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MATH 594 Theory of Special Functions

Homework 2 (EHT and the Rodrigues formula)

Exercise 1 Verify the formula in (1.22).

Exercise 2 Use *Rodrigues* formula in (1.23) to determine explicitly the polynomials $y_0(z)$, $y_1(z)$, $y_2(z)$ and $y_3(z)$. Write down each EHT having these polynomials as a particular solution.

Exercise 3 Show that the possible forms of $\rho(z)$ in $(1.15)_{\mathrm{a}}$ are

$$\rho(z) = \begin{cases} (b-z)^\alpha (z-a)^\beta \\ (z-a)^\alpha \mathrm{e}^{\beta z} \\ \mathrm{e}^{\alpha z^2 + \beta z} \end{cases}$$

corresponding to the possible degrees of $\sigma(z)$, *i.e.*

$$\sigma(z) = \begin{cases} (b-z)(z-a) \\ (z-a) \\ 1 \end{cases}$$

respectively. Show also that $\sigma(z)$ and $\rho(z)$ can be reduced (up to unimportant constant multipliers) to the *canonical* forms

$$\rho(z) = \begin{cases} (1-z)^\alpha (1+z)^\beta & \text{for} \quad \sigma(z) = 1 - z^2 \\ z^\alpha \mathrm{e}^{-z} & \text{for} \quad \sigma(z) = z \\ \mathrm{e}^{-z^2} & \text{for} \quad \sigma(z) = 1 \end{cases}$$

Find $\tau(z)$ in each case.

Exercise 4 Consider the canonical forms of the EHT in Exercise 3. Then determine the condition leading to polynomial solutions in each case.