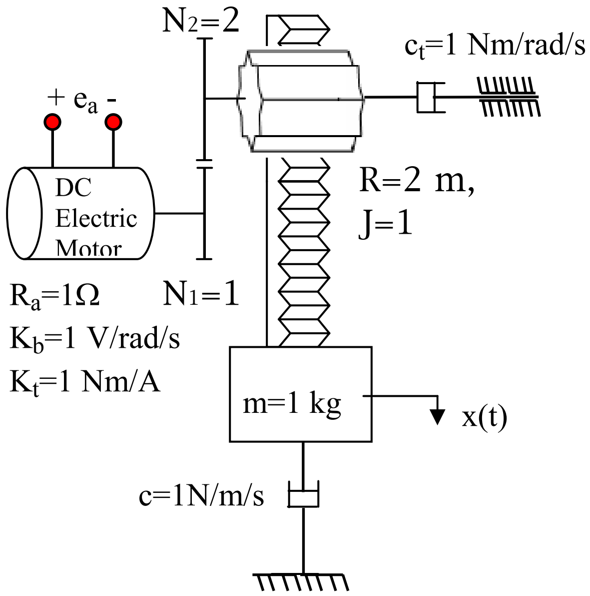
d)



e)

PROBLEM III-6: (See Nise, Problem 2.46)

Consider the electro-mechanical system illustrated in the figure. A fixed current is applied to the field circuit of the electric motor. You can neglect the inertia of the armature rotor and the inductance of the armature circuit.

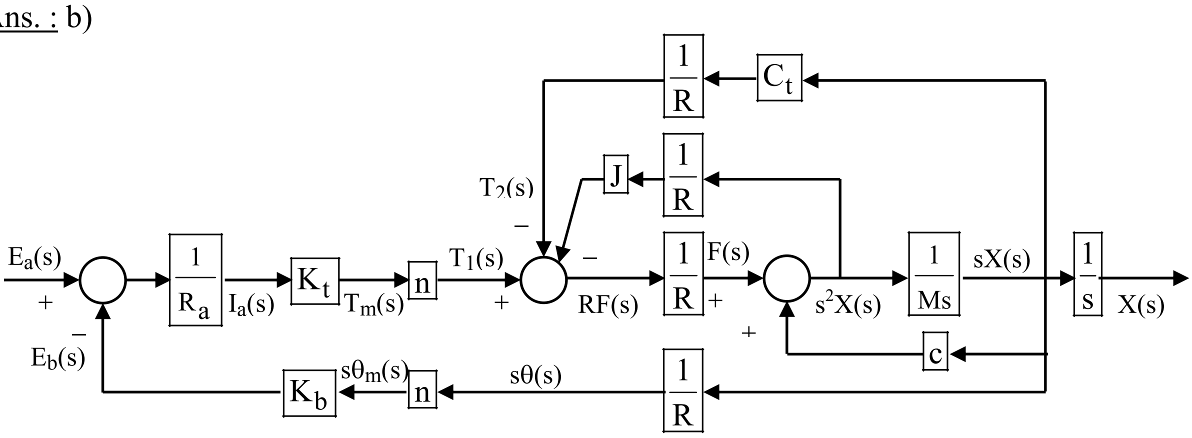


a) Write the elemental equations.

b) Draw the corresponding detailed block diagram of the system. Use symbols only in the blocks. Do not insert numerical values yet !

c) Insert the numerical values for the parameters and find the overall transfer function $X(s)/E_a(s)$ – which should not contain any symbols with the exception of Laplace variable "s".

Ans. : b)

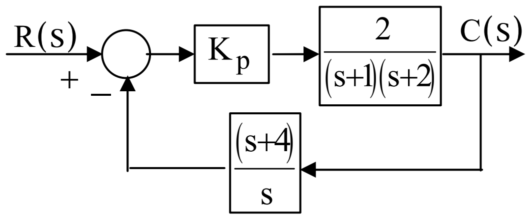


c) $G(s) = \dfrac{\frac{4}{5}}{s^2 + \frac{9}{4}s}$

IV/ Stability

PROBLEM IV-1:

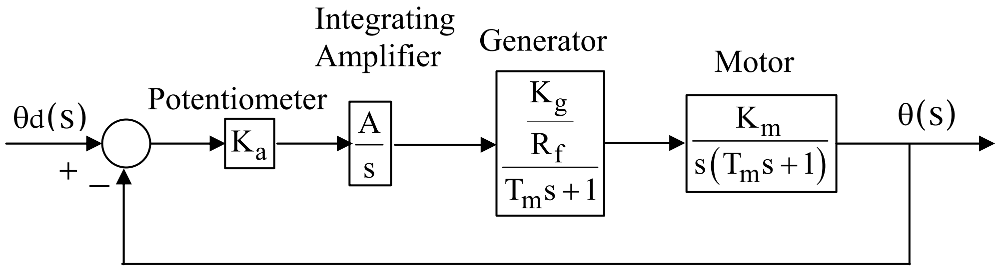
Write down the characteristic equation for the system with the block diagram shown in the figure.



Ans. : $s^3 + 3s^2 + 2(1 + K_p)s + 8K_p = 0$

PROBLEM IV-2:

Block diagram of a position control system for a space camera is given. Show that the system is unstable and determine the number of poles with positive real parts. All constants take positive values only.



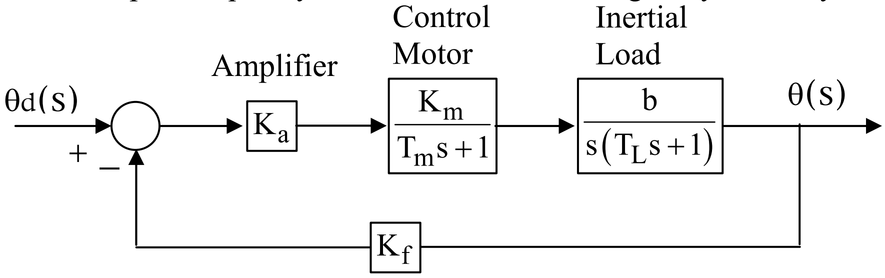
Ans. : There are two roots with positive real parts.

PROBLEM IV-3:

Consider the control system for an inertial load, as represented by the block diagram shown.

a) Determine the value of the open loop gain (**$K=bK_aK_mK_f$**) which results in a marginally stable system, if the time constants for the motor (**T_m**) and for the load (**T_L**) are specified to be 0.05 and 0.20 seconds, respectively.

b) What is the undamped frequency of oscillation of the marginally stable system ?



Ans. : a) 0<K<25, b) for K=25 the system poles are -25, -10j, 10j, which yield asymptotic stability and the undamped frequency of oscillations of the marginally stable system 10 rad/s; for K=0 the system poles are -20, -5, 0, which again result in asymptotic stability and in this case the response is not oscillatory.

PROBLEM IV-4:

For the two characteristic equations given, use Routh's array to determine whether there are any roots with positive real parts, and if there are, the number of such roots.

a) $D(s)= s^5+7s^4+19s^3+25s^2+16s+4$
b) $D(s)= s^6+2s^5+8s^4+12s^3+20s^2+16s+16$

Ans. : a) None, b) None.

PROBLEM IV-5:

For the characteristic equations listed below, use Matlab to define the characteristic equation by a row vector of the coefficient of powers of s as illustrated. Then print the command roots(p) and press enter to find the roots. Comment on the stability of the systems.

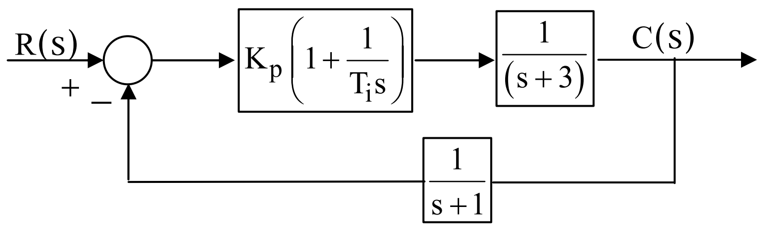
$s^3+12s^2+46s+52=0$ → p=[1, 12, 46, 52] → roots(p)

a) $3s^4+10s^3+5s^2+8s+8=0$
b) $s^6+2s^5+8s^4+12s^3+20s^2+16s+16=0$

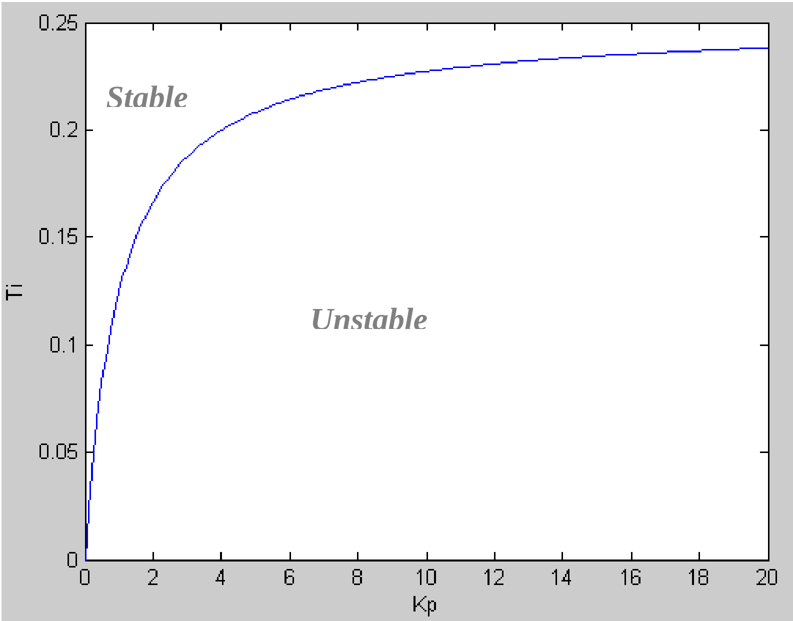
Ans. : a) -2.9729, 0.2533 + 0.9851i, 0.2533 - 0.9851i, -0.8670, b) -0.0000 + 2.0000i, -0.0000 - 2.0000i, -1.0000 + 1.0000i, -1.0000 - 1.0000i, 0.0000 + 1.4142i, 0.0000 - 1.4142i

PROBLEM IV-6:

Consider the block diagram for a control system. Determine the ranges of the control parameters $\mathbf{K_p}$ and $\mathbf{T_i}$ permissable for a stable system and plot the permissable range on the parameter plane.



Ans. : $T_i > \dfrac{K_p}{4(3+K_p)}$, $T_i > 0$, $K_p > 0$

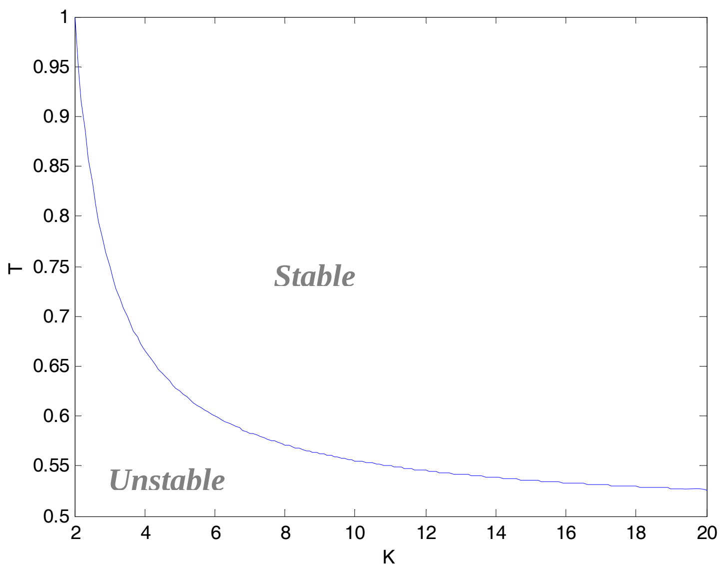


PROBLEM IV-7:

The characteristic polynomial of a control system is given as

$$D(s) = Ts^3 + 4Ts^2 + (K+3)Ts + K$$

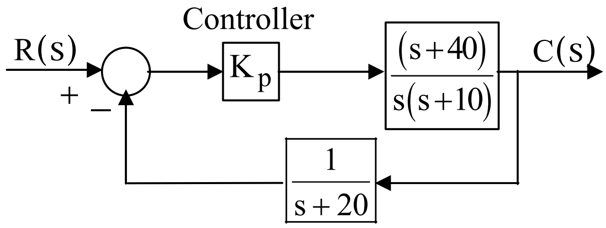
Find and plot the admissable ranges of T and K on the parameter plane such that the system has a stability margin of at least 1.



Ans. : $T > \dfrac{K}{2(K-1)}$, $T > 0$

PROBLEM IV-8:

Consider the system with the block diagram as shown.



a) Determine the value(s) of the controller gain $\mathbf{K_p}$ that will make the system marginally stable.

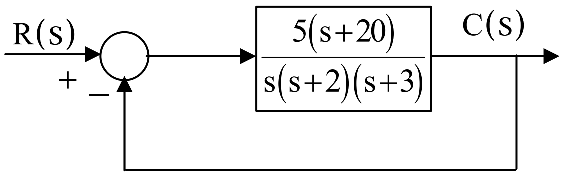
b) What will be the frequency of oscillations of the unforced response of the marginally stable system to an initial condition ?

c) If the value of $\mathbf{K_p}$ is selected as 200, determine if the stability margin of the system is less or more than 1.

Ans. : a) K_p=600 and K_p=0, b) 28.3 rad/s, c) More than 1.

PROBLEM IV-9:

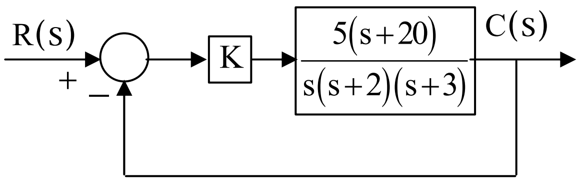
Consider the unity feedback system with the block diagram as shown.



a) Show that the system is unstable. Find the number of unstable poles.

Assume that a proportional controller of gain K>0 is added to the system.

b) Check if it is possible to stabilize the system by a proper choice of the proportional gain K. If the answer is yes, determine the range of values of K for which the system is stable.



c) For which value(s) of K will the system be marginally stable? Determine the frequency of oscillations of the unforced response of the marginally stable system.

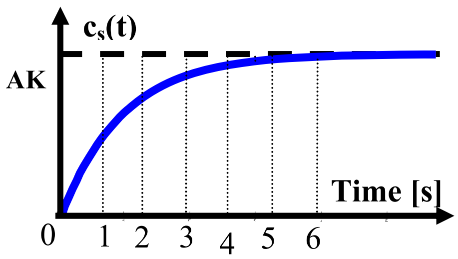
d) Determine the range of values of K such that the stability margin of the system is at least 0.5.

Ans. : a) Two poles with positive real parts, b) $0<K<2$, c) K=2, 2.83 rad/s, d) $0.096<K<0.500$.

V/ Transient Response

PROBLEM V-1:

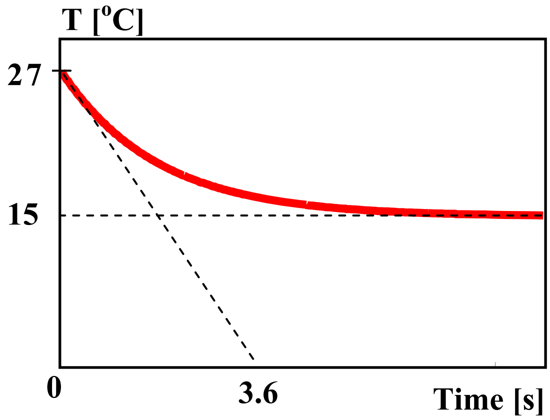
Estimate the time constant (illustrate on the figure) for a first order system with the step response as shown in the figure.



Ans. : ~2 seconds

PROBLEM V-2:

A thermometer, reading 27°C initially, is tested by dipping it in water at a constant temperature of 15°C. The variation of the thermometer reading is recorded and plotted as shown in the figure.



The same temperature brought back to steady state once more such that it reads 27 °C just above the surface of a lake.

It is then is lowered into the lake at a constant speed of 0.3 m/s. Water temperature in the lake decreases by 1.8°C for each meter below the surface. Determine the thermometer reading when the thermometer reaches a depth of 3 meters. What is the reading error for the thermometer ?

Ans. : 22.46 °C, -0.86 °C

PROBLEM V-3:

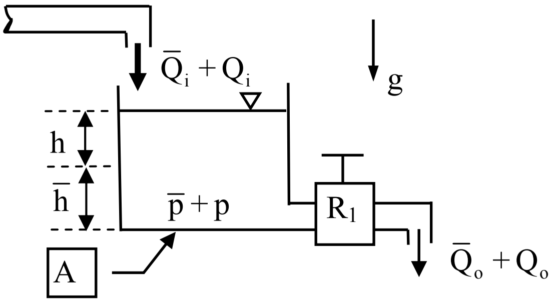
Transfer functions for two different first order systems are given below. Which of these two systems will have a faster response ? Explain.

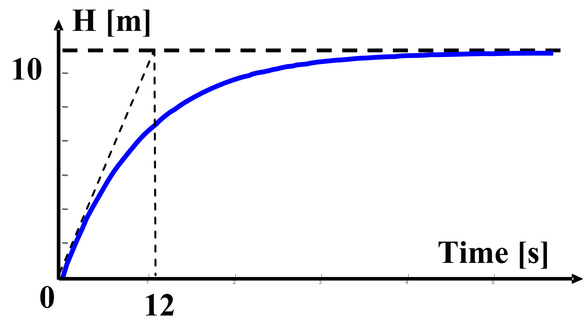
$$G_1(s) = \frac{4}{3s+1} \qquad G_2(s) = \frac{0.1}{s+0.2}$$

Ans. : System 1

PROBLEM V-4:

The water level in the tank, shown in the figure, as well as the volume flow rates in and out of the tank are initially constant at $\bar{h}, \bar{Q}_i$, and $\bar{Q}_o$. The volume flow rate into the tank is suddenly increased by 2 m^3/s. The increase in water level in the tank is plotted versus time. Determine the effective cross-sectional area of the tank.



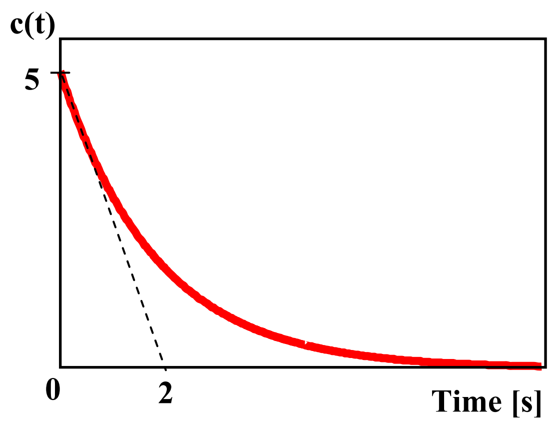


Hint : Write the input-output relation originally at steady state and then after the step input in flow rate. Be careful with the units - make sure you write the units for all calculated values !

Ans. : 2.4 m^2

PROBLEM V-5:

The response of a system, to an impulse of magnitude (strength) 0.1, is measured and plotted as shown in the figure. Estimate the response of the same system, 3 seconds after the application of a step input of magnitude 0.2.



Ans. : 15.5

PROBLEM V-6:

If the overall transfer function of a third order system is given as

$$G(s) = \frac{32}{(s+12)(s^2+4s+20.25)}$$

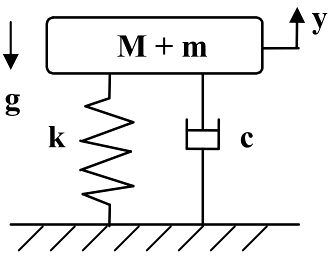
calculate an approximate value for the undamped natural frequency.

Hint : Consider the locations of the poles

Ans. : 4.5 rad/s

PROBLEM V-7:

Consider the single degree of freedom mass-spring-damper system illustrated in the figure. Assume that a load of mass m is placed on the mass M. Model the presence of the load as an initial condition on the displacement of the mass. Find an equivalent transfer function, the unit step response of which will represent the motion of the mass when the load is suddenly lifted off.



M	=	250	kg
m	=	70	kg
k	=	12	kN/m
c	=	800	N/m/s

Hint : Examine solved example 5-9 in Ogata.

Ans. : $G(s) = -\dfrac{(0.05717s^2 + 0{,}18294s)}{s^2 + 3.2s + 48}$

PROBLEM V-8:

Consider a standard second order system.

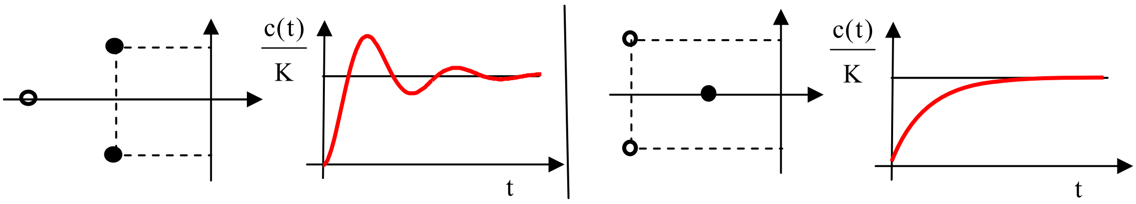
a) Starting with the expression for the general unit step response, obtain an expression to calculate the rise time.

b) Calculate the delay time for the system specified by an undamped natural frequency of 2.5 Hz and a damping ratio of 0.5.

Ans. : 0.082 s

PROBLEM V-9:

The unit step response due to black poles only are given. Superimpose the unit step response when all the poles are considered.



PROBLEM V-10:

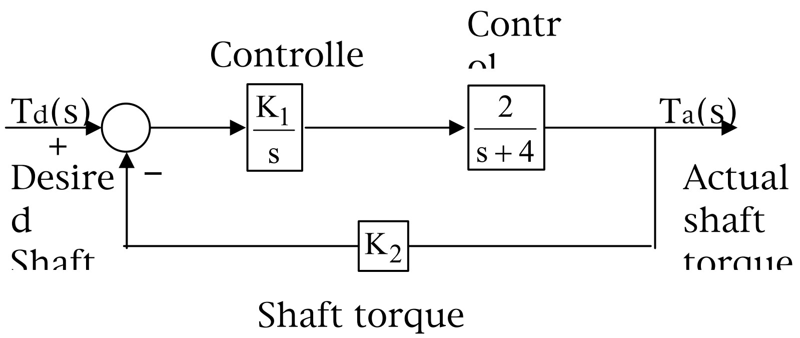
A control system designed to maintain a desired torque on a shaft is represented by the block diagram shown. The transient response specifications to be provided are selected as follows.

Rise time < 5 s,

Maximum Overshoot < 12 %,

2 % settling time < 15 s

Determine the acceptable ranges for the parameters $\mathbf{K_1}$ and $\mathbf{K_2}$.



Ans. : $2.16 \le K_1 \cdot K_2 \le 6.4$

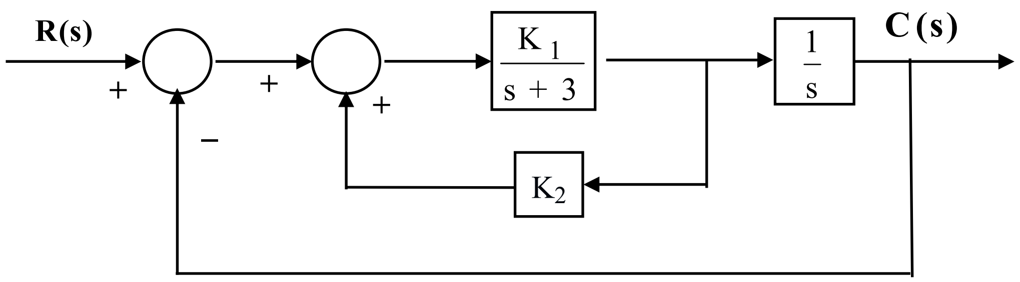
PROBLEM V-11:

Write a script m-file to determine the unit step response of a system represented by the block diagram below with $\mathbf{K_1}$=1.32 and $\mathbf{K_2}$=1.14.

a) Plot the response.

b) Calculate the values of all the transient response specifications.

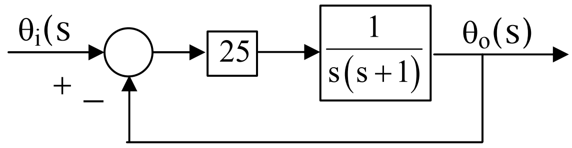
c) Compare the calculated maximum overshoot and the 5% settling time values with
   - the values read from the plot, and
   - the specifications of the example solved in class.



Ans. : t_d=1.2 s, t_r=2.6s, M_p=1.07 (t_{Mp}=3.58s), t_s=4.25s

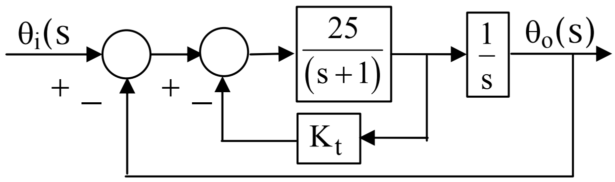
PROBLEM V-12:

Consider the control system represented by the block diagram shown.



a) Calculate the maximum overshoot and 2 % settling time of the system.

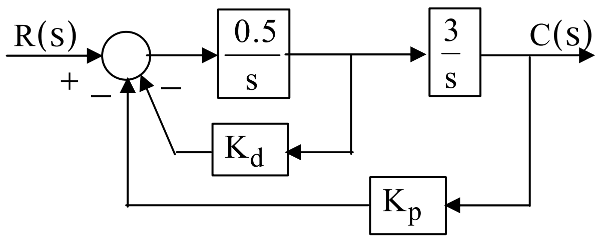
b) A velocity feedback is added to the system, as shown in the block diagram, to improve the transient response of the system. Determine the value $\mathbf{K_t}$ such that the system will have a damping ratio of 0.7.



Ans. : a) $t_s(2\%) \cong 8\,s$, b) K_t=0.24

PROBLEM V-13:

Consider the block diagram of a space telescope pointing control system. $\mathbf{K_p}$ and $\mathbf{K_d}$ are positive parameters to be used to obtain the desired performance.



a) Determine the relation between the parameters $\mathbf{K_p}$ and $\mathbf{K_d}$ such that the damping ratio is critical.

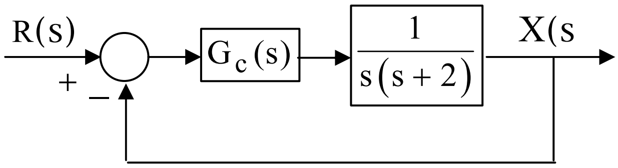
Plot $\mathbf{K_p}$ versus $\mathbf{K_d}$ and clearly indicate the regions in which the system is underdamped and overdamped.

b) Select ranges of values for the parameters $\mathbf{K_p}$ and $\mathbf{K_d}$ such that for a step input, the maximum overshoot is 4% and the peak time is less than 2 seconds. In specifying the required parameter ranges, make sure that you use the proper equality and/or inequality signs.

Ans. : a) $K_p = 0.042K_d^2$ or $K_d^2 = 24K_p$, c) K_p>3.375, K_d>6.45

PROBLEM V-14:

Transient response specifications for a unit step input for the system with the block diagram shown in the figure are specified as :



Peak time = 1 s

Percent overshoot = 5 %

a) Show, without any calculations that these two specifications cannot be satisfied simultaneously by using proportional control $G_c(s)=K_p$.

b) What type of controller would you use to satisfy both specifications simultaneously without reducing the stability ? Explain!

Ans. : a) $K_p = 2.1$ or $K_p = 18.8$ required !.

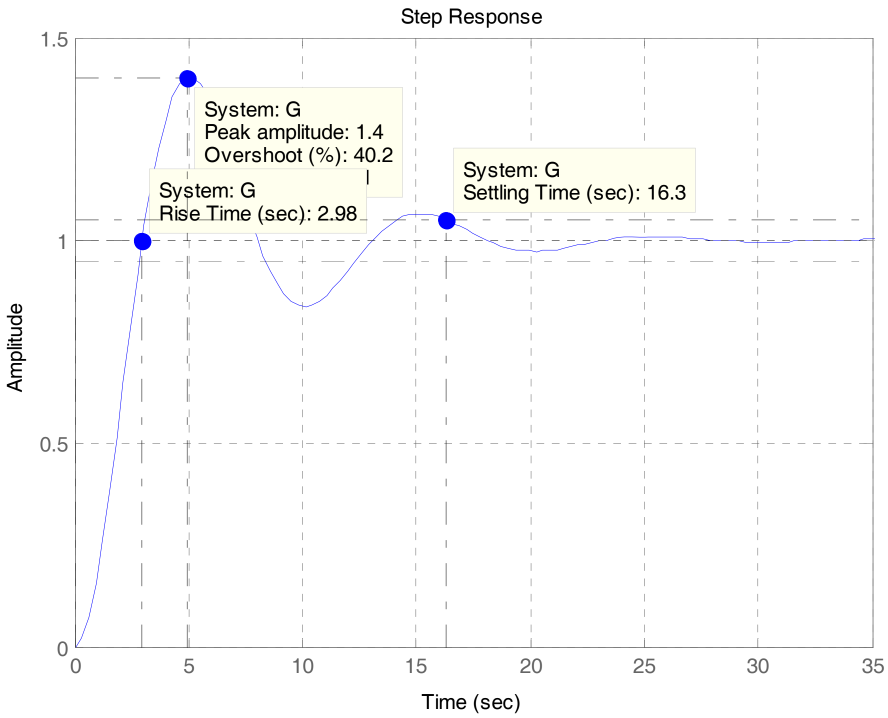
PROBLEM V-15:

Unit step response of a control system is obtained and presented below.

a) Estimate (clearly indicate which values you take from the figure)

- Rise time (0 to 100 %),
- Maximum overshoot (percent),
- Peak time,
- Settling time (5 %).

b) Assume that the system is a second order system without numerator dynamics and determine the undamped natural frequency and damping ratio.

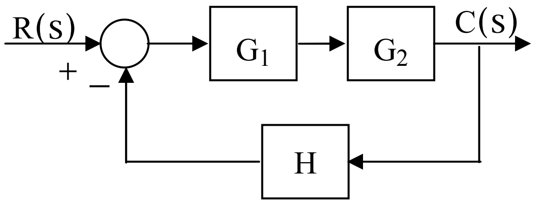


Ans. : a) 3.0 s, 40 %, 4.9 s, 17 s, b) $\xi = 0.28$, $\omega_n = 0.65\,\text{rad/s}$.

VI/ Steady State Response and Error

PROBLEM VI-1:

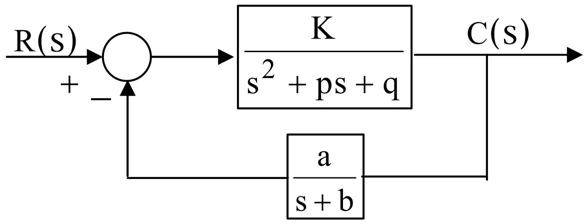
Write down the open loop transfer function OLTF(s) of the closed loop system with the block diagram as shown.



Ans. : $G=G_1G_2H$

PROBLEM VI-2:

A unit step input is applied to the system represented by the block diagram shown. Determine the time domain response, i.e. c(t), as time goes to infinity, i.e. the steady state response, of the system.

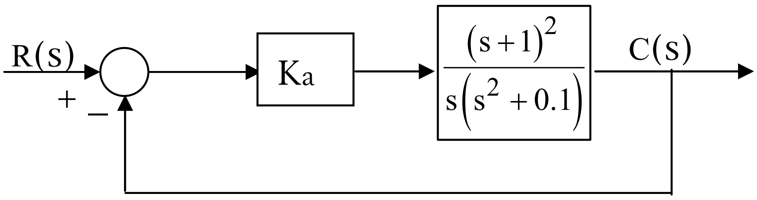


Ans. : $c_{ss} = \frac{Kb}{(qb + Ka)}$ (check stability first !)

PROBLEM VI-3:

Consider the block diagram of a system designed to control the depth of a submarine. The actual depth is measured by a pressure transducer and is compared with the desired depth. The error is amplified and is then used to drive the stern plane actuator.

Determine the value of the amplifier gain such that the steady state error for a unit ramp input will be less than 5 %.



Ans. : $K_a > 2$ (check stability first !)

PROBLEM VI-4:

What is the steady state error of the unity feedback system with an open loop transfer function

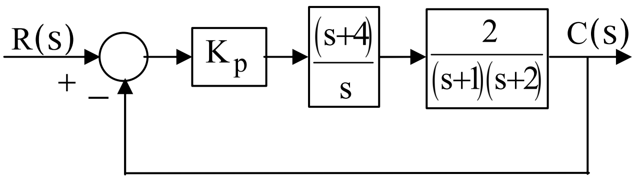
$$G(s) = K_p\left(\frac{1}{Ts+1}\right)\frac{1}{s^2}$$

to an acceleration input of magnitude R ?

Hint : Is it defined ?

PROBLEM VI-5:

a) If the steady state error for a unit ramp input of the unity feedback system, represented by the block diagram shown, is required to be equal to or less than 2%, determine the minimum value of $\mathbf{K_p}$.



b) Is it possible to use this value for actual operation ? If not, calculate the maximum value of $\mathbf{K_p}$ that can be used and the resulting steady state error.

Ans. : a) $K_p \geq 12.5$, b) $K_p<3$, $e_{ss} > 0.08$ or 8%

PROBLEM VI-6:

Consider the position control of an inertial load with damping. A tachometer is built-in the electric motor and provides a feedback proportional to the angular velocity of the load.

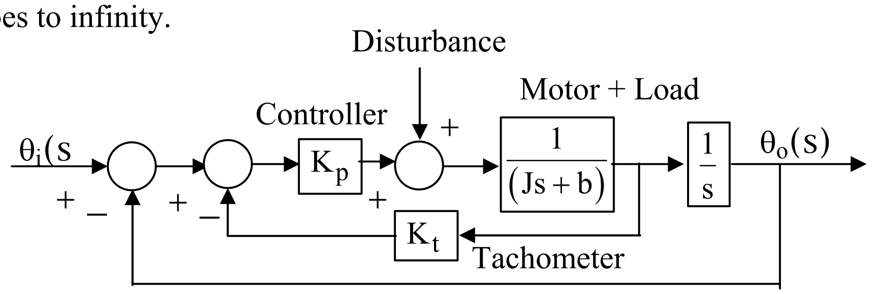
a) Determine the values of the control parameters $\mathbf{K_p}$ and $\mathbf{K_t}$ such that the undamped natural frequency of the system is 1 rad/s, and the system is critically damped.

b) Calculate the values of the control parameters $\mathbf{K_p}$ and $\mathbf{K_t}$ such that the steady state error due to a unit ramp input is less than 0.03 when the damping ratio is 0.5 and the disturbance is zero.

c) What should be the values of $\mathbf{K_p}$ and $\mathbf{K_t}$ if the steady state error due to a unit step disturbance is to be less than 0.01. Note that, in this case, the steady state error is defined as the steady state value of the output itself as the desired input is now zero. Thus, you should re-draw the block diagram with the disturbance as the input and evaluate the output as time goes to infinity.

J = 15 kgm2

b = 3 Nm/rad/s



Ans. : a) $K_t = 1.8$, b) $K_p > 16667$, $K_t < 0.0298$, c) $K_p > 100$ (check stability first !)

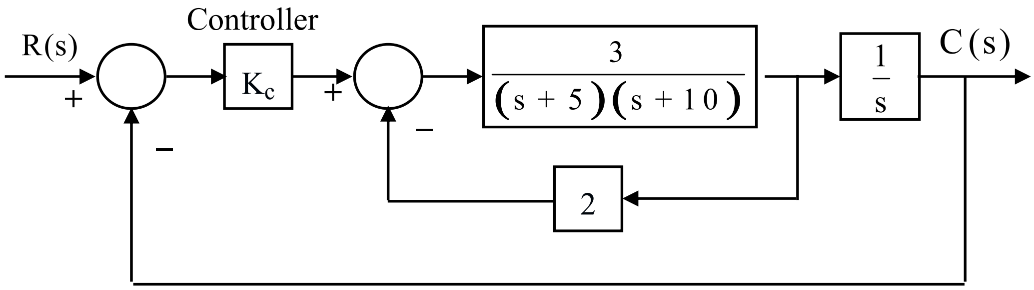
PROBLEM VI-7:

The block diagram of a unity feedback speed control system is shown in the figure.

a) Determine the type of the system. Explain how you have decided.

b) Obtain the open loop gain **K** of the system.

c) Decide if it is possible to obtain a steady state error less than **0.04** by a proper choice of the controller gain $\mathbf{K_c}$, if the input is specified as **r(t)=0.9t**.



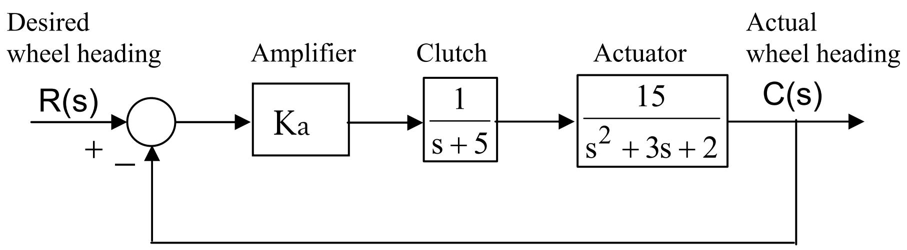
Ans. : a) Type 1, b) $K=3K_c/56$, c) No.

PROBLEM VI-8:

Consider the block diagram of a nose-wheel steering system for medium size aircraft, shown below.

a) Write down the open loop transfer function and determine the type and open loop gain **K** the of the system.

b) Calculate the amplifier gain $\mathbf{K_a}$, to result in the minimum possible steady state error in response to a unit step input. Further calculate the value of the minimum achievable steady state error.

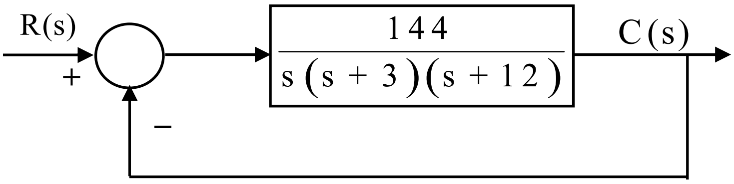


Ans. : a) $T(s)=\frac{15K_a}{(s+1)(s+2)(s+5)}$, type zero, open loop gain=$3K_a/2$,

b) $K_a<8.4$, e_{ss} => 0.106

PROBLEM VI-9:

Consider the block diagram of a unity feedback system shown in the figure.



a) Determine the system type and open loop gain. Obtain the steady state error for a unit ramp input.

b) Add a proportional controller of gain $K_p>0$ to the system. Determine the minimum steady state error for a ramp input achievable with the controlled system.

c) Which controller type can be used to reduce the steady state error of this system to a ramp input to zero. Show that you actually get zero steady state error to a ramp input with your controller.

Ans. : a) Type 1, K=144/36=4, b) $K_p<3.75$, $e_{ss}=0.067$.

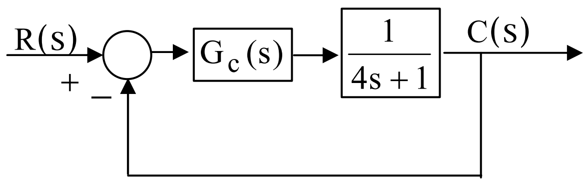
VIII/ Basic Control Actions and Controllers

PROBLEM VIII-1:

Consider the system represented by the block diagram given.

a) If proportional control $G_c(s)=K_p$ is used, determine the value of proportional gain such that the steady state error of the system due to unit step input is equal to or less than 1%.

b) It is decided to use a proportional+ integral (PI) control.



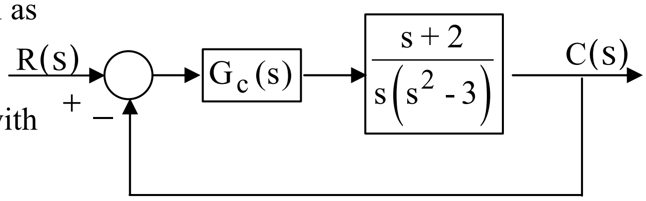
$$G_c(s) = K_p\left(1+\frac{1}{T_i s}\right)$$

instead of proportional only control. Determine the values of the proportional gain and the integral time such that the 2% settling time is less than or equal to 4 seconds, and the steady state error for unit ramp input is equal to or less than 1%.

Ans. : a) $K_p \geq 99$, b) $K_p \geq 7$, $T_i \leq 0.07$

PROBLEM VIII-2:

Consider the system with the block diagram as shown.



a) Show that the uncontrolled system (with no controller : $G_c(s)=1$) is always unstable.

b) Try proportional control, i.e. $G_c(s)=K_p$. Can the system be stabilized for some value of the parameter K_p ?

c) Try now PI control with :

$$G_c(s) = K_p + \frac{K_i}{s}$$

Can you find some values of the parameters K_p and K_i for which the system is stabilized ?

d) Finally use PD control with :

$$G_c(s) = K_p + K_d s$$

Find and sketch the region for the stable operation of the system in the parameter plane.

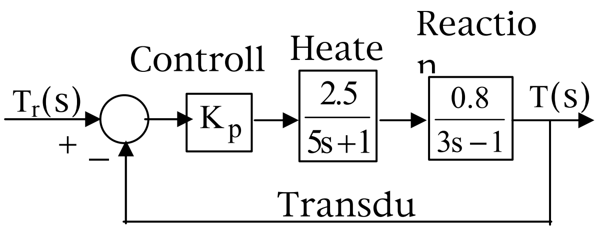
Ans. : d) $K_p > \dfrac{3-2K_d}{(1-\frac{2}{K_d})}$

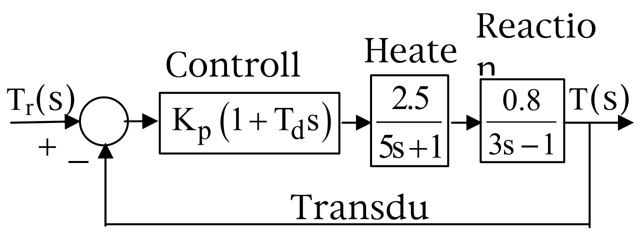
PROBLEM VIII-4:

A proportional control system is used for controlling the temperature of an exothermal (unstable) chemical reaction.

a) Determine if the system is stable for a proportional gain of 100.

b) To improve the stability of the system, derivative control is added to the proportional control. Determine the value of derivative gain $\mathbf{T_d}$ for a stability margin of at least 1, if the value of $\mathbf{K_p}$ is 100.

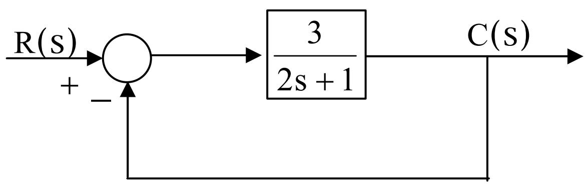




<u>Ans. :</u> a) Unstable, b) $0.16<T_d < 1.08$

PROBLEM VIII-3:

Consider the system with the block diagram as shown.



a) Estimate the time required to reach steady state.

b) What is the steady state error for a unit step input ?

Now insert a PI controller with the transfer function

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s}\right)$$

in the control system, and re-draw the block diagram. If the values of the proportional gain and the integral time are selected as 4 and 0.15 seconds, Use Matlab to :

c) Calculate the 2% settling time.

d) Determine the steady state error for a unit step input.

e) Calculate the values of the proportional gain and the integral time to be used if the steady state error for a unit ramp input is to be less than 1 % and the 2% settling time is to be less than 1 second.

PROBLEM VIII-4:

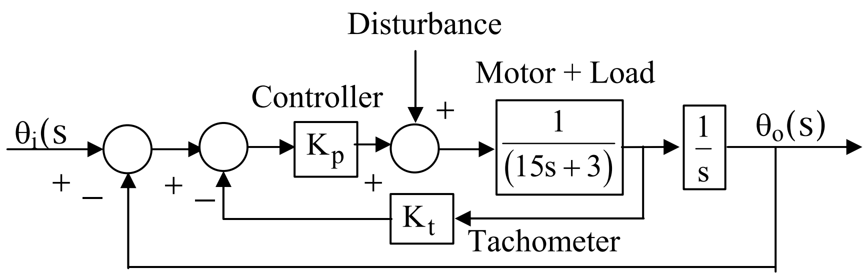
Consider the position control of an inertial load with a proportional controller of gain $\mathbf{K_p}$. A tachometer is built-in the electric motor and provides a feedback proportional to the angular velocity of the load.

a) Determine the values of the control parameters $\mathbf{K_p}$ and $\mathbf{K_t}$ such that the undamped natural frequency of the system is at least 2 rad/s, and the system is critically damped.

b) Calculate the values of the control parameters $\mathbf{K_p}$ and $\mathbf{K_t}$ such that the steady state error due to a unit ramp input is less than 5% when the damping ratio is 0.5 and the disturbance is zero.

c) What should be the values of $\mathbf{K_p}$ and $\mathbf{K_t}$ if the steady state error due to a unit step **disturbance** is to be less than 2.5%

Note : In specifying the required parameter values, make sure that you use the proper equality and/or inequality signs.



Ans. : a) $K_p \geq 60$ b) $K_p > 6000$, $K_t \leq 0.05 - \dfrac{3}{K_p}$ c) $K_p > 40$

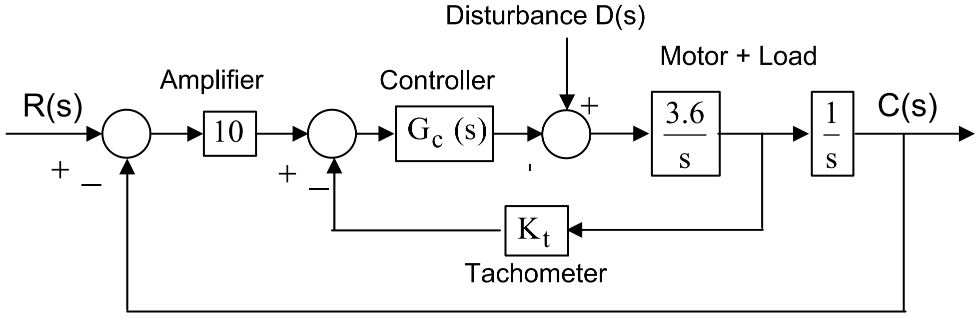
PROBLEM VIII-5:

Consider the position control of an inertial load with the block diagram as shown in the figure. A tachometer is built-in the electric motor. Disregard disturbance and

a) Assume that a **proportional controller**, $G_c(s)=K_p$, is used. If the undamped natural frequency is to be at least 2.4 rad/s and the damping ratio is to be set to 0.72, determine the (range of) values of the parameters $\mathbf{K_p}$ and $\mathbf{K_t}$.

b) Now, consider the **disturbance**, D(s), as the only input to the system. Redraw the block diagram accordingly. What are the conditions on the values of $\mathbf{K_p}$ and $\mathbf{K_t}$ if the steady state error due to a unit step disturbance is to be less than 0.025 ?

c) Finally, assume that a **PI controller**, $G_c(s) = K_p\left(1+\frac{1}{T_i s}\right)$, is used. If $\mathbf{K_t}$=0.25, check the conditions on the selection of the parameters $\mathbf{K_p}$ and $\mathbf{T_i}$ for stability. If necessary plot $\mathbf{K_p}$ versus $\mathbf{T_i}$ and clearly mark the stable and unstable regions.



Ans. : a) $K_p \geq 0.16, K_t \geq 6$, b) $K_p > 4$, c) $K_p > \frac{10}{0.9 + 36T_i}$

IX/ Frequency Response **PROBLEM IX-1:**

Calculate the steady state response, x(t), of the system with the overall transfer function

$$T(s) = \frac{X(s)}{Y(s)} = \frac{s}{0.2s+1}$$

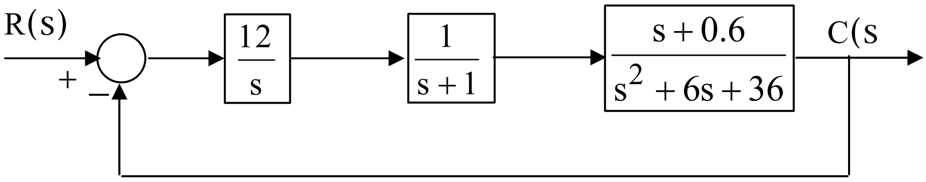
to a sinusoidal input

y(t)=0.5sin2t.

Ans. : $x(t) \cong 0.93\sin\left(2t + 68^{o}\right)$

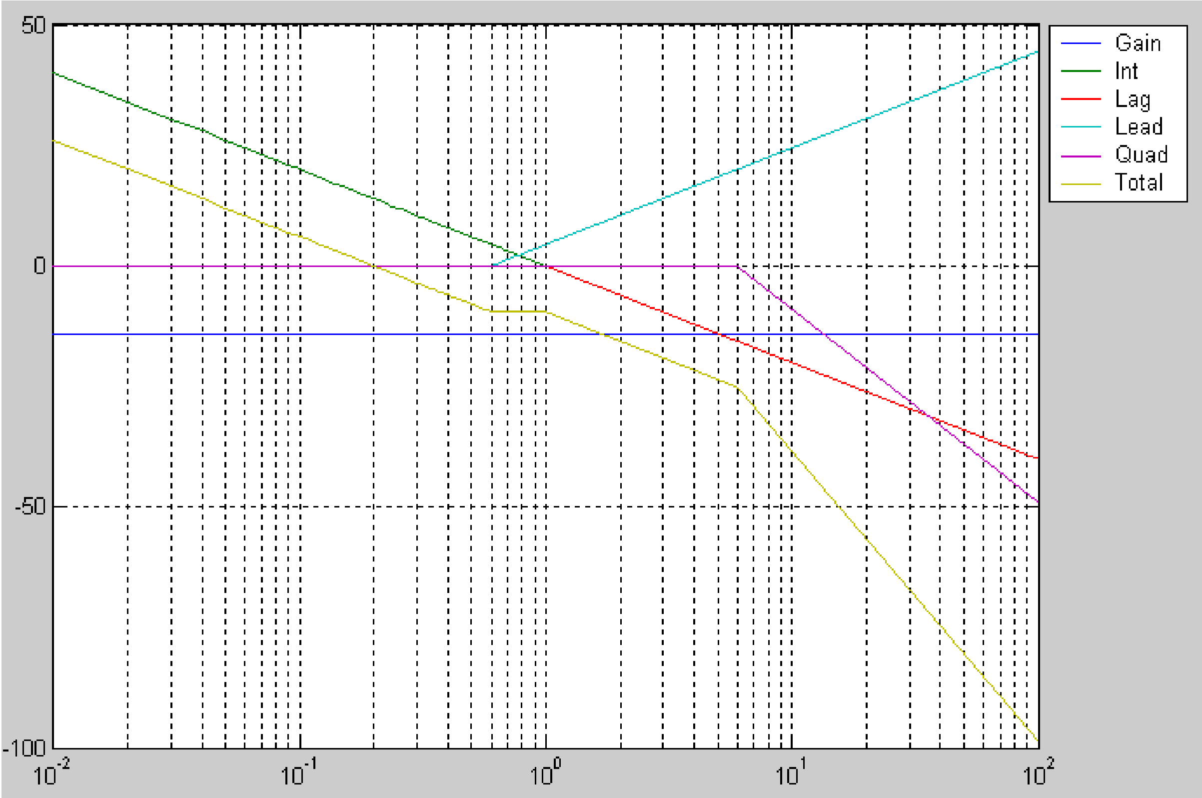
PROBLEM IX-2:

Consider the unity feedback control system with the block diagram shown. Sketch the approximate Bode plots for the open loop transfer function of the control system.

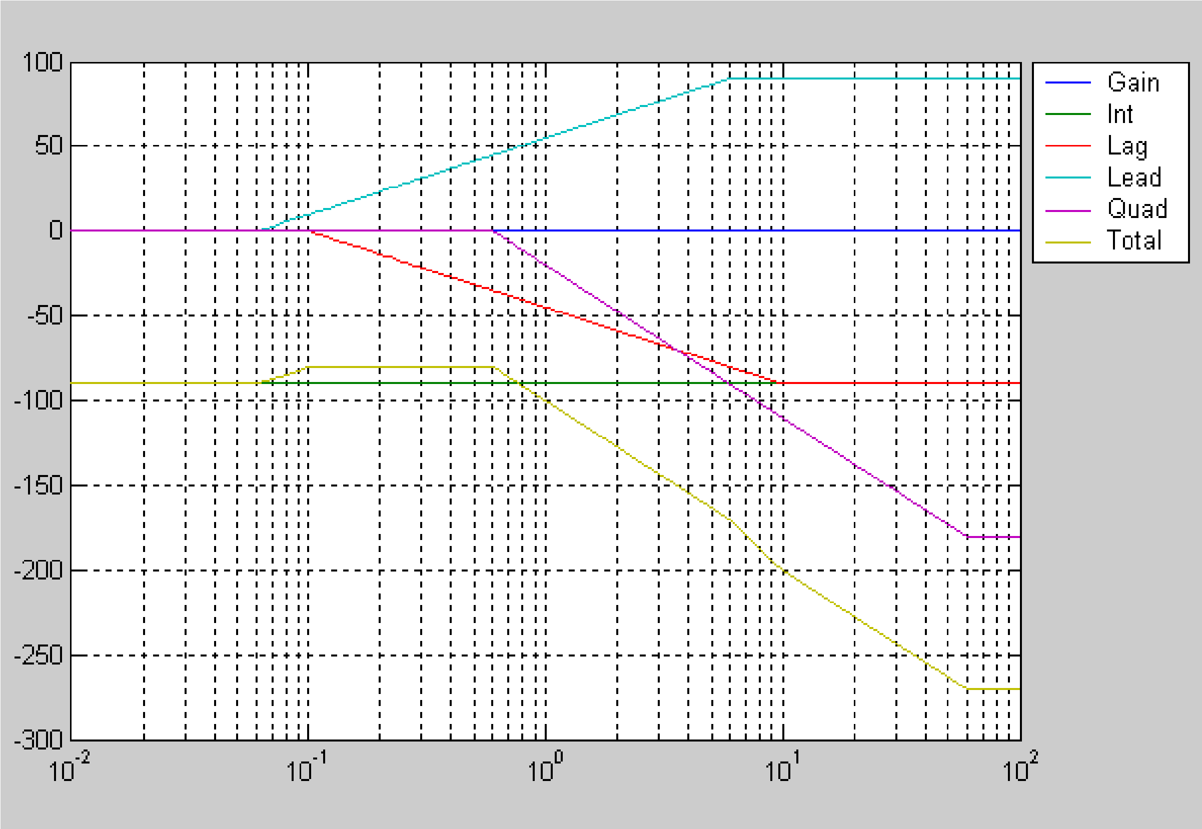


Ans. :

Approximated Magnitude of Bode Plot



Approximated Phase of Bode Plot



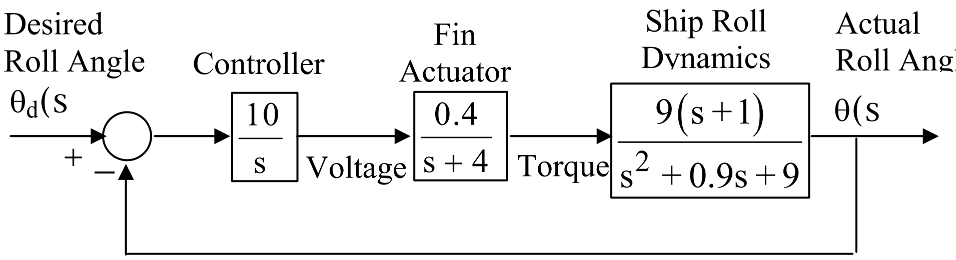
PROBLEM IX-3:

The block diagram of a control system for the stabilization of the roll motion of a ship is given.

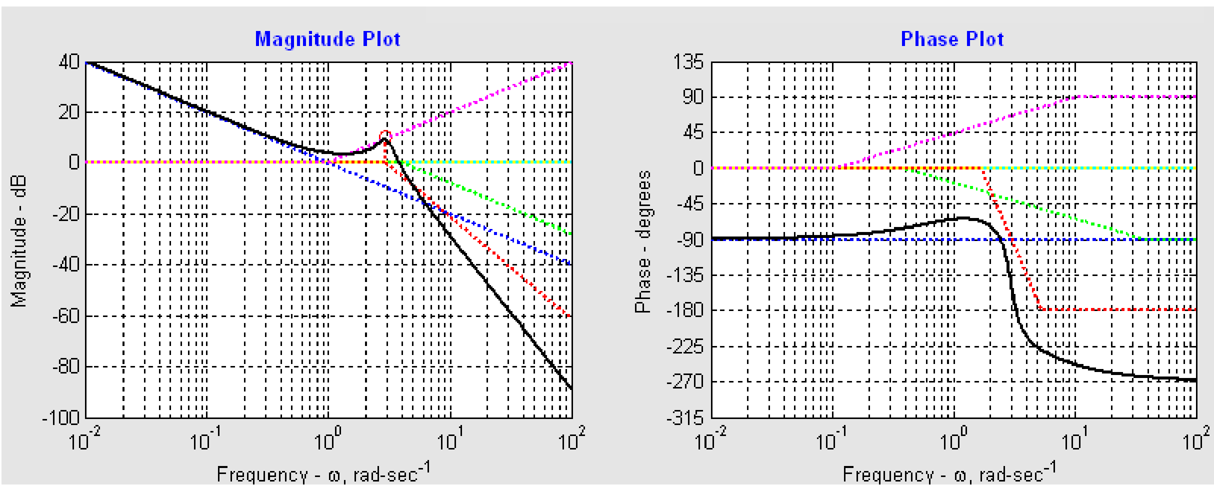
a) Identify the basic factors for the <u>open loop</u> transfer function of the system.

b) Identify the corner frequencies separately for the magnitude and phase plots. Remember that a corner frequency is defined as the intersection of two straight line approximations !

c) Plot first the straight line approximations for the basic factors and then sketch the approximate Bode plots for the <u>open loop</u> transfer function of the system by graphically adding the contributions of the basic factors. Make sure that the plots are clear and readable.



<u>Ans. :</u>



PROBLEM IX-4:

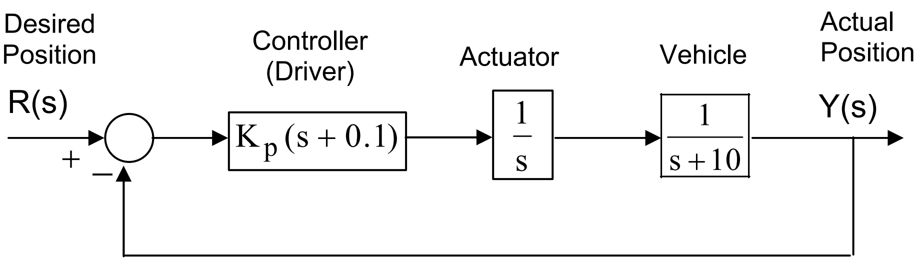
The block diagram of a control system for the human steering of an automobile is given.

a) Identify the basic factors for the open loop transfer function of the system.

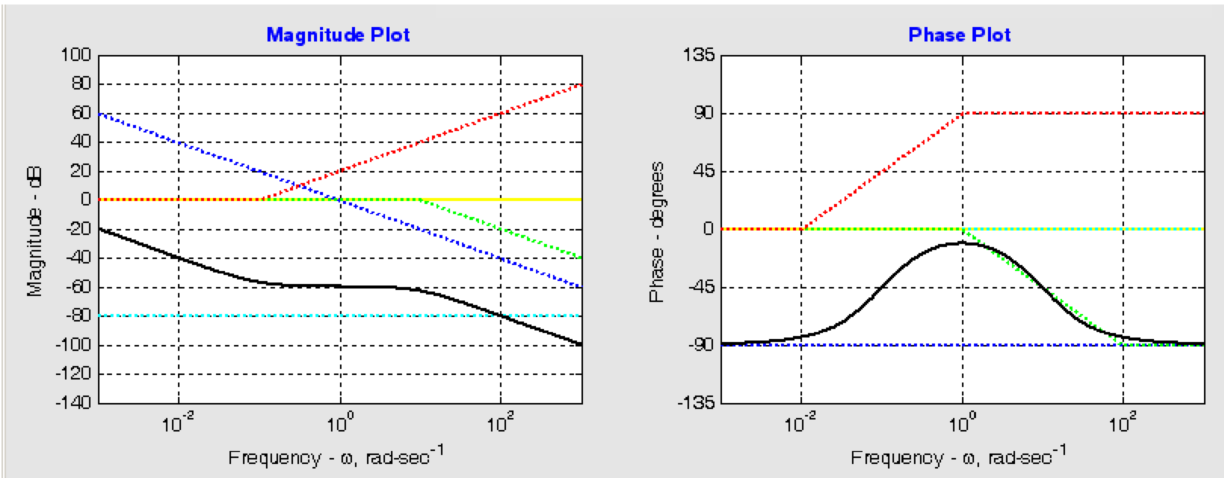
b) Identify the corner frequencies separately for the magnitude and phase plots. Remember that a corner frequency is defined as the intersection of two straight line approximations !

c) Take K_p=1 and plot first the straight line approximations for the basic factors. Then sketch the approximate Bode plots for the open loop transfer function of the system by graphically adding the contributions of the basic factors. Mark all the lines and make sure that the plots are clear and readable.

d) What should be the value of K_p if a magnitude of 0 [dB] is desired at a frequency of 1 [rad/s] ?



Ans. : c)



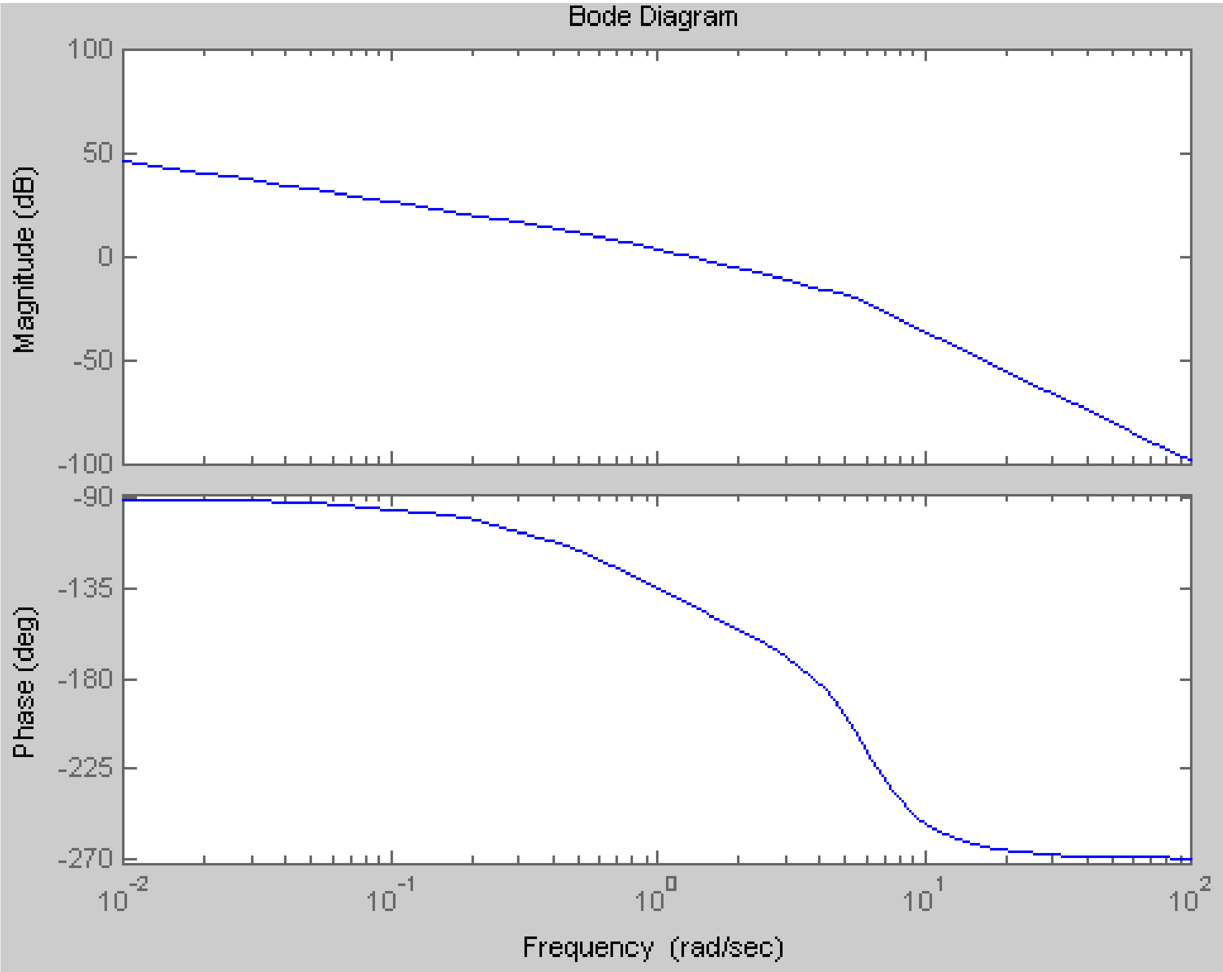
d) 1000

PROBLEM IX-5:

Use Matlab to obtain the exact Bode plots for the open loop transfer function of the control system in problem IX-2 and compare with the approximate plots. You can use the Matlab command "bode(num, den)" which provides both the magnitude and phase plots. "num" and "den" are the row vectors for the coefficients of the numerator and denominator polynomials of the open loop transfer function. Use "help Bode" for additional information.

Ans. :

```
% Define s as the Laplace variable
s=tf('s');
% Define transfer function of the system
sys=12*(s+6)/(s*(s+1)*(s^2+6*s+36));
% Draw Bode diagram
BODE(sys)
```



X/ Sensitivity

PROBLEM X-1:

Consider a unity feedback control system with the feedforward transfer function

$$G(s) = \frac{K}{as+1}$$

Determine the sensitivity of the closed loop transfer function T(s) with respect to the gain K at steady state.

Ans. : $S_K^T\big|_{ss} = \frac{1}{1+K}$

PROBLEM X-2:

Consider a feedback control system with the feedforward and feedback transfer functions G(s) and H(s), respectively.

Determine the sensitivity of the closed loop transfer function T(s) with respect to G(s) and H(s). Can you reduce them both by choosing large controller gains, i.e. $G(s)H(s) \rightarrow \infty$?

Ans. : No !

PROBLEM X-3:

Consider the unity feedback sytem with the block diagram shown.

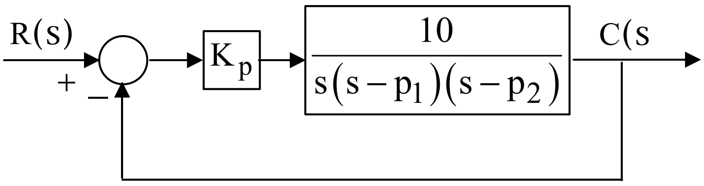
a) Obtain the steady state sensitivity of the transfer function with respect to controller gain $\mathbf{K_p}$ and the plant poles $\mathbf{p_1}$ and $\mathbf{p_2}$.

b) Calculate and plot, simultaneously, the magnitude of the dynamic sensitivity function of the transfer function for this system with respect to controller gain $\mathbf{K_p}$ and the plant poles $\mathbf{p_1}$ and $\mathbf{p_2}$, in the frequency range from 0 to 3 rad/s.

c) Considering the sensitivity function magnitudes at 2 rad/s, order the magnitude of the sensitivities with respect to each parameter, from the least to the most.

Nominal values :

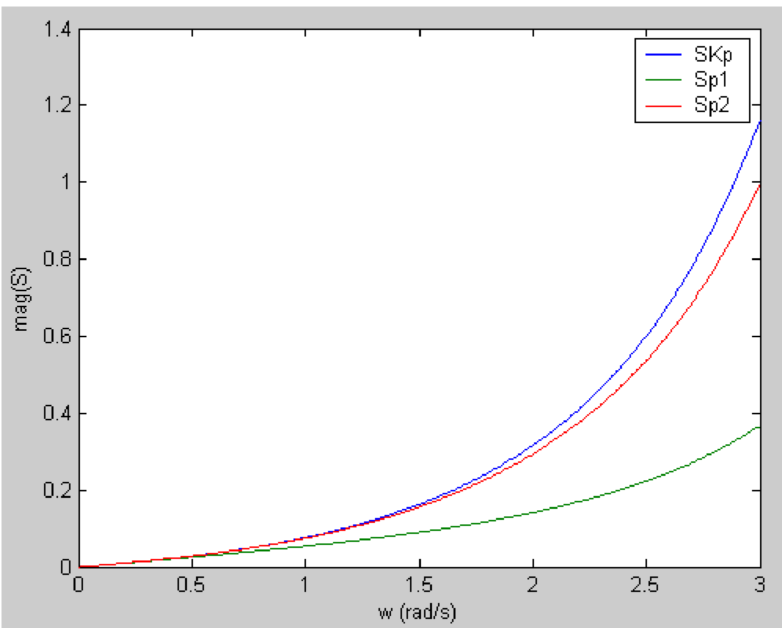
$\mathbf{K_p} = 10$
$\mathbf{p_1} = -1$
$\mathbf{p_2} = -5$



Ans. : a) $S^T_{K_p} = \frac{[s(s-p_1)(s-p_2)]}{[s(s-p_1)(s-p_2)+10K_p]}$, $S^T_{p_1} = \frac{p_1[s(s-p_2)]}{[s(s-p_1)(s-p_2)+10K_p]}$,

$$S^T_{p_2} = \frac{p_2[s(s-p_1)]}{[s(s-p_1)(s-p_2)+10K_p]}$$

c)



XI/ ROOT LOCUS

PROBLEM XI-1:

The open loop transfer function of a system is given as:

$$G(s) = \frac{K(2s+1)}{s(s+1)(s+2)(5s+1)}$$

a) How many branches of the root locus are there ? Explain.

b) Where do they start and where do they end ?

Ans. : a) 4 branches, b) 1 finite zero at -0.5 and 3 zeroes at infinity. Root loci will start from poles at 0, -1, -2, and -0.2. One will end at -0.5 and three at infinity.