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MATH 594 Theory of Special Functions

Homework 1 (Generalized equation of the hypergeometric type)

Exercise 1 Verify the expression (1.11) in Remark 1.1(i).

Exercise 2 Consider the possibility (ii) in Remark 1.1, *i.e.*

$$u'' + \tilde{\tau}(z)u' + \tilde{\sigma}(z)u = 0$$

where $\tilde{\sigma}(z) - \frac{1}{4}\tilde{\tau}^2(z)$ is linear. In this case, choose $\tau(z) = 0$ in (1.5) and show that the generalized EHT in (1.6) takes the simple form

$$y'' + (az + b)y = 0 \qquad a, b \in \mathbb{C}.$$

Then use the substitution $s = az + b$ to obtain a special case of the equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}s^2} + \frac{1-2\alpha}{s}\frac{\mathrm{d}y}{\mathrm{d}s} + \left[(\beta\gamma s^{\gamma-1})^2 + \frac{\alpha^2 - \nu^2\gamma^2}{s^2}\right] y = 0$$

known as Lommel's equation. Check your result whether it is indeed a particular Lommel equation. (**Note:** The Lommel equation can be transformed to the Bessel equation so that its solutions are closely related to the Bessel functions.)

Exercise 3 Following Example 1.1, find out the other forms of the Bessel equation transformed into EHT.

Exercise 4 Verify the differential equation (1.12) and the definitions (1.13) in Theorem 1.1.