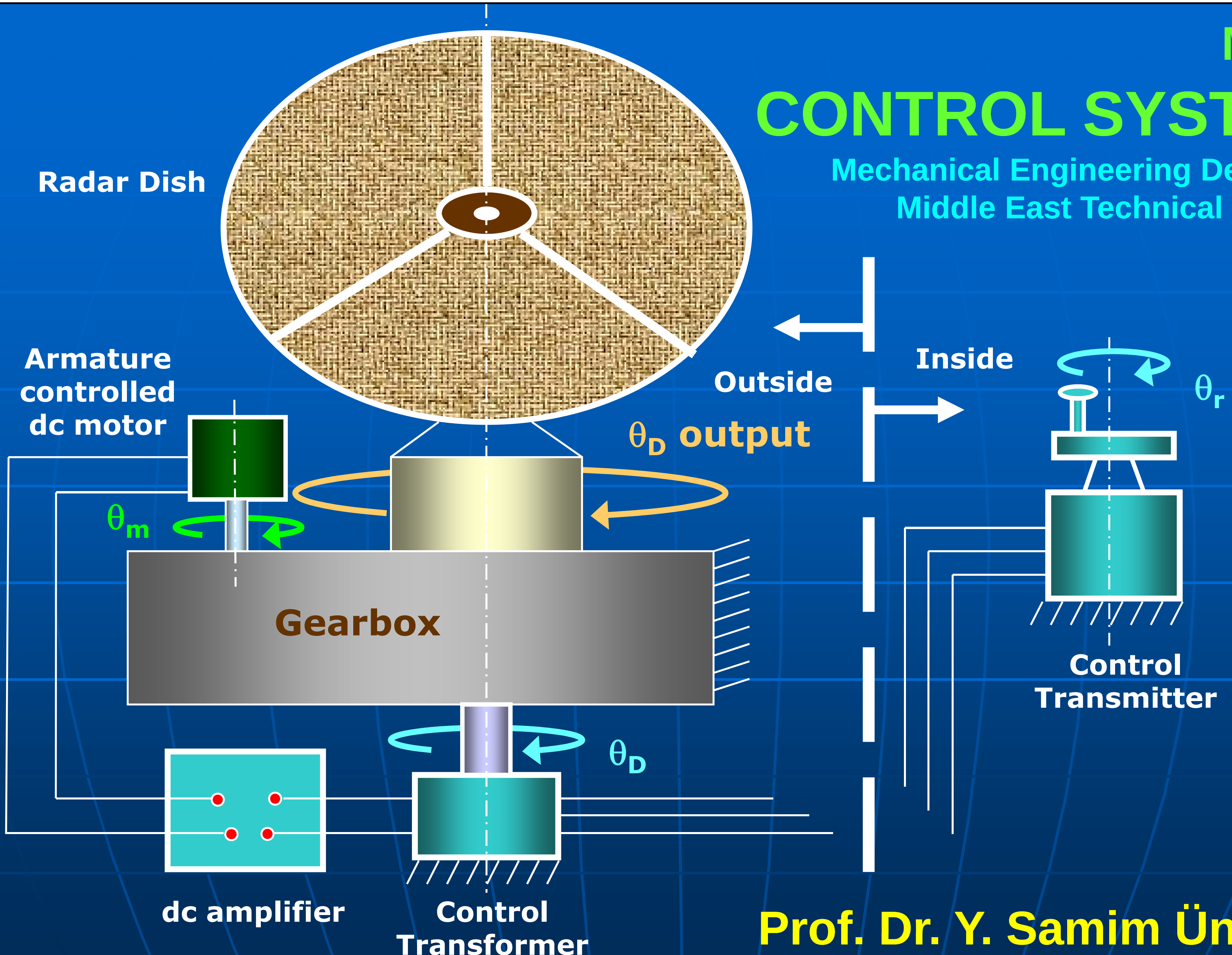
ME 304
CONTROL SYSTEMS

Mechanical Engineering Department,
Middle East Technical University



Prof. Dr. Y. Samim Ünlüsoy

CH VIII

COURSE OUTLINE

CONTROL ACTIONS & CONTROLLERS
OBJECTIVES

In this chapter :

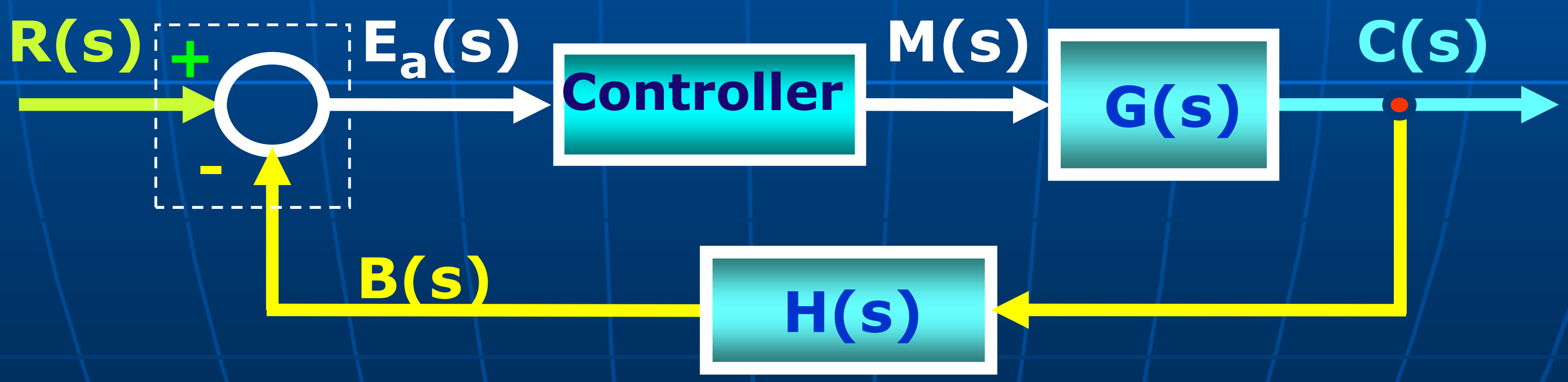
- **Basic control actions and controllers, and their characteristics will be examined.**
- **Their applications will be introduced.**

CONTROL ACTIONS & CONTROLLERS

- A **controller** is a device which produces a control signal that results in the reduction of the deviation of controlled output from the desired value.

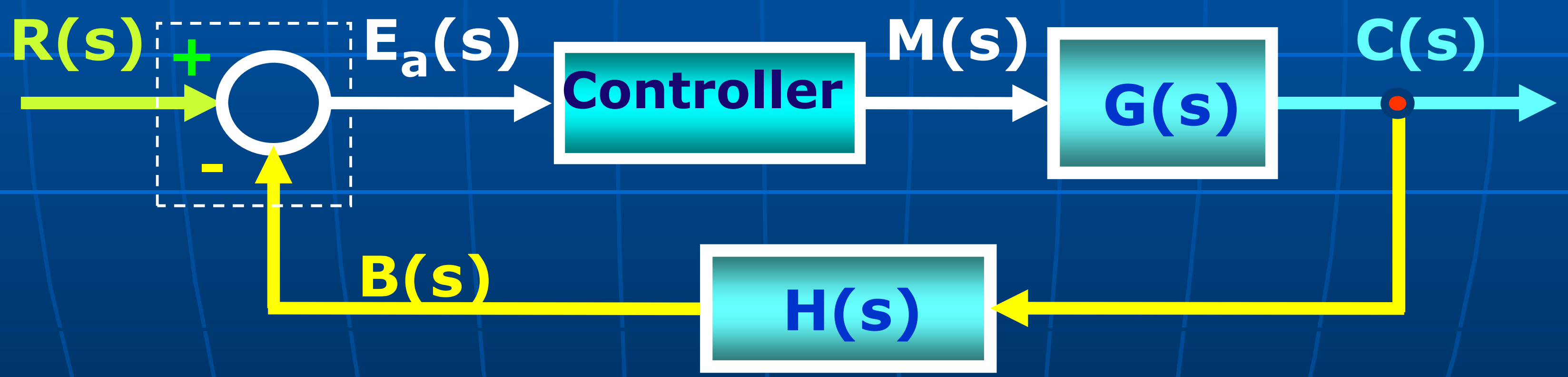
CONTROL ACTIONS & CONTROLLERS

- **The input to the controller is the actuating error $E_a(s)$, which is the difference between**
  - **the system response B(s), as measured by a sensor, and**
  - **the reference signal R(s), which represents the desired system response.**



CONTROL ACTIONS & CONTROLLERS

- **Controller produces the manipulated input at its output to be applied to the actuator.**



CONTROL ACTIONS & CONTROLLERS

- **A controller is required to shape the error signal such that certain control criteria or specifications, are satisfied.**
- **These criteria may involve :**

  - Transient response characteristics,

  - Steady-state error,

  - Disturbance rejection,

  - Sensitivity to parameter changes.

CONTROL ACTIONS & CONTROLLERS

Basic Control Actions :

- A controller produces a control signal (manipulated input) according to its input which is the actuating error.
- The algorithm that relates the error and the control signal is called the control action (law)(strategy).

CONTROL ACTIONS & CONTROLLERS

The most commonly used Control Actions are :

1. Two position (on-off, bang-bang),
2. Proportional (P-control),
3. Derivative (D-control),
4. Integral (I-control).

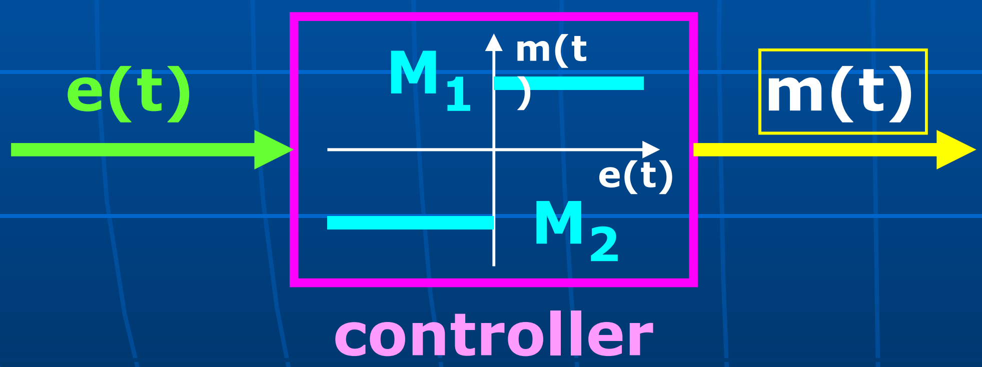
Controllers to provide these control actions (with the exception of D-control) and the combinations of the last three are commercially available.

TWO POSITION CONTROL

(Ogata pp. 63-64)

Two Position Control

For this type of control, the output of the controller can have only one of the two possible values, M_1 or M_2.

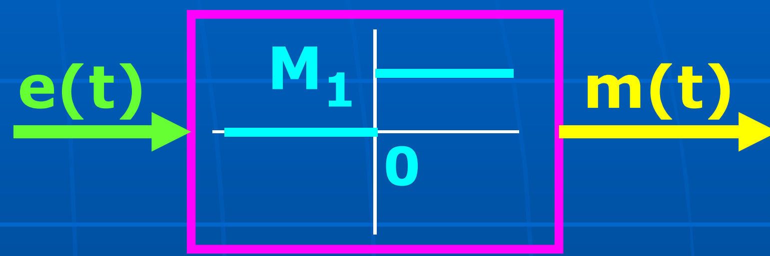


$$m(t) = \begin{cases} M_1 & \text{for} \quad e(t) > 0 \\ M_2 & \text{for} \quad e(t) < 0 \end{cases}$$

TWO POSITION CONTROL

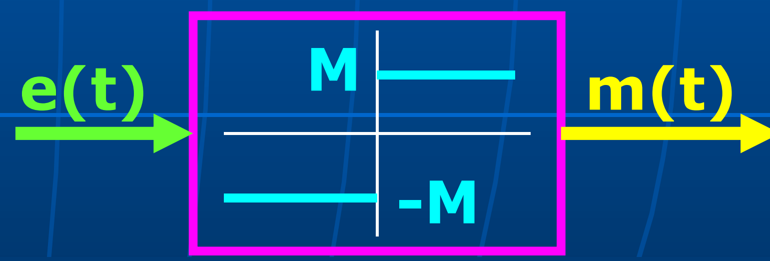
The value of M_2 is usually either

i) **zero**, in which case the controller is called the <u>on-off controller</u>, or



On-off controller

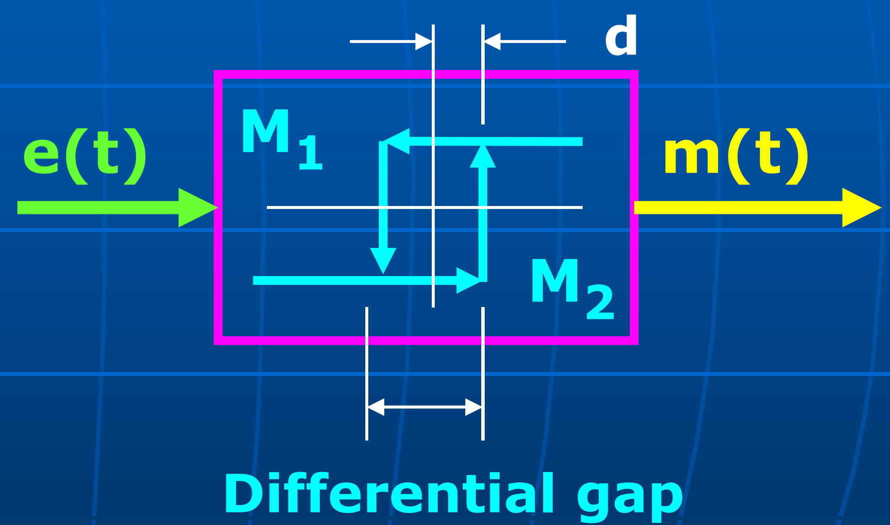
ii) **equal to - M_1**, in which case, the controller is called the <u>bang-bang controller</u>.



Bang-bang controller

TWO POSITION CONTROL

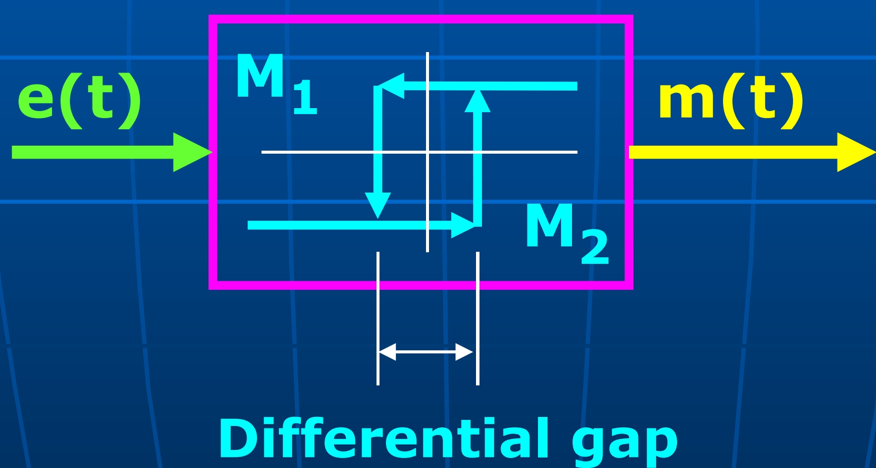
In practice, the two position controllers exhibit somewhat significant hysteresis (usually due to friction), sometimes introduced intentionally to prevent premature wear-out of the switching hardware.



In this case the output switches to M_1 only after the actuating error becomes positive by an amount d. Similarly it switches back to M_2 only after the actuating error becomes equal to -d.

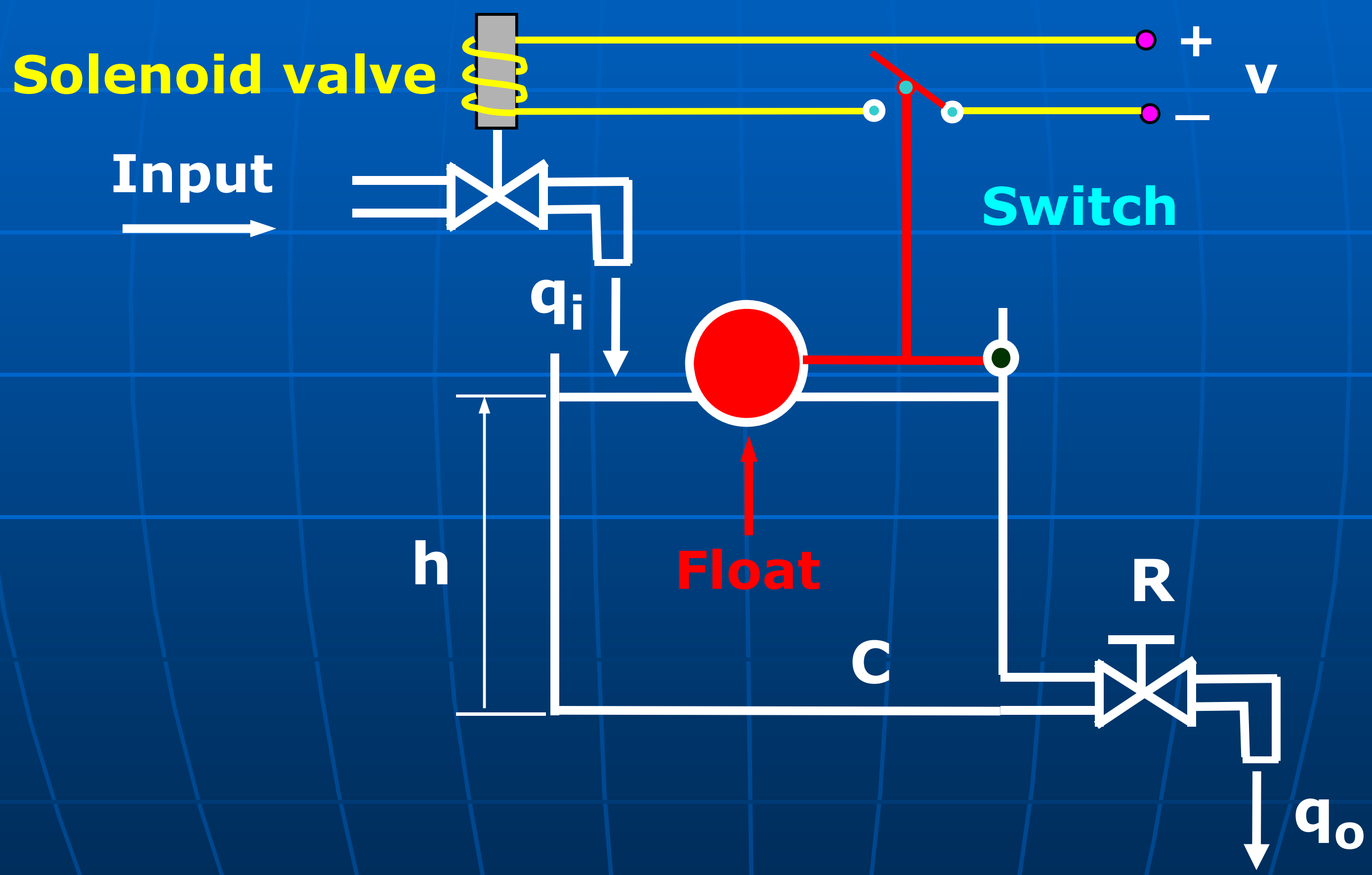
TWO POSITION CONTROL

The existence of a differential gap reduces the accuracy of the control system, but it also reduces the frequency of switching which results in longer operational life.



TWO POSITION CONTROL – EXAMPLE a

Consider the water level control system illustrated in the figure.

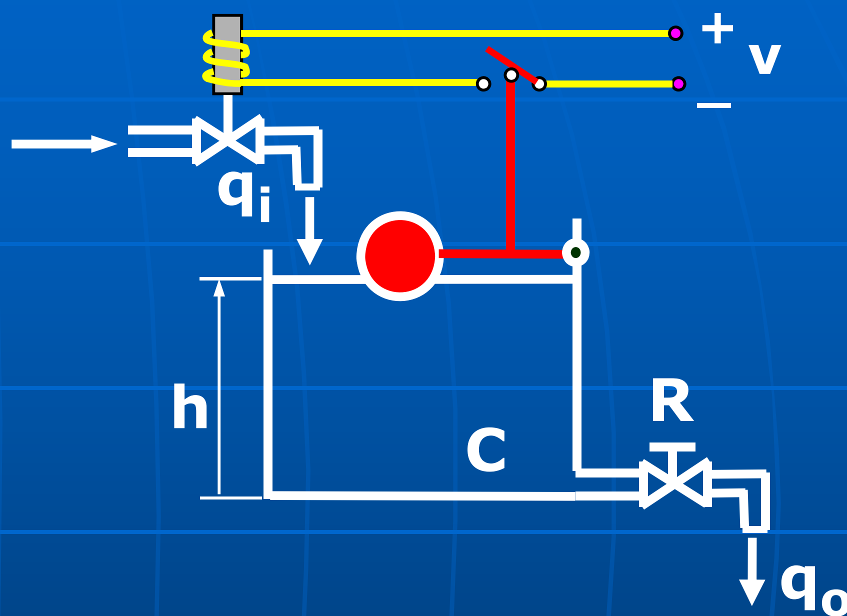


TWO POSITION CONTROL – EXAMPLE b

Assume at first that the tank is empty.

In this case, the solenoid will be energized opening the valve fully.

Hence provided that $q_i > q_o$, the water level will rise.



$$q_i - q_o = C\frac{dh}{dt}$$

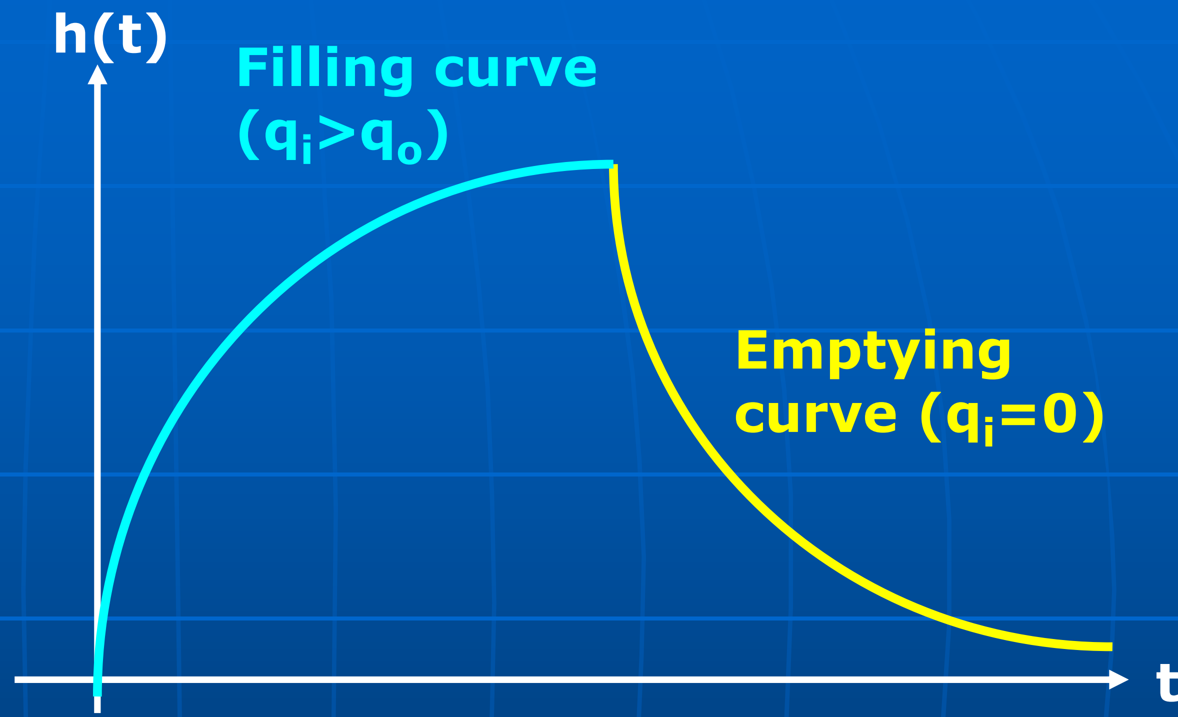
$$h = Rq_o$$

$$RC\frac{dh}{dt} + h = Rq_i$$

TWO POSITION CONTROL – EXAMPLE c

The system is of first order and the variation of water level versus time is shown by the filling curve.

$$RC\frac{dh}{dt}+h=Rq_i$$



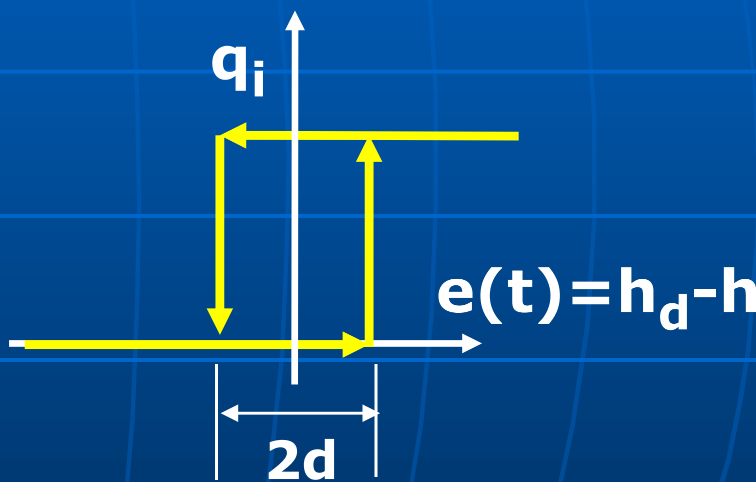
If, at some time t_o, the solenoid is de-energized closing the valve completely, $q_i=0$, then the water in the tank will drain off. The variation of the water level in the tank is now shown by the emptying curve.

$$RC\frac{dh}{dt}+h=0$$

TWO POSITION CONTROL – EXAMPLE d

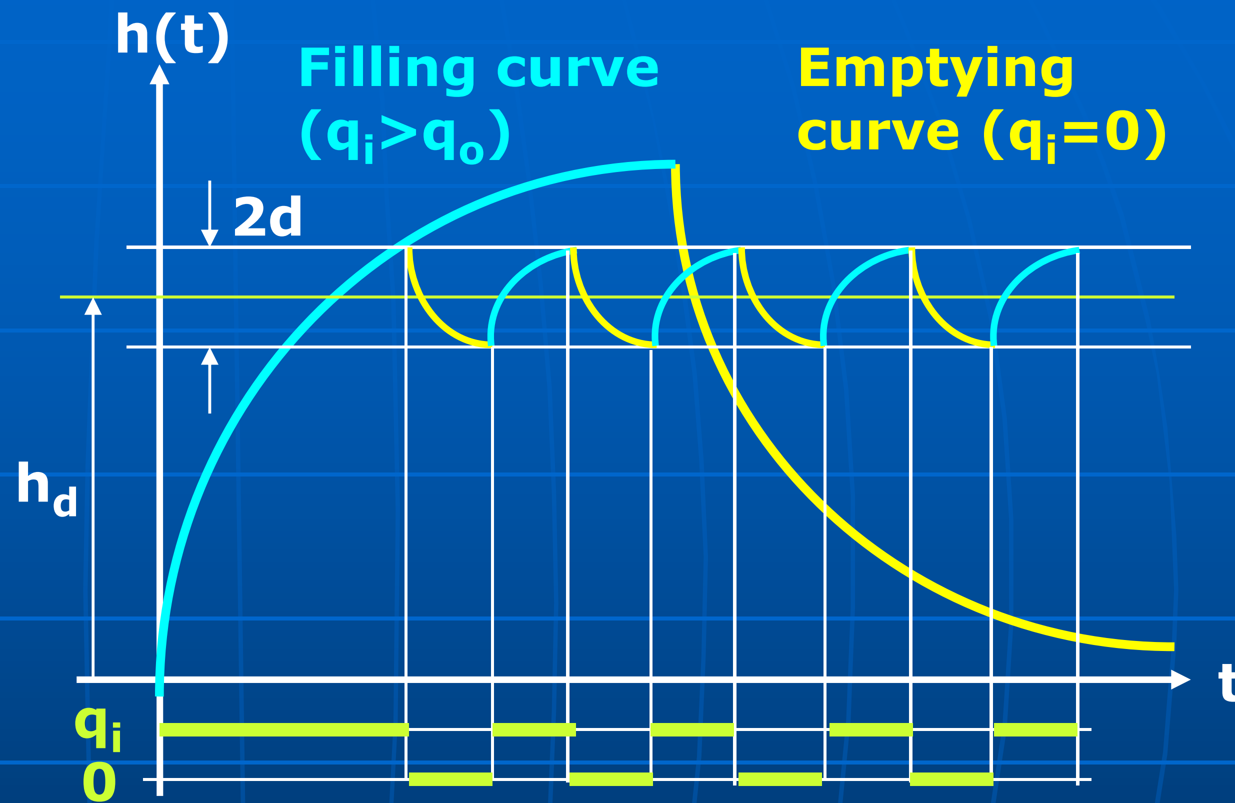
If the switch is adjusted for a desired water level, the input q_i will be on or off (either a positive constant or zero) depending on the difference between the desired and the actual water levels.

If the clearances and friction in the pivot points are considered, the switching characteristics are likely to have a certain amount of differential gap.



TWO POSITION CONTROL – EXAMPLE e

Therefore during the actual operation, input will be on until the water level exceeds the desired level by half the differential gap.

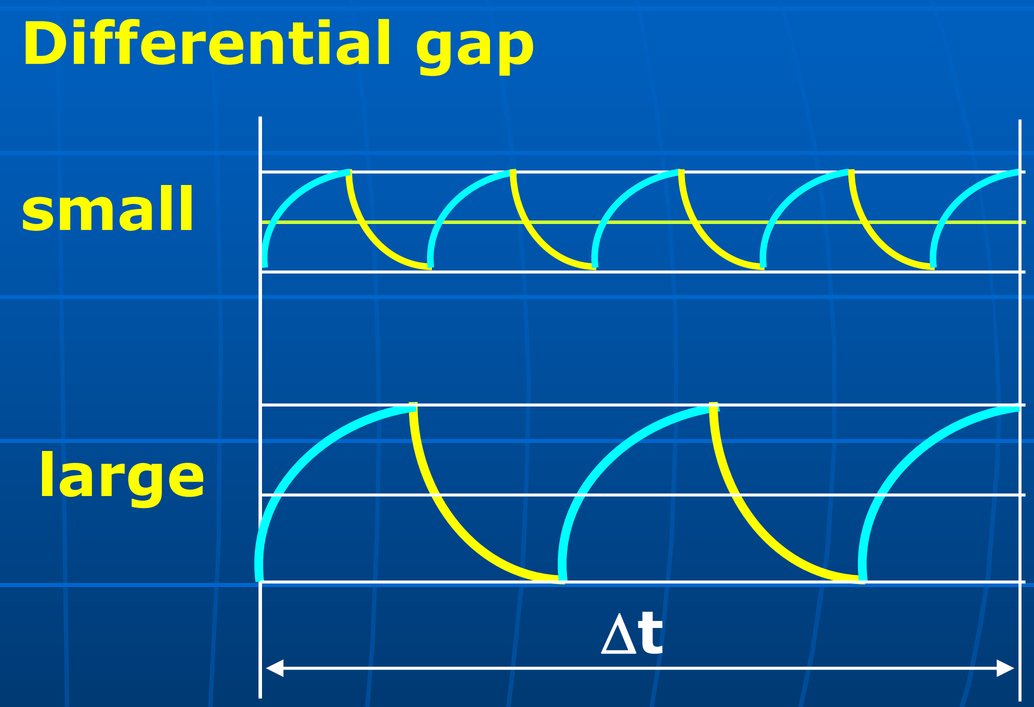


Then the solenoid valve will be shut off until the water level drops below the desired level by half the differential gap. The water level will continuously oscillate about the desired level.

TWO POSITION CONTROL – EXAMPLE f

It should be noted that, the smaller the differential gap is, the smaller is the deviation from the desired level.

But on the other hand, the number of switch on and off's increases.



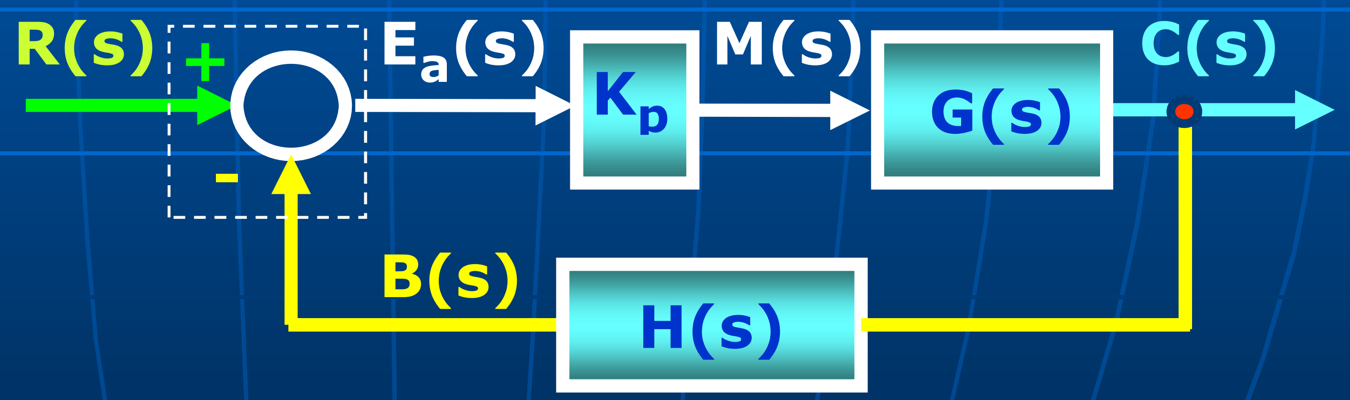
PROPORTIONAL CONTROL

Ogata p.65

Proportional Control (P-control)

- With this type of control action, a control signal proportional to error signal is produced.

$$m(t) = K_p e_a(t) \quad \Rightarrow \quad \frac{M(s)}{E_a(s)} = K_p$$



K_p : proportional gain.

PROPORTIONAL CONTROL

- A **proportional controller** is essentially <u>an amplifier with an adjustable gain</u>.
- The value of K_p should be selected to satisfy the requirements of
  - stability,
  - accuracy, and
  - satisfactory transient response, as well as
  - satisfactory disturbance rejection characteristics.

PROPORTIONAL CONTROL

- **In general,**
  - **For small values of K_p, the corrective action is slow particularly for small errors.**
  - **For large values of K_p, the performance of the control system is improved. But this may lead to instability.**
- **Usually, a compromise is necessary in selecting a proper gain. If this is not possible, then proportional control action is used with some other control action(s).**

DERIVATIVE CONTROL

Derivative Control (D-control)

- In this case the control signal of the controller is proportional to the derivative (slope) of the error signal.

$$m(t) = K_d \frac{de_a(t)}{dt} \quad \Rightarrow \quad \frac{M(s)}{E_a(s)} = K_d s$$

- **Derivative control action is never used alone**, since it does not respond to a constant error, however large it may be.

K_d : derivative gain (adjustable).

DERIVATIVE CONTROL

- **Derivative control** action responds to the rate of change of error signal and can produce a control signal before the error becomes too large.
- As such, **derivative control** action anticipates the error, takes early corrective action, and tends to increase the stability of the system.

DERIVATIVE CONTROL

- **Derivative control action has no direct effect on steady state error.**
- **But it increases the damping in the system and allows a higher value for the open loop gain K which reduces the steady state error.**

DERIVATIVE CONTROL

Derivative control, however, has disadvantages as well.

- It amplifies noise signals coming in with the error signal and may saturate the actuator.
- It cannot be used if the error signal is not differentiable.

<u>**Thus derivative control is used only together with some other control action!**</u>

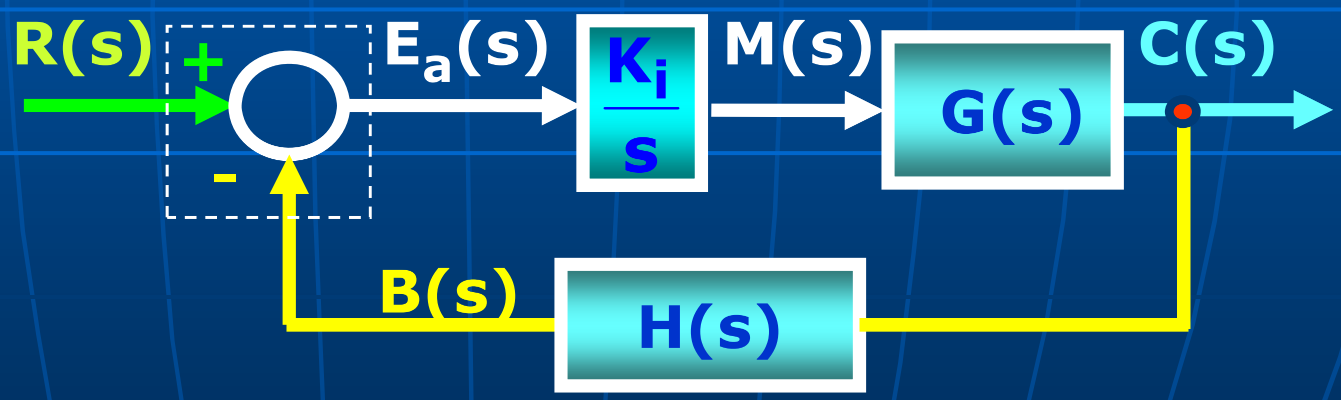
INTEGRAL CONTROL

Ogata p.65

Integral Control (I-control)

- With this type of control action, control signal is proportional to the integral of the error signal.

$$m(t) = K_i \int e_a(t)dt \quad \Rightarrow \quad \frac{M(s)}{E_a(s)} = \frac{K_i}{s}$$



K_i : integral gain (adjustable).

INTEGRAL CONTROL

- **It is obvious that even a small error can be detected, since integral control produces a control signal proportional to the area under the error signal.**
- **Hence, integral control increases the accuracy of the system.**
- **Integral control is said to look at the past of the error signal.**

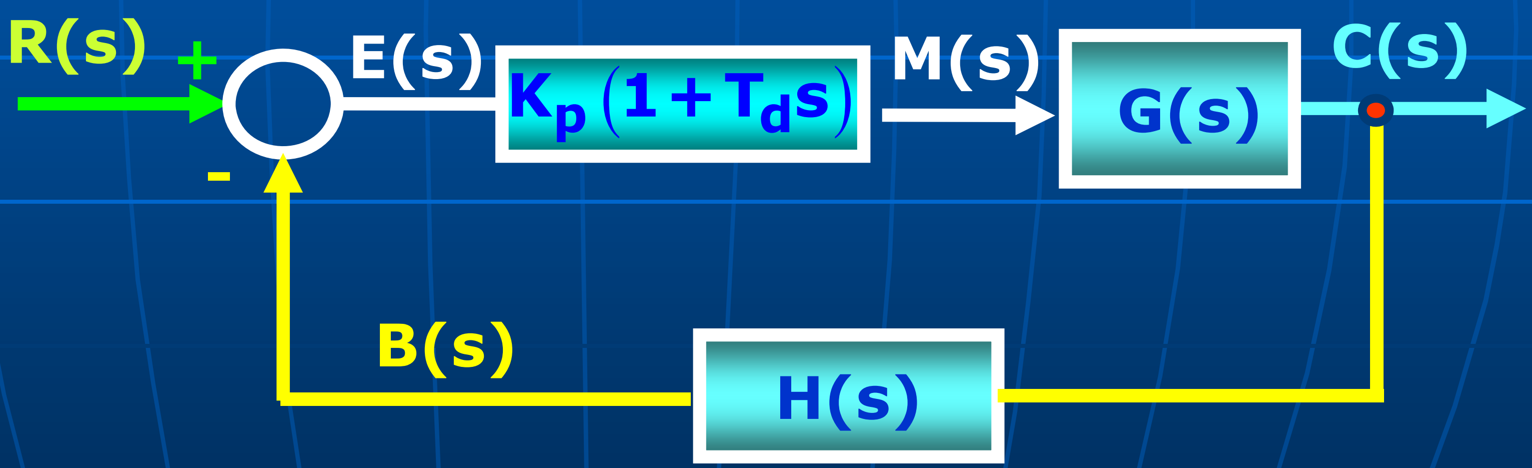
INTEGRAL CONTROL

- **Remember that each s term in the denominator of the open loop transfer function increases the type of the system by one, and thus reduces the steady state error.**
- **The use of integral controller will increase the type of the open loop transfer function by one.**
- **Note, however, that for zero error signal, the integral control may still produce a constant control signal which may in turn lead to instability.**

PROPORTIONAL + DERIVATIVE (PD) CONTROL

- Control action with **proportional + derivative** (PD) control is given by

$$m(t) = K_p e(t) + K_p T_d \frac{de(t)}{dt} \quad \Rightarrow \quad \frac{M(s)}{E(s)} = K_p \left(1 + T_d s\right)$$

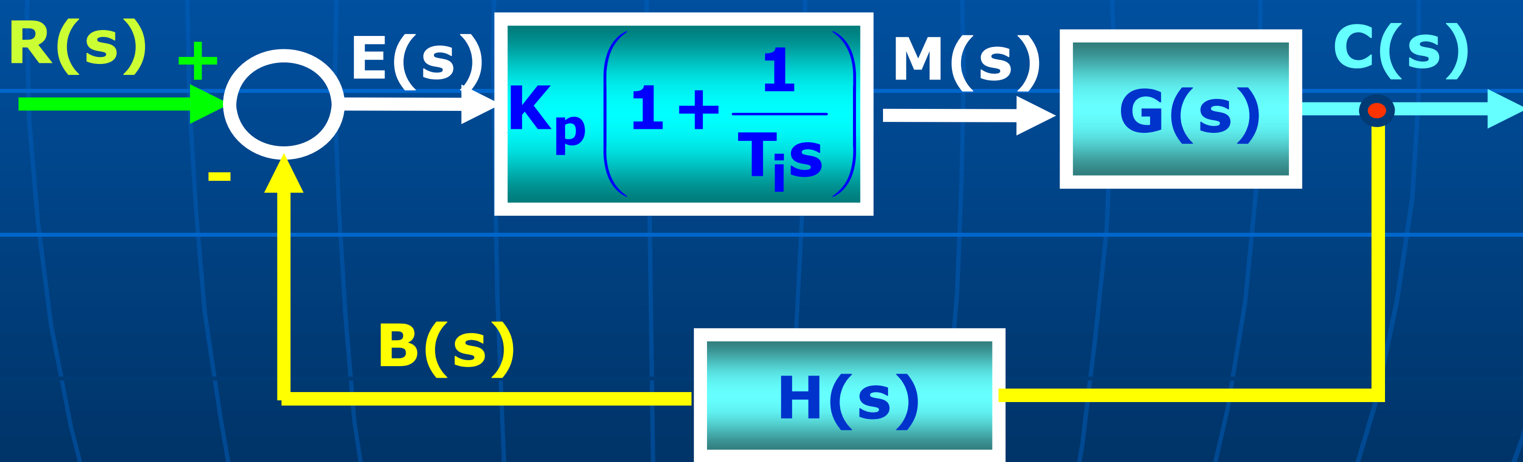


T_d : derivative time.

PROPORTIONAL + INTEGRAL (PI) CONTROL

- **Control action with proportional + integral (PI) control is given by**

$$m(t) = K_p e(t) + K_p \frac{1}{T_i} \int e(t)\,dt \quad \Rightarrow \quad \frac{M(s)}{E(s)} = K_p \left(1 + \frac{1}{T_i s} \right)$$

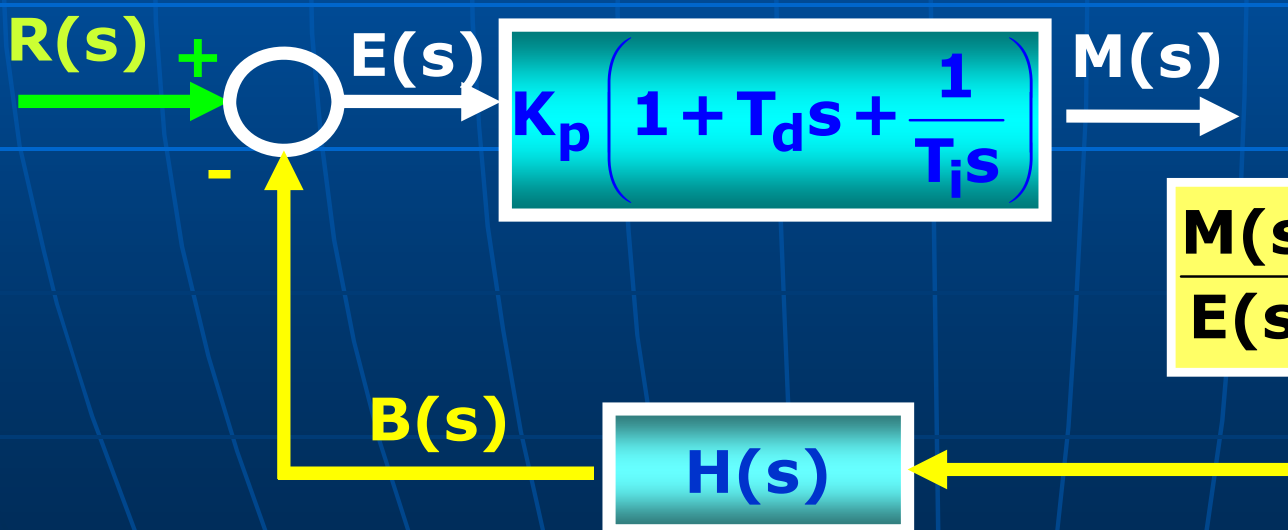


T_i : integral time

PROPORTIONAL + INTEGRAL + DERIVATIVE (PID) CONTROL

- **Control action with proportional + integral + derivative (PID) control is given by**

$$m(t) = K_p e(t) + K_p T_d \frac{de(t)}{dt} + \frac{K_p}{T_i} \int e(t)\,dt$$

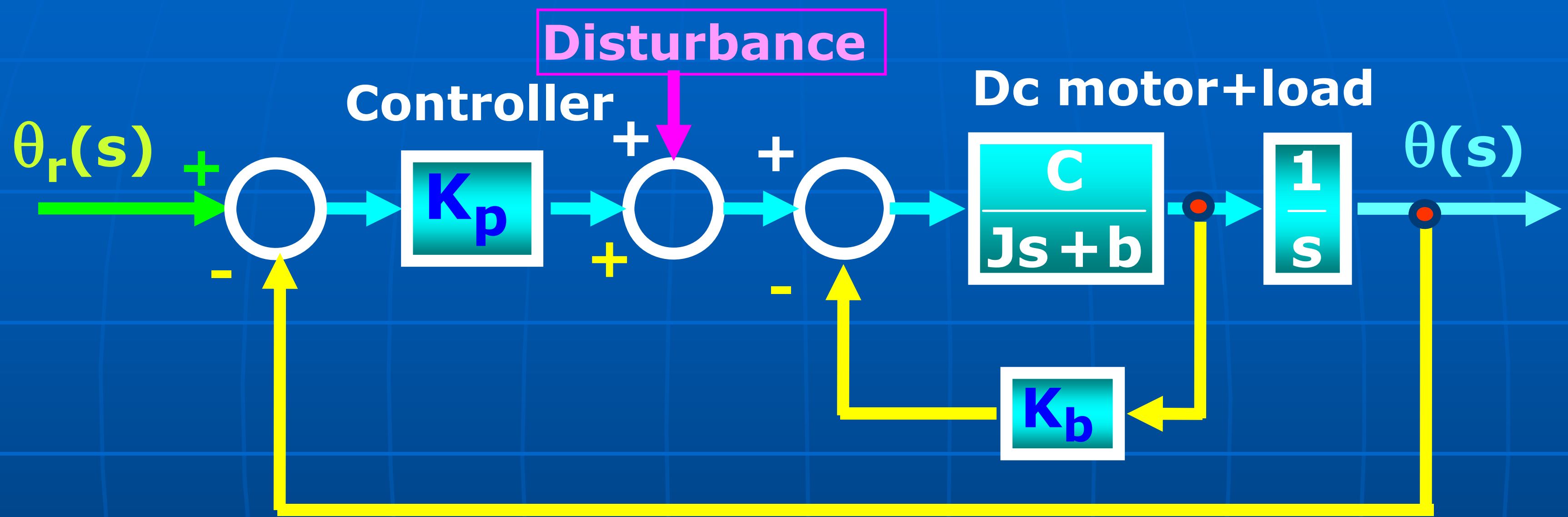


$$\frac{M(s)}{E(s)} = K_p\left(1 + T_d s + \frac{1}{T_i s}\right)$$

REALIZATION OF CONTROLLERS

- **Components in most control systems can be mechanical, electrical, hydraulic or pneumatic. In some cases, sensors, controllers, and actuators may be obtained from combinations such as electro-mechanical or electro-hydraulic components.**
- **See Ogata pp. 162-188 for various realizations of electrical, hydraulic, and pneumatic controllers.**

- **Position Control of an Inertial Load**



K : motor torque constant,

R_a : armature resistance,

K_b : motor back emf constant,

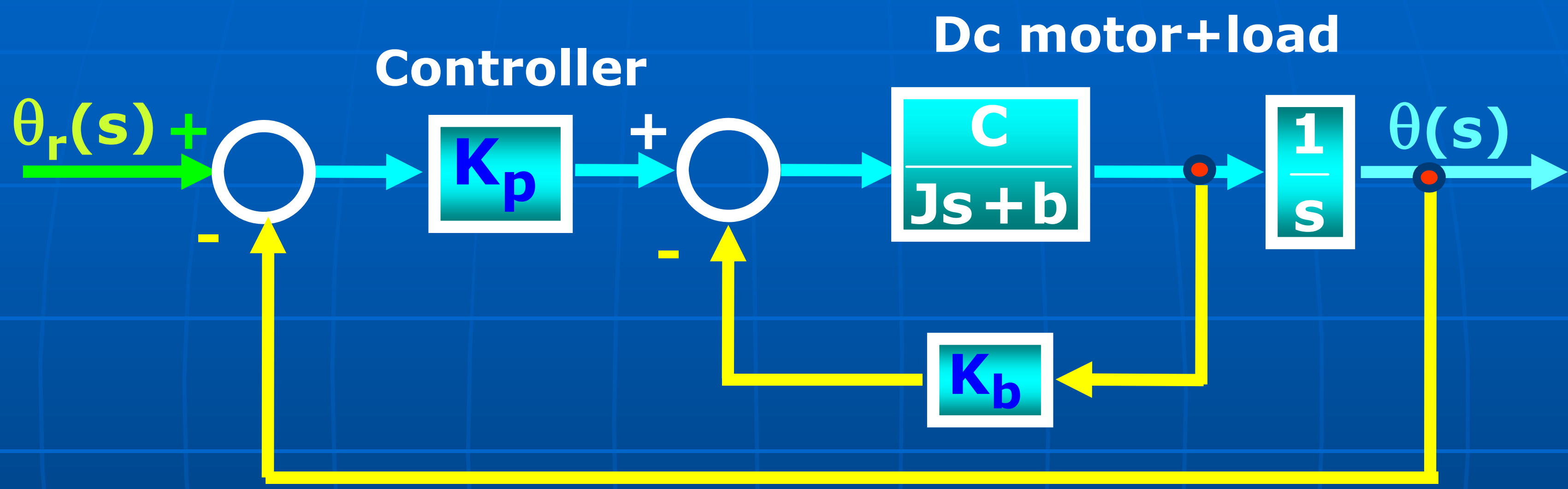
J : motor + load inertia,

b : viscous damping coefficient.

$C = K / R_a$

CONTROL ACTIONS – Proportional Control

- **Initially neglect disturbance, i.e. D(s)=0.**



$$\frac{\theta(s)}{\theta_r(s)} = \frac{\frac{K_pC}{J}}{s^2 + \frac{b+K_bC}{J}s + \frac{K_pC}{J}}$$

$$\omega_n^2 = \frac{K_pC}{J}$$

$$2\xi\omega_n = \frac{b+K_bC}{J}$$

$$\xi = \frac{b+K_bC}{2\sqrt{JC}}\,\frac{1}{\sqrt{K_p}}$$

$$\omega_n^2 = \frac{K_p C}{J}$$

$$\xi = \frac{b + K_b C}{2\sqrt{JC}} \frac{1}{\sqrt{K_p}}$$

CONTROL ACTIONS

Proportional Control

$$t_r = \frac{\pi - \beta}{\omega_d}$$

$$\beta = \tan^{-1} \frac{\omega_d}{\xi \omega_n}$$

$$t_p = \frac{\pi}{\omega_d}$$

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

$$M_p = e^{-\pi\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)}$$

$$t_s(5\%) = \frac{3}{\xi \omega_n}$$

$$t_s(2\%) = \frac{4}{\xi \omega_n}$$

- **It is noted that increasing the proportional gain :**

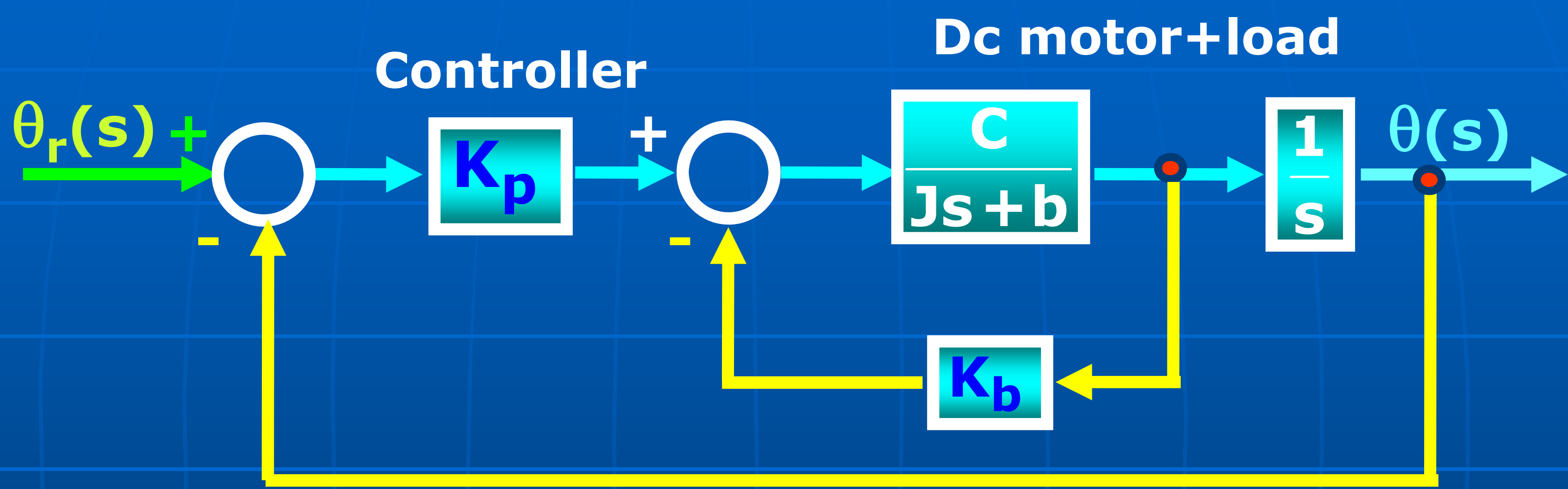
  i) Undamped natural frequency of the system is increased, thus the speed of response is increased.

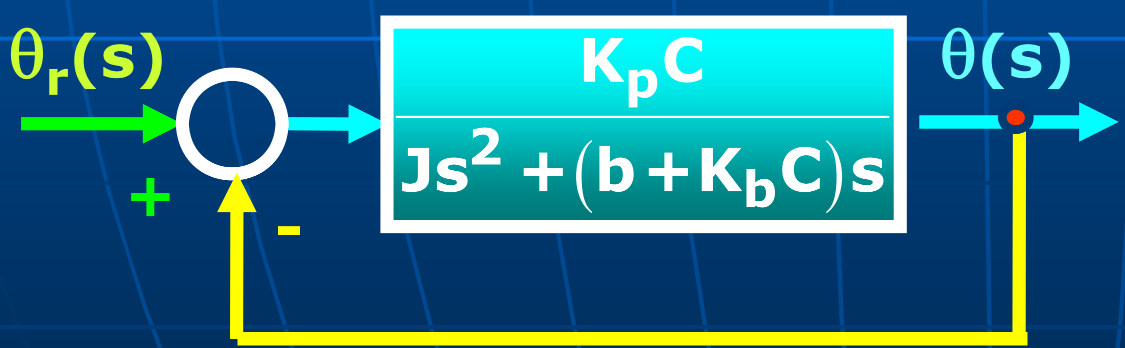
  ii) The damping ratio is decreased, thus the relative stability of the system is reduced.

Note : $\xi\omega_n$ is not affected by K_p !

CONTROL ACTIONS - Proportional Control

- **Now, let us examine the steady state response.**





Open loop transfer function :

$$G(s)=\frac{K_p C}{s\left[Js+\left(b+K_b C\right)\right]}$$

- Classify the open loop transfer function. First convert to standard form.

$$G(s)=\frac{K_pC}{s\left[Js+\left(b+K_bC\right)\right]}$$

$$G(s)=\frac{\dfrac{K_pC}{b+K_bC}}{s\left[\dfrac{J}{b+K_bC}s+1\right]}$$

Thus,

- System type is 1.
- The open loop gain is

$$K=\frac{K_pC}{b+K_bC}$$

CONTROL ACTIONS – Proportional Control

System type is 1.

The open loop gain is $K = \dfrac{K_p C}{b + K_b C}$

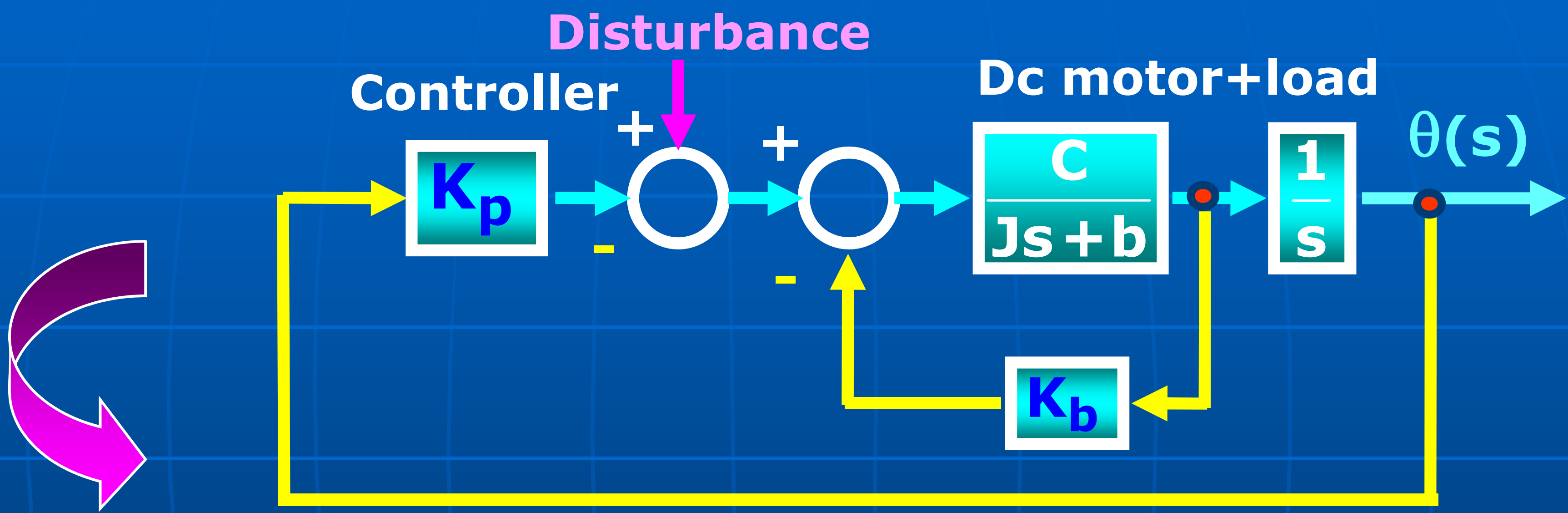
- Thus the steady state error due to unit step input will be zero.
- The steady state error due to unit ramp input will be finite.

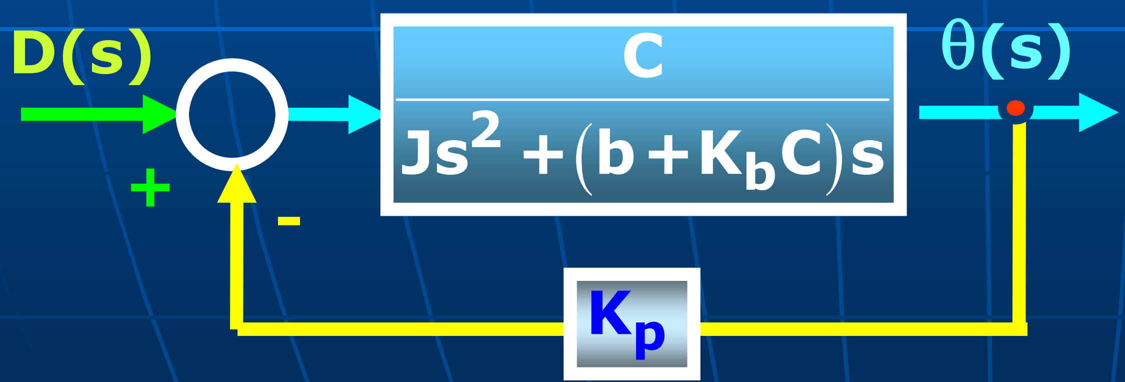
$$e_{ss} = \frac{1}{K} = \frac{b + K_b C}{C} \frac{1}{K_p}$$

- The steady state error due to unit ramp input can be minimized by using a larger value of the proportional gain.

CONTROL ACTIONS – Proportional Control

- Now let us consider disturbance input only.

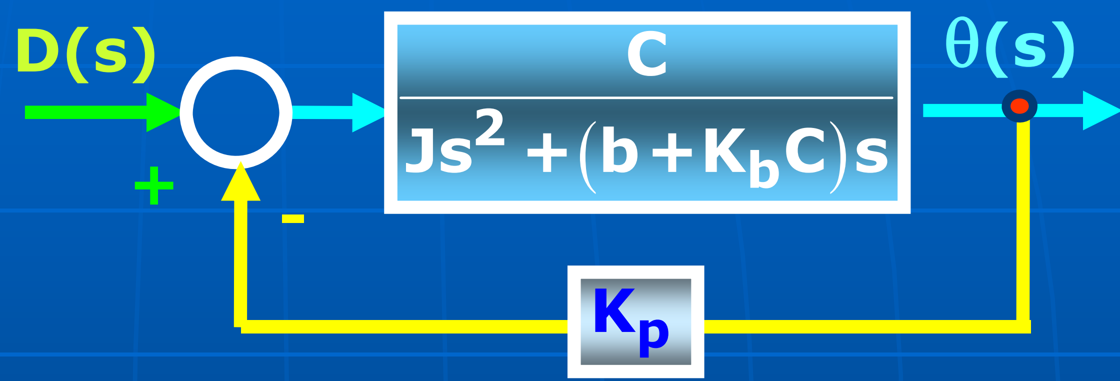




- In this case, the output itself will be the error.

CONTROL ACTIONS – Proportional Control

- In this case, the output itself will be the error.

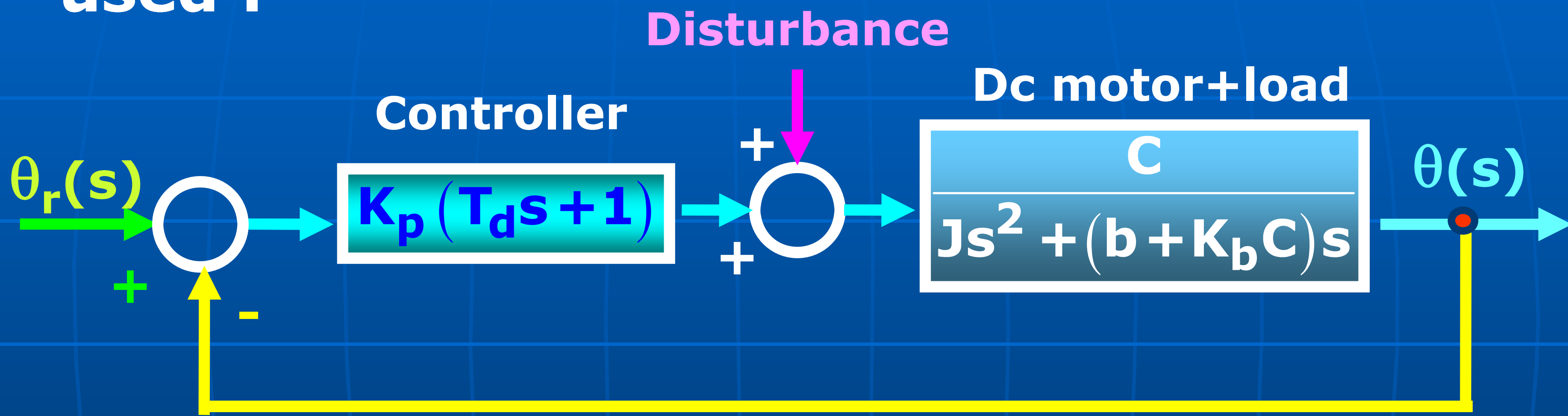


$$e_{ss} = \lim_{s \to 0} s \left(\frac{C}{Js^2 + (b + K_b C)s + K_p C} \right) \frac{1}{s} = \frac{1}{K_p}$$

- It is observed that for minimum steady state error due to a step disturbance, and a ramp input, K_p must be as large as stability requirements allow.

CONTROL ACTIONS
Proportional + Derivative Control

- **If a proportional + derivative controller is used :**



- **Neglecting disturbance**

$$\frac{\theta(s)}{\theta_r(s)}=\frac{K_pC(T_ds+1)}{Js^2+(b+K_bC+K_pT_dC)s+K_pC}$$

CONTROL ACTIONS

Proportional + Derivative Control

$$\frac{\theta(s)}{\theta_r(s)} = \frac{\frac{K_p C}{J}\left(1+T_d s\right)}{s^2 + \frac{b+K_b C+K_p T_d C}{J}s + \frac{K_p C}{J}}$$

$$\omega_n^2 = \frac{K_p C}{J}$$

$$2\xi\omega_n = \frac{b+\left(K_b + K_p T_d\right)C}{J}$$

$$\xi = \frac{b+K_b C}{2\sqrt{JC}}\frac{1}{\sqrt{K_p}} + \frac{1}{2}\sqrt{\frac{CK_p}{J}}T_d$$

Note that :

- **Derivative control does not affect the undamped natural frequency. (Meaning ?)**
- **Derivative time T_d can be selected to improve damping ratio.**

CONTROL ACTIONS

Proportional + Derivative Control

$$G(s)=\frac{K_pC(1+T_ds)}{s\left[Js+(b+K_bC)\right]}=\frac{\dfrac{K_pC(1+T_ds)}{b+K_bC}}{s\left[\dfrac{J}{b+K_bC}s+1\right]}$$

System type is still 1, thus the steady state error for unit step input is zero.

- The steady state error for unit ramp input is given as :

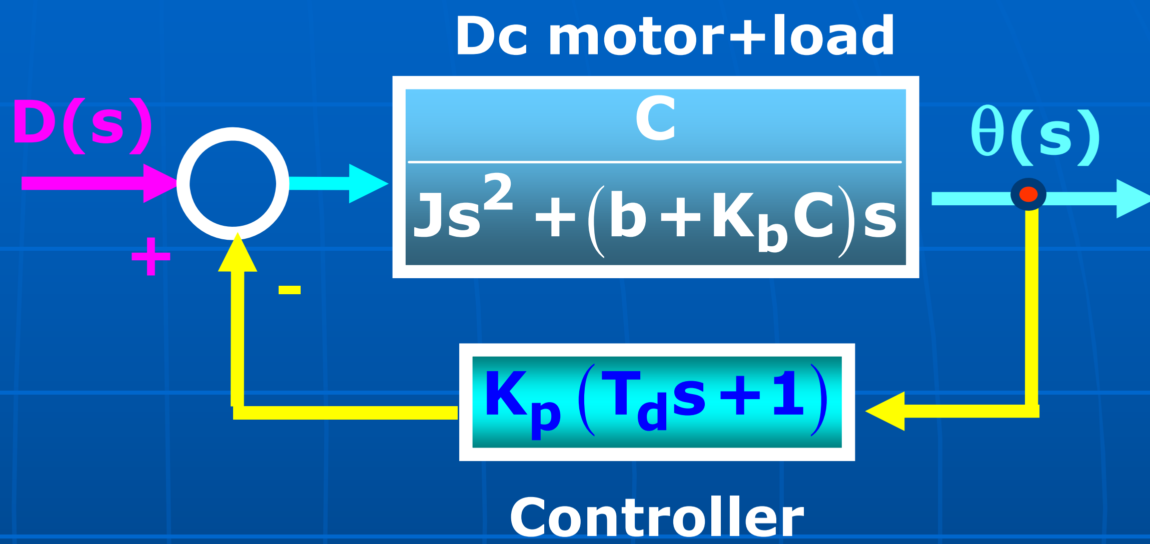
$$e_{ss}=\frac{1}{K}=\frac{b+K_bC}{C}\frac{1}{K_p}$$

- Therefore, the derivative control has no effect on the steady state error.

CONTROL ACTIONS

Proportional + Derivative Control

If the disturbance is taken to be the only input, then the output $\theta(s)$ will be the error.



$$e_{ss}=\lim_{s\to 0}s\left(\frac{C}{Js^2+(b+K_bC+K_pT_dC)s+K_pC}\right)\frac{1}{s}=\frac{1}{K_p}$$

CONTROL ACTIONS
Proportional + Derivative Control

It is observed that, if the disturbance is taken to be the only input, the steady state error (equal to the steady state value of the output) for a unit step input also is unaffected by the inclusion of the derivative control.

$$e_{ss}\Big|_{\text{unit step}} = \frac{1}{K_p}$$

CONTROL ACTIONS
Proportional + Derivative Control

- **<u>In conclusion</u>, the addition of derivative control action to proportional control action**

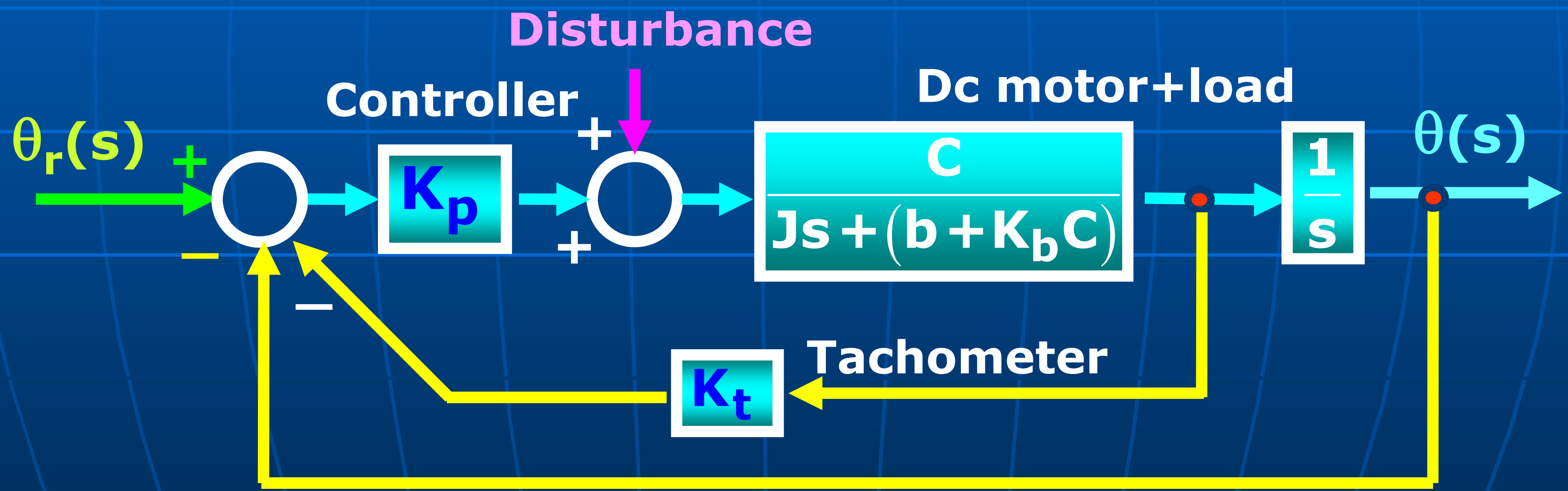
  - is useful to provide additional control on damping and hence improves relative stability,

  - has no direct effect on steady state error.

- **It may, however, be useful in reducing steady state error indirectly by allowing the use of larger values of K_p without causing stability problems.**

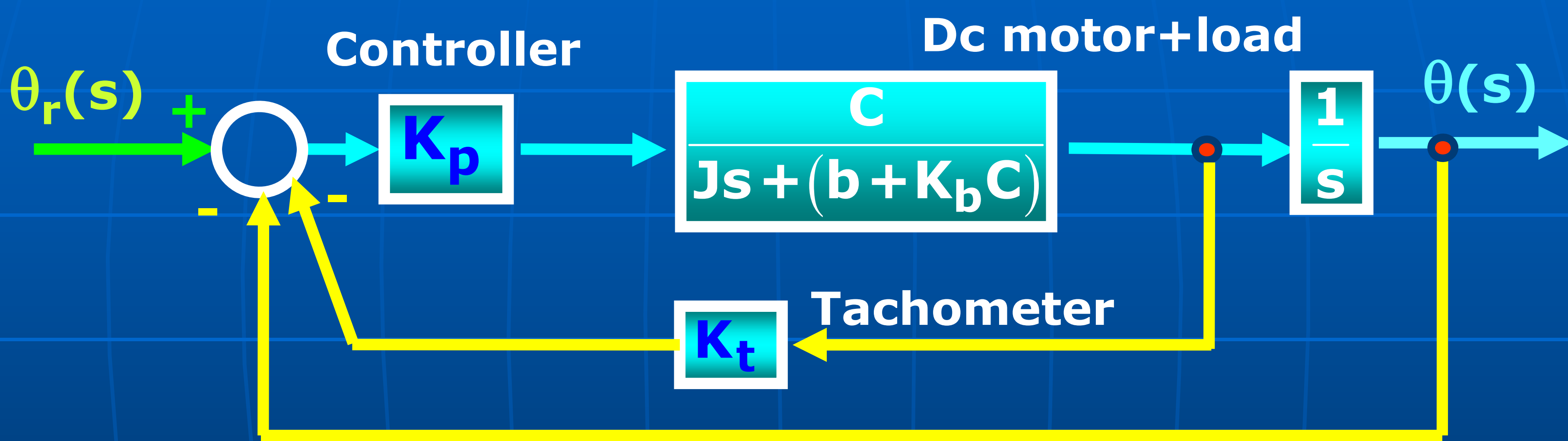
CONTROL ACTIONS
Velocity (Tachometer) Feedback

- **Velocity or tachometer feedback can be employed together with proportional control to provide characteristics similar to that of PD-control.**



CONTROL ACTIONS
Velocity (Tachometer) Feedback

- **Consider the case without disturbance.**



$$\frac{\theta(s)}{\theta_r(s)} = \frac{K_pC}{Js^2 + (b + K_bC + K_pK_tC)s + K_pC}$$

CONTROL ACTIONS
Velocity (Tachometer) Feedback

- Clearly, if you replace T_d by K_t, the characteristic equation is of the same form as that of the PD-control case.
- Thus the undamped natural frequency and damping ratio expressions are again the same, if one replaces T_d by K_t.

$$\frac{\theta(s)}{\theta_r(s)} = \frac{K_p C}{Js^2 + \left(b + K_b C + K_p K_t C\right)s + K_p C}$$

$$\omega_n^2 = \frac{K_p C}{J}$$

$$\xi = \frac{b + K_b C}{2\sqrt{JC}} \frac{1}{\sqrt{K_p}} + \frac{1}{2}\sqrt{\frac{CK_p}{J}} K_t$$

CONTROL ACTIONS
Velocity (Tachometer) Feedback

$$G(s) = \frac{K_pC}{s\left[Js + \left(b + K_bC + K_pK_tC\right)\right]} = \frac{\dfrac{K_pC}{b + K_bC + K_pK_tC}}{s\left[\dfrac{J}{b + K_bC + K_pK_tC}s + 1\right]}$$

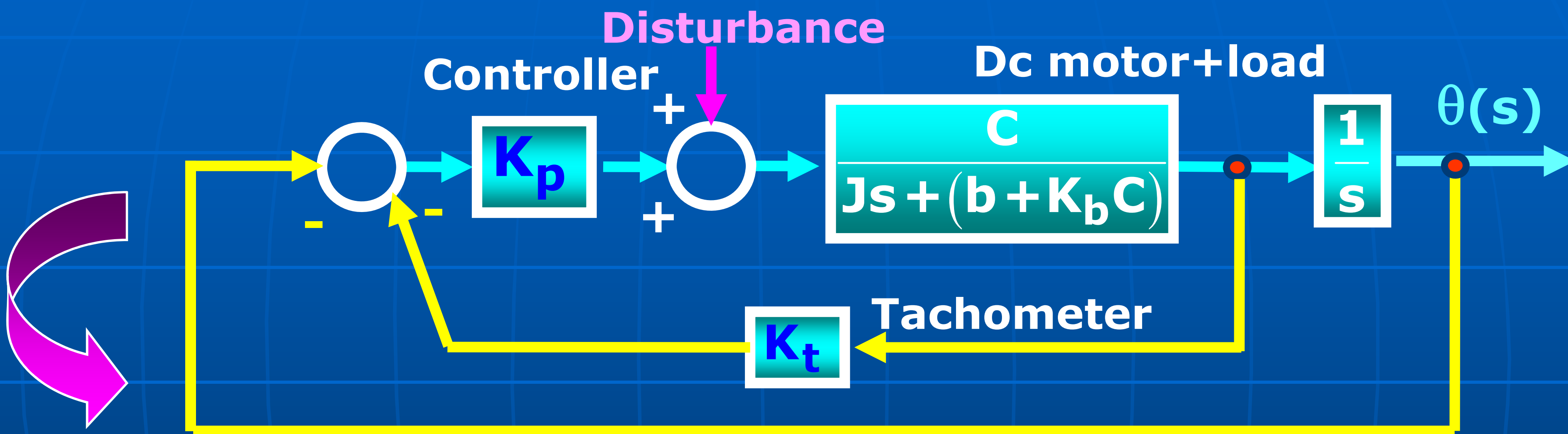
System type is 1, thus the steady state error for unit step input is zero.

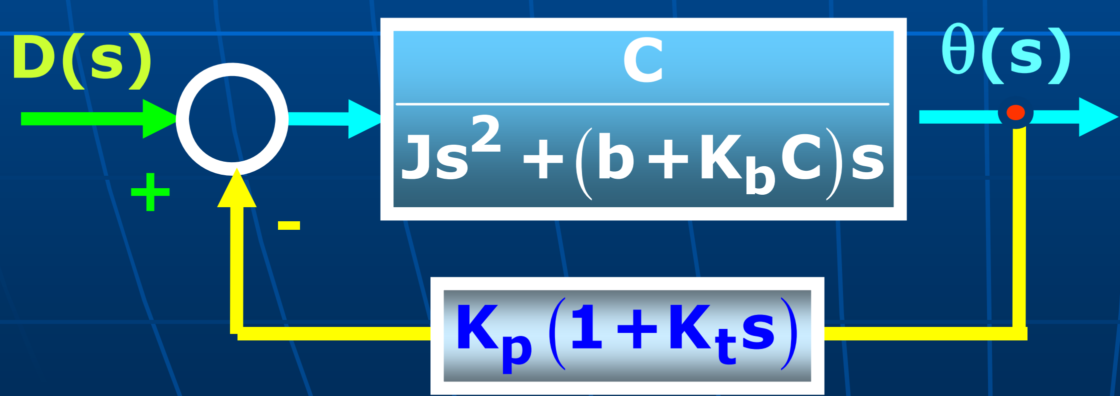
- The steady state error for unit ramp input is given as :

$$e_{ss} = \frac{1}{K} = \frac{b + K_bC + K_pK_tC}{C}\frac{1}{K_p} = \left(\frac{b + K_bC}{C}\right)\frac{1}{K_p} + K_t$$

CONTROL ACTIONS
Velocity (Tachometer) Feedback

- **Now let us consider disturbance input only.**



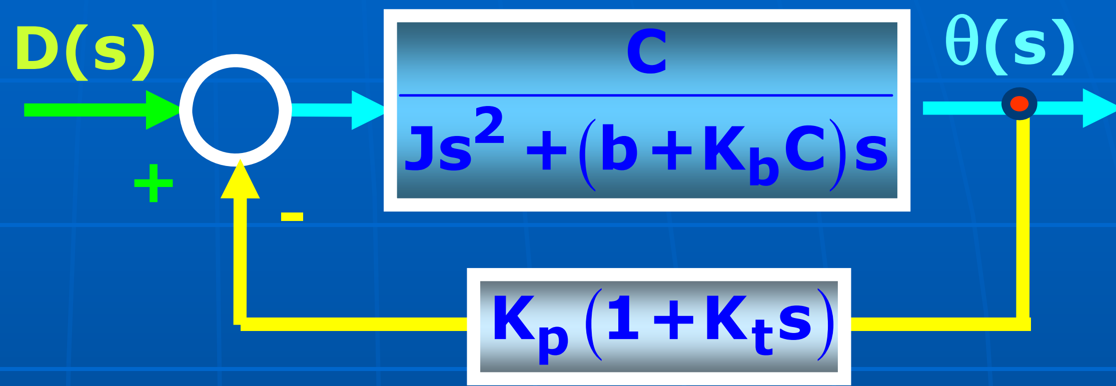


- **In this case, the output itself will be the error.**

CONTROL ACTIONS

Velocity (Tachometer) Feedback

- In this case, the output itself will be the error.

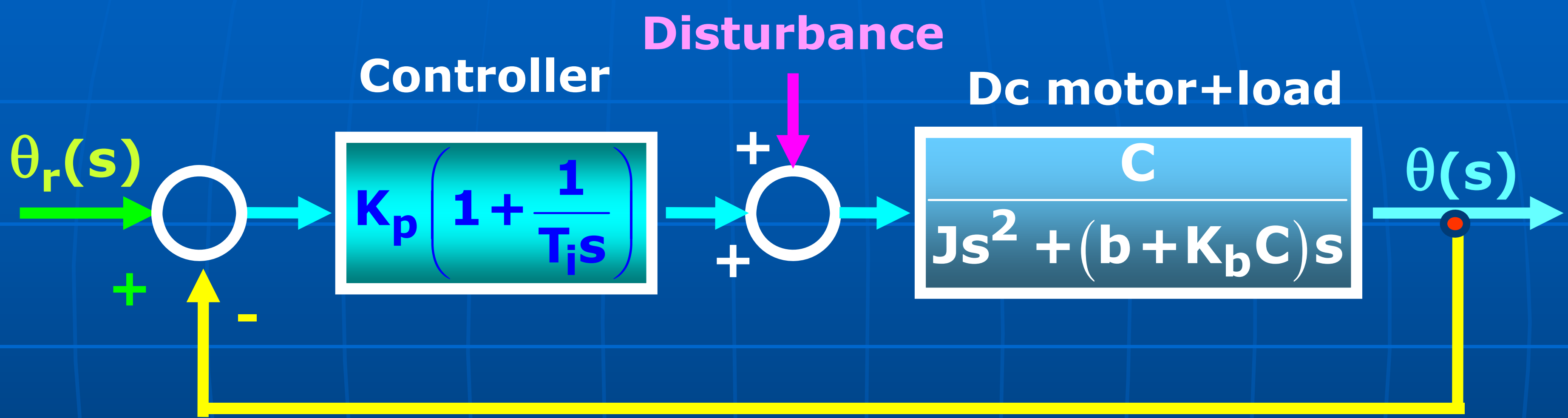


$$e_{ss}=\lim_{s\to 0} s\frac{C}{Js^2+\left[b+C\left(K_b+K_pK_t\right)\right]s+K_pC}\frac{1}{s}=\frac{1}{K_p}$$

- It is observed that disturbance response of proportional control + velocity feedback is the same as that of proportional + derivative control.

CONTROL ACTIONS
Proportional + Integral Control

- **If a proportional + integral controller is used :**



- **Neglecting disturbance**

$$\frac{\theta(s)}{\theta_r(s)}=\frac{K_pC(T_is+1)}{T_iJs^3+(b+K_bC)s^2+K_pT_iCs+K_pC}$$

CONTROL ACTIONS
Proportional + Integral Control

$$\frac{\theta(s)}{\theta_r(s)} = \frac{K_p C (T_i s + 1)}{T_i J s^3 + T_i (b + K_b C) s^2 + K_p T_i C s + K_p C}$$

Note that :

- **Addition of integral control action to proportional control action increases the order of the system.**
- **Thus, transient and steady response as well as the stability and disturbance rejection characteristics of the system will be changed.**

CONTROL ACTIONS

Proportional + Integral Control

$$G(s)=\frac{K_pC(T_is+1)}{s^2T_i\left[Js+(b+K_bC)\right]}=\frac{\dfrac{K_pC(T_is+1)}{T_i(b+K_bC)}}{s^2\left[\dfrac{J}{b+K_bC}s+1\right]}$$

System type is now 2, thus the steady state error for unit step and ramp inputs are zero.

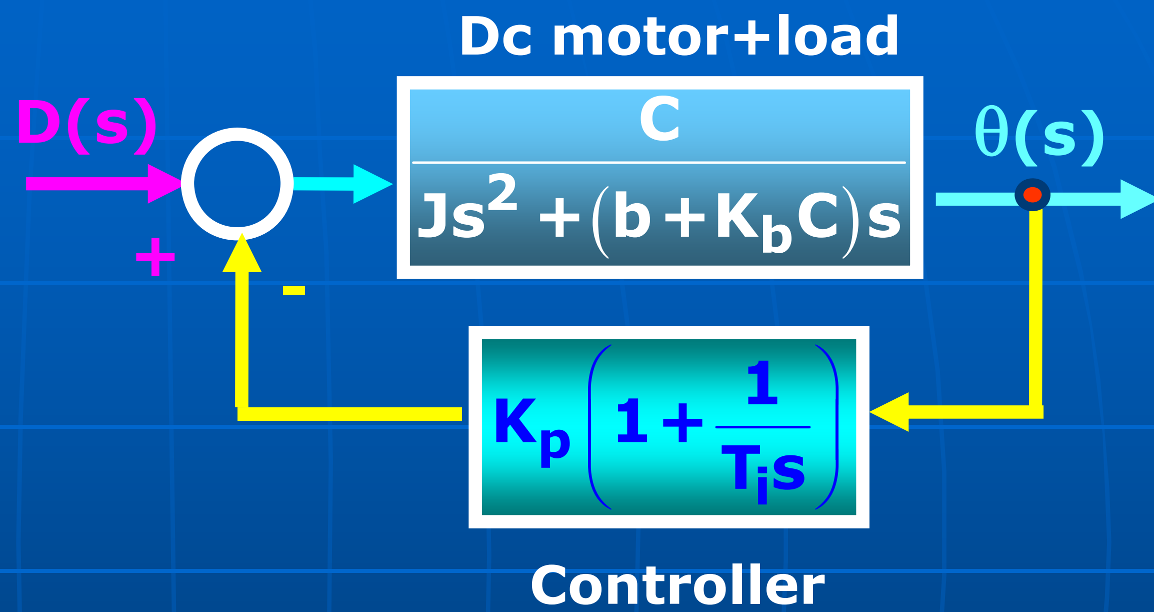
- **The steady state error for unit acceleration input is given as :**

$$e_{ss}=\frac{1}{K}=\frac{T_i(b+K_bC)}{C}\frac{1}{K_p}$$

- **Therefore, the integral control can be used to reduce steady state error.**

CONTROL ACTIONS
Proportional + Integral Control

If the disturbance is taken to be the only input, then the output $\theta(s)$ will be the error.



$$e_{ss}=\lim_{s\to 0} s\left(\frac{T_iCs}{T_iJs^3+T_i(b+K_bC)s^2+K_pT_iCs+K_pC}\right)\left(\frac{1}{s}\right)=0$$

$$e_{ss}=\lim_{s\to 0} s\left(\frac{T_iCs}{T_iJs^3+T_i(b+K_bC)s^2+K_pT_iCs+K_pC}\right)\left(\frac{1}{s^2}\right)=\frac{T_i}{K_p}$$

CONTROL ACTIONS
Proportional + Integral Control

If the disturbance is taken to be the only input, the steady state error (equal to the steady state value of the output) for a unit step disturbance input is zeroed and the steady state error for a unit ramp disturbance input becomes finite by the inclusion of the integral control.

$$e_{ss}\Big|_{step} = 0$$

$$e_{ss}\Big|_{unit\ ramp} = \frac{T_i}{K_p}$$

CONTROL ACTIONS
Proportional + Integral Control

- **<u>In conclusion</u>, it is observed that the addition of integral control action to proportional control action**

  - is useful in reducing the steady state error by increasing the type of the open loop transfer function by one,

  - changes the transient response characteristics of the system by increasing the order of the system, making the system more susceptible to instability.