COURSE OUTLINE

I. INTRODUCTION & BASIC CONCEPTS
II. MODELING DYNAMIC SYSTEMS
III. CONTROL SYSTEM COMPONENTS
IV. STABILITY
V. TRANSIENT RESPONSE
VI. STEADY STATE RESPONSE
VII. DISTURBANCE REJECTION
VIII. BASIC CONTROL ACTIONS & CONTROLLERS
IX. FREQUENCY RESPONSE ANALYSIS
X. SENSITIVITY ANALYSIS
XI. ROOT LOCUS ANALYSIS
### Sensitivity - Objectives

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<th>Getting familiar with the concept of sensitivity.</th>
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<td>Applying sensitivity analysis to basic components of control systems.</td>
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<td>Investigation of sensitivity of control systems to parameter variations.</td>
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There are many uncertainties encountered in the design and analysis of control systems.

In modeling and parameter selection imperfections and inaccuracies always exist.

Parameters may also vary in time due to age, wear, etc.
One of the basic design criteria for control systems is to minimize the sensitivity of the response to modelling inaccuracies and parameter variations.

In control system terminology, a system that is insensitive to external disturbances and parameter variations is called a robust system.
The sensitivity function of a system is defined as the ratio of the percentage variation of the system transfer function, $M$, to the percentage variation of a system parameter, $k$.

\[
S^M_k = \frac{dM}{M \frac{dk}{k}} = \frac{\text{percentage change in } M \text{ (due to a change in } k)}{\text{percentage change in } k}
\]
- Consider the open loop control system.

\[ C(s) = G(s)R(s) \]

- Sensitivity of \( C(s) \) w.r.t. \( G(s) \)

\[ S_{CG} = \frac{dC/C}{dG/G} \]
Therefore, any change or uncertainty in the transfer function of the open loop system is directly reflected in the output of the system.
Consider the general (canonical) form of the closed loop control system.

- Sensitivity of $C(s)$ w.r.t. $G(s)$
- Sensitivity of $C(s)$ w.r.t. $H(s)$
Express the output in terms of the input.

\[
\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}
\]

\[
\frac{dC}{C} = \left[ \frac{dG - G^2dH}{(1+GH)^2} \right] \left( \frac{1+GH}{G} \right) \frac{1}{R} = \frac{dG - G^2dH}{G(1+GH)}
\]

\[
dC = \frac{1}{1+GH} \left( \frac{dG}{G} - \frac{GH}{1+GH} \right) dH
\]
Let $dH=0$:

\[
\frac{dC}{C} = \frac{1}{1+GH} \frac{dG}{G} - \frac{GH}{1+GH} \frac{dH}{H}
\]

\[
S^C_G = \frac{dC/C}{dG/G} = \frac{1}{1+GH}
\]

Let $dG=0$:

\[
S^C_H = \frac{dC/C}{dH/H} = -\frac{GH}{1+GH}
\]
If large controller gains are chosen so that:

\[ \lim_{s \to \infty} G(s)H(s) \to \infty \]

Then

\[ S_G^C = \frac{dC/C}{dG/G} = \frac{1}{1 + GH} \]

which is very good!

\[ \lim_{GH \to \infty} S_G^C = 0 \]

\[ S_H^C = \frac{dC/C}{dH/H} = -\frac{GH}{1 + GH} \]

which is not so good!

\[ \lim_{GH \to \infty} S_H^C = -1 \]
Therefore by choosing $G(s)H(s) \rightarrow \infty$

The effects of parameter variations in feedforward elements (i.e. the controller, the actuator, the plant) can be suppressed.

The effects of the parameter variations in feedback elements (sensors) cannot be suppressed and their variations will be fully reflected in the variation of the output.

**Conclusion:** Quality of a feedback control system depends on the quality of measurement.
Let us determine the sensitivity of the transfer function of the system represented by the block diagram shown, to variations in each of the parameters $K_1$, $K_2$, and $p$.

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$
- Sensitivity of the transfer function w.r.t. variations in the feedforward gain $K_1$.

\[ T(s) = \frac{K_1}{s(s+p) + \frac{K_1K_2s}{s(s+p)}} = \frac{K_1}{s(s+p + K_1K_2)} \]

\[ S^T_{K_1}(s) = \frac{K_1}{K_1} \frac{dT}{dT} = \frac{1}{K_1} \frac{1}{s(s+p + K_1K_2)} \left( s + p + K_1K_2 \right) - \frac{K_1(K_2)}{s(s+p + K_1K_2)^2} \]

\[ S^T_{K_1}(s) = \frac{s + p}{s + p + K_1K_2} \]
Sensitivity of the transfer function w.r.t. variations in the feedback gain $K_2$.

$$T(s) = \frac{K_1}{s(s+p)(1 + \frac{K_1K_2s}{s(s+p)})} = \frac{K_1}{s(s+p+K_1K_2)}$$

$$S^T_{K_2}(s) = \frac{K_2}{T} \frac{dT}{dK_2} = \frac{K_2}{K_1} \frac{1}{s(s+p+K_1K_2)^2}$$

$$S^T_{K_2}(s) = \frac{-K_1K_2}{s+p+K_1K_2}$$
Sensitivity of the transfer function w.r.t. variations in the open loop pole $p$.

$$T(s) = \frac{K_1}{s(s+p)} = \frac{K_1}{s(s+p+K_1K_2)}$$

$$S_p^T(s) = \frac{p}{T} \frac{dT}{dp} = \frac{p}{K_1 \frac{s(s+p+K_1K_2)}{s(s+p+K_1K_2)}} \frac{1}{s} \frac{-K_1(1)}{(s+p+K_1K_2)^2}$$

$$S_p^T(s) = \frac{-p}{s+p+K_1K_2}$$
It is noted that the sensitivity expressions are in general functions of s.

The steady state sensitivities can be obtained by letting s=0 in those expressions.
The dynamic sensitivity is obtained by letting

\[ s = j\omega \]

in the sensitivity expressions, and then by plotting the sensitivity versus frequency.

It is also possible to determine the sensitivity of the roots of the characteristic equation for small variations of a parameter in a similar manner.
- Study Examples 7.10-7.12, and solve Exercise 7.21 and problems 7.44-7.45 in Nise.