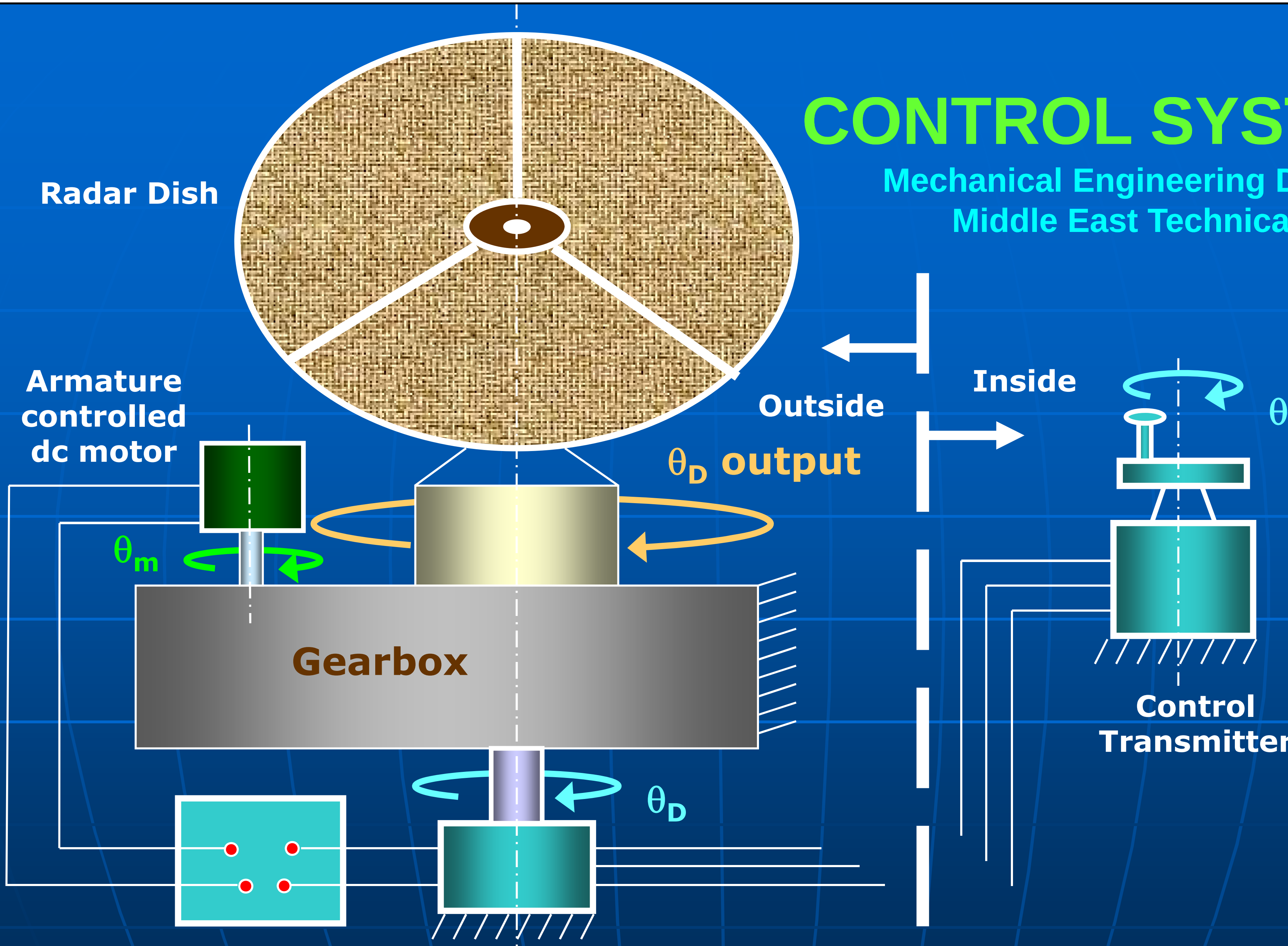
ME 304

CONTROL SYSTEMS

Mechanical Engineering Department,
Middle East Technical University



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CH XI

COURSE OUTLINE

I. INTRODUCTION & BASIC CONCEPTS
II. MODELING DYNAMIC SYSTEMS
III. CONTROL SYSTEM COMPONENTS
IV. STABILITY
V. TRANSIENT RESPONSE
VI. STEADY STATE RESPONSE
VII. DISTURBANCE REJECTION
VIII. BASIC CONTROL ACTIONS & CONTROLLERS
IX. FREQUENCY RESPONSE ANALYSIS
X. SENSITIVITY ANALYSIS

XI. ROOT LOCUS ANALYSIS

ROOT LOCUS - OBJECTIVES

Getting familiar with the

- **definition of root locus, and**
- **concept of root locus analysis.**

Understanding the

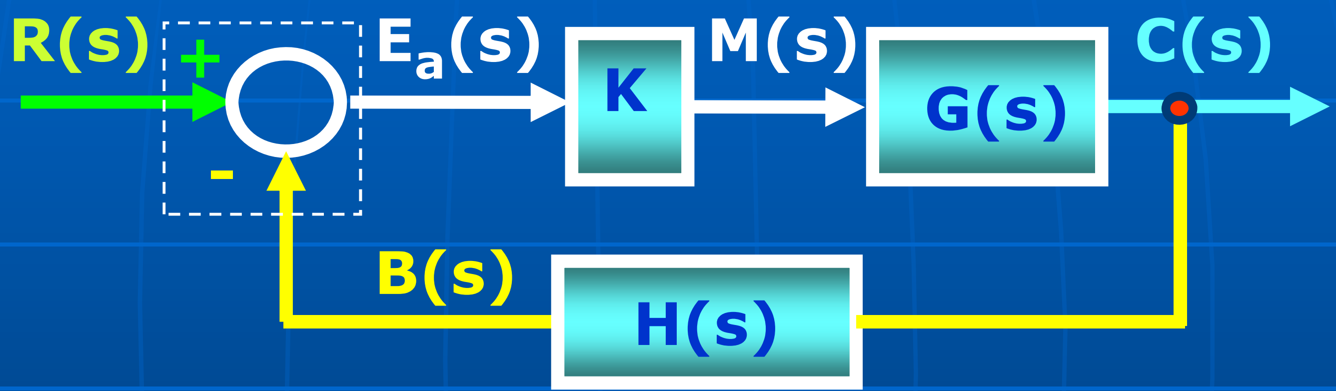
- **use of root locus to find the poles of a closed loop system,**
- **selection of a design parameter to meet transient response and stability requirements of a control system.**

ROOT LOCUS - Definition

- **Root Locus** is the trajectory of the closed loop poles of a system in the s (complex) plane, as the value of a design parameter (such as open loop gain) is varied.
- Thus, the variation of the dynamic behaviour of the system with different values of the design parameter can be observed and a suitable value can be selected for use.

ROOT LOCUS

- Consider a typical closed loop system.



- Open Loop Transfer Function

$$\frac{B(s)}{R(s)} = KG(s)H(s)$$

Closed Loop Transfer Function

$$T(s) = \frac{C(s)}{R(s)} = \frac{KG(s)}{1 + KG(s)H(s)}$$

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$$T(s) = \frac{C(s)}{R(s)} = \frac{KG(s)}{1+KG(s)H(s)}$$

ROOT LOCUS

- It is noted that, the transient response and stability of the system is dependent on the poles of the closed loop transfer function T(s).
- The poles of the closed loop transfer function (poles of the open loop transfer function are independent of K) vary as the parameter K varies. The characteristic equation is given by :

$$1+KG(s)H(s)=0$$

$$1+KG(s)H(s)=0$$

ROOT LOCUS

- If one writes the open loop transfer function in the form

$$G(s)H(s)=\frac{N(s)}{D(s)}$$

Then

$$1+K\frac{N(s)}{D(s)}=0$$

and

$$\frac{D(s)}{K}+N(s)=0$$

or

$$D(s)+KN(s)=0$$

$$G(s)H(s)=\frac{N(s)}{D(s)}$$

$$1+K\frac{N(s)}{D(s)}=0$$

ROOT LOCUS

$$D(s)+KN(s)=0$$

- It is clear that as $K \Rightarrow 0$, the roots of the characteristic equation (closed loop poles) are given by

$$D(s)=0$$

i.e. the poles of the open loop transfer function.

$$G(s)H(s)=\frac{N(s)}{D(s)}$$

$$G(s)H(s)=\frac{N(s)}{D(s)}$$

$$1+K\frac{N(s)}{D(s)}=0$$

ROOT LOCUS

$$\frac{D(s)}{K}+N(s)=0$$

- Similarly, if $K \Rightarrow \infty$ then the roots of the characteristic equation (closed loop poles) are given by

$$N(s)=0$$

i.e. the zeroes of the open loop transfer function.

$$G(s)H(s)=\frac{N(s)}{D(s)}$$

ROOT LOCUS

- **Thus, as K varies from 0 to ∞, the roots of the characteristic equation (closed loop poles)**
  - **start as the poles of the open loop transfer function and**
  - **end as the zeroes of the open loop transfer function.**
- **For an nth order system, there will be n closed loop poles.**

ROOT LOCUS

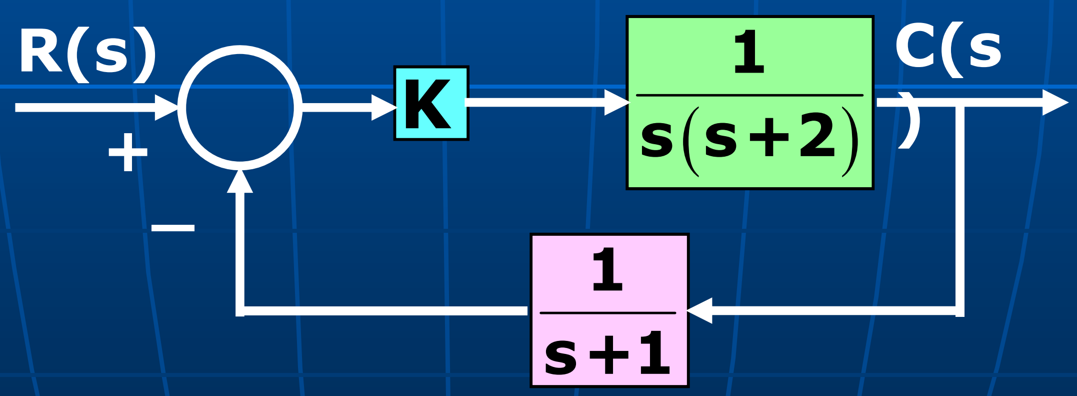
- **Let the order of the numerator and denominator polynomials of the open loop transfer function be m and n, respectively. Thus the open loop transfer function will have m zeroes and n poles.**
- **Then, n closed loop poles will start from the open loop poles and only m will end at open loop zeroes.**
- **The remaining n-m closed loop poles will end at infinity, i.e. open loop zeroes at infinity.**

ROOT LOCUS

- Thus, **the root locus will consist of n branches**; each starting from an open loop pole and ending either at an open loop zero or at infinity.
- Corresponding to each value of the gain K, there will be n closed loop poles; one on each branch of the root locus.
- Hence, by proper selection for the gain K, the poles of the system can be located such that the system response is close to the desired response.

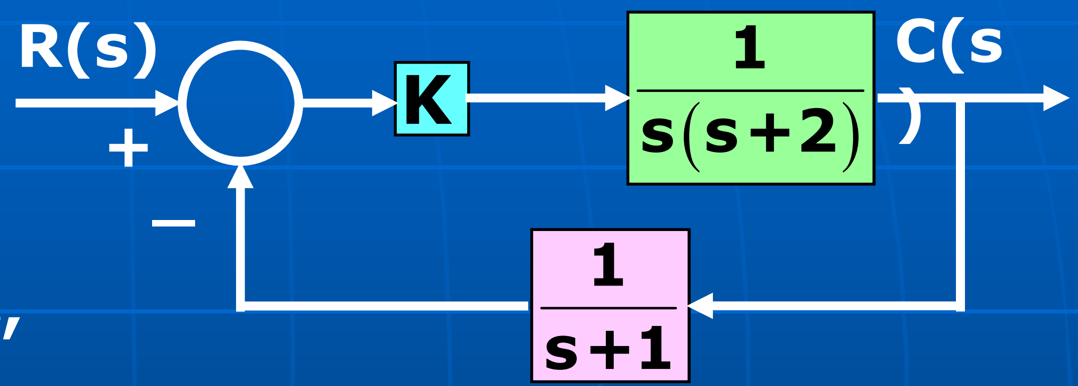
ROOT LOCUS Example 1a

- Draw the root locus for the following system.
- Select a value for the gain K such that the system has an undamped natural frequency of at least 0.5 Hz and a damping ratio between 0.5 to 0.8.



ROOT LOCUS Example 1b

- To draw the root locus, first determine the open loop transfer function.



- 3 open loop poles, (0,-1,-2)
- 0 open loop zeroes, (3 open loop zeroes at infinity)

$$T(s) = \frac{K}{s(s+1)(s+2)}$$

Thus the root locus will have 3 branches starting from the open loop poles at 0, -1, and -2 and all of them will go to infinity.

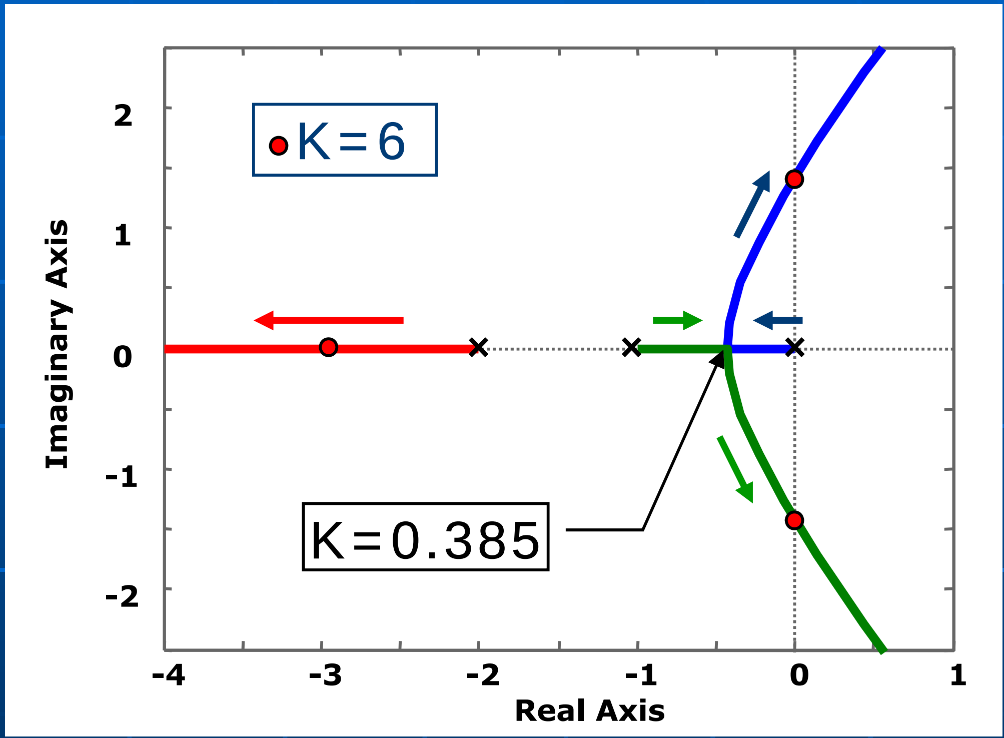
$$T(s)=\frac{K}{s(s+1)(s+2)}$$

ROOT LOCUS Example 1c

- Enter the Matlab commands :

```
s=tf('s');
T=1/s/(s+1)/(s+2)
rlocus(T)
axis([-4 1 -2.5 2.5])
```

K	Closed loop poles
0	0, -1, -2
0.385	-0.423, -0.423, -2.155
6	$-3, \pm\sqrt{2}j$
10	$-3.3, 0.155 \pm 1.73j$



Remember that for a complex pole : $s_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$

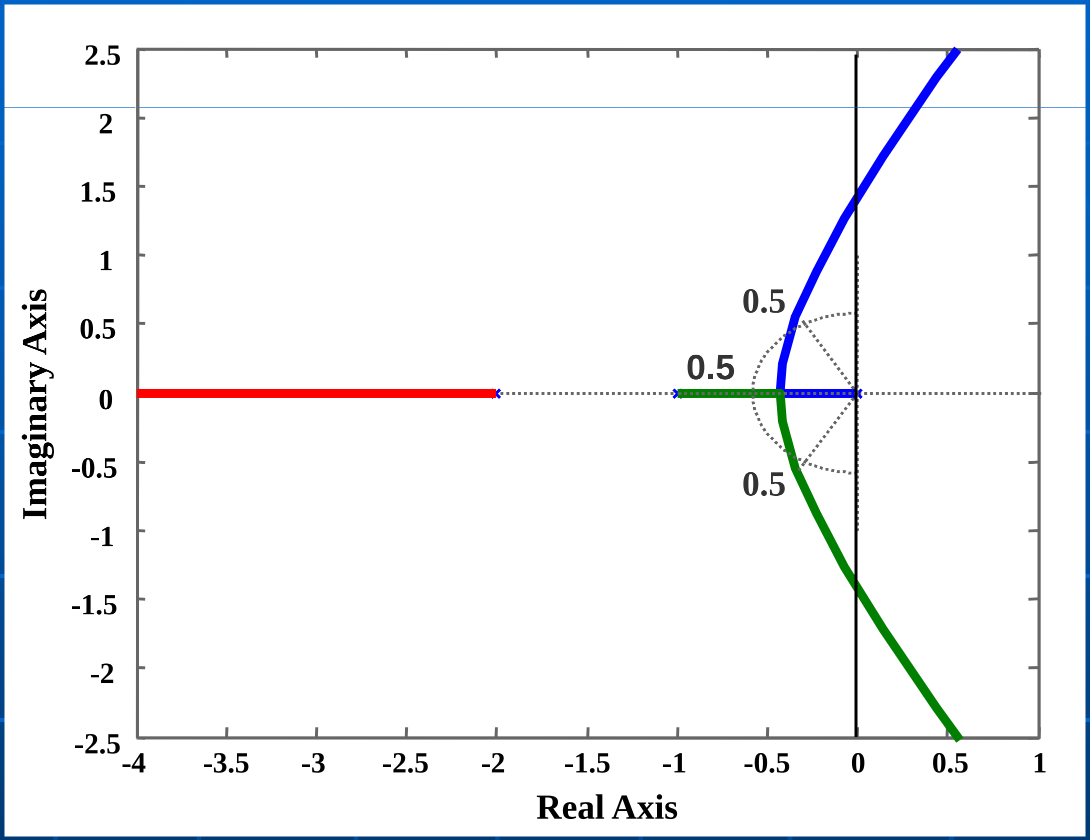
$$T(s) = \frac{K}{s(s+1)(s+2)}$$

ROOT LOCUS Example 1d

- Add 3 more Matlab commands.

```
s=tf('s');
T=1/s/(s+1)/(s+2)
rlocus(T)
axis([-4 1 -2.5 2.5])

zeta=0.5;
wn=0.5;
sgrid(zeta, wn)
```



to get the constant undamped natural frequency and damping ratio lines.

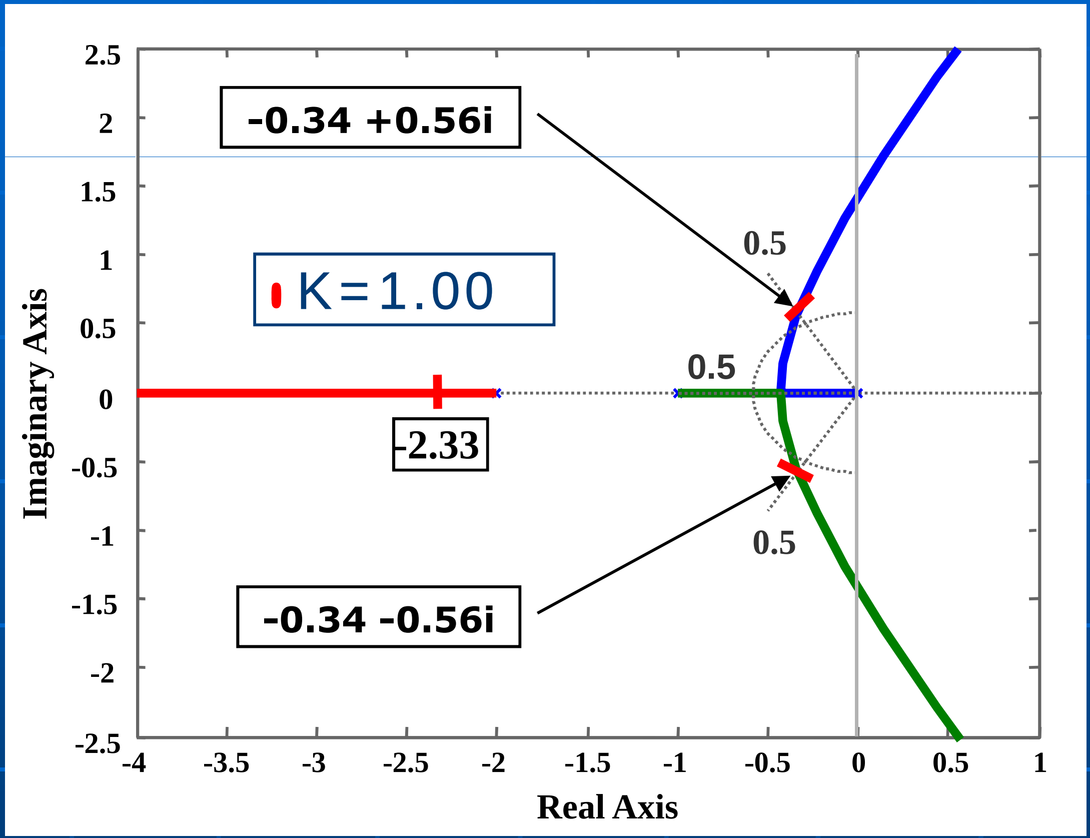
$$T(s) = \frac{K}{s(s+1)(s+2)}$$

ROOT LOCUS Example 1e

- Add 1 more Matlab command.

```
s=tf('s');
T=1/s/(s+1)/(s+2)
rlocus(T)
axis([-4 1 -2.5 2.5])

[K,poles] = rlocfind(T)
```



- Click on the crosshair cursor in the graphics window put by **rlocfind** to select a pole location on an existing root locus. The root locus gain associated with this point is returned in **K** and all the system poles for this gain are returned in **poles**.

ROOT LOCUS Reading

- Read Nise 8.1-8.4, 8.7
- Dorf & Bishop 7.1-7.2

The END

(for this term !)