# HELP SHEET

**Differentibility**: Let $f(z)=u(x,y)+iv(x,y)$ be defined throughout some neighborhood of a point $z_0=x_0+iy_0$. For f to be differentible at $z_0$, it is necessary that the Cauchy-Riemann equations $u_x=v_y$ and $u_y=-v_x$ be satisfied at $x_0,y_0$.

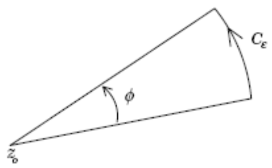
**Analyticity**: Suppose $f(z)$ is differentible at $z_0$ and throughout some neighborhood of $z_0$, then it is analytic at $z_0$. If it is not analytic at $z_0$, it is singular there. If it is analytic at each point of a region D, then it is analytic in D.

**Cauchy's Theorem**: If f(z) is analytic in a simply connected domain D, then $\oint_C f(z)dz=0$ for every piecewise smooth simple closed curve C in D.

**Cauchy Integral Formula**: Let $f(z)$ be analytic in a simply connected domain D, C be a piecewise smooth simple closed curve in D oriented anticlockwise, and $z_0$ be any point within C. Then $\oint_C \frac{f(z)}{z-z_0}dz=2\pi i f(z_0)$ and $\oint_C \frac{f(z)}{(z-z_0)^{n+1}}dz=\frac{2\pi i}{n!}f^{(n)}(z_0)$.

**Laurent Series**: Let D be a closed region between, and including circles with their centers at $z=a$. If $f(z)$ is analytic in D, then it admits the Laurent series representation $f(z)=\sum_{n=-\infty}^{\infty}c_n(z-a)^n$ with $c_n=\frac{1}{2\pi i}\oint_C \frac{f(\zeta)d\zeta}{(\zeta-a)^{n+1}}$ where C is any piecewise smooth simple closed anticlockwise path in D.

**Residue Theorem**: Let C be a piecewise smooth simple closed curve oriented anticlockwise and let $f(z)$ be analytic inside and on C except at finitely many isolated points $z_1,...,z_k$ within C. If $c_{-1}^{(j)}$ denotes the residue of f at $z_j$, then $\oint_C f(z)dz=2\pi i\sum_{j=1}^{k}c_{-1}^{(j)}$ where the reside at an Nth order pole is calculated by $c_{-1}=\frac{1}{(N-1)!}\lim_{z\to a}\left\{\frac{d^{N-1}}{dz^{N-1}}\left[(z-a)^N f(z)\right]\right\}$.




**(Indentation) Theorem**: Suppose $f(z)$ has a simple pole at $z_0$ with residue $\mathrm{Res}(f(z);z_0)=c_{-1}$. Then for the contour $C_\varepsilon$, $\lim_{\varepsilon\to 0}\oint_{C_\varepsilon} f(z)dz=i\phi c_{-1}$ where the integration is carried out in the counterclockwise sense.

**Bilinear Transformation**: is defined by $w=S(z)=\dfrac{az+b}{cz+d}$, $ad-bc\neq 0$. Its inverse is given by $z=S^{-1}(w)=\dfrac{dw-b}{-cw+a}$. If $z_1,z_2,z_3$ are distinct points, then the cross ratio, $\dfrac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)}=\dfrac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$ determines the map where $w_i=S(z_i)$.

**Fundamental Lemma of the Calculus of Variations** states that if $g(x)$ is a continuous function in $[x_0,x_1]$ and if $\int_{x_0}^{x_1}g(x)h(x)\,dx=0$ where $h(x)$ is an arbitrary function with $h(x_0)=h(x_1)=0$, then $g(x)=0$ at every point in the interval.

**The Functional (Frechet) Derivative** of a functional $I[u(x)]$ is defined by $\left.\frac{dI}{d\varepsilon}\right|_{\varepsilon=0}=\lim_{\varepsilon\to 0}\frac{I[u+\varepsilon\eta]-I[u]}{\varepsilon}$ where $\eta(x)$ denote suitable functions with $\eta(x_0)=\eta(x_1)=0$ such that the "variation of u" is given by $\delta u\equiv\varepsilon\eta(x)\equiv\hat{u}(x)-u(x)$.

**A Cauchy-Euler ODE** $a_n x^n y^{(n)}+a_{n-1}x^{n-1}y^{(n-1)}+...+a_1 xy'+a_0 y=0$ admits solutions of the form $y=x^m$, while **A Constant-Coefficient ODE** $a_n y^{(n)}+a_{n-1}y^{(n-1)}+...+a_1 y'+a_0 y=0$ admits solutions of the form $y=e^{mx}$, where m is to be determined and the coefficients $a_n,...,a_1,a_0$ are constants.

**Fredholm Integral Equation** of the 2nd kind $g(s)=f(s)+\lambda\int K(s,t)g(t)dt$ with a degenerate or separable kernel $K(s,t)=\sum_{i=1}^{n}a_i(s)b_i(t)$ has its inverse is defined by $g(s)=f(s)+\lambda\int\Gamma(s,t:\lambda)f(t)dt$ where the function $\Gamma(s,t;\lambda)$ is the resolvent (or reciprocal) kernel that is given by

$$\Gamma(s,t;\lambda)=\begin{vmatrix}0 & a_1(s) & a_2(s) & \cdots & a_n(s)\\ b_1(t) & 1-\lambda a_{11} & -\lambda a_{12} & \cdots & -\lambda a_{1n}\\ b_2(t) & -\lambda a_{21} & 1-\lambda a_{22} & \cdots & -\lambda a_{2n}\\ \vdots & & & & \\ b_n(t) & -\lambda a_{n1} & -\lambda a_{n2} & \cdots & 1-\lambda a_{nn}\end{vmatrix}\div\begin{vmatrix}1-\lambda a_{11} & -\lambda a_{12} & \cdots & -\lambda a_{1n}\\ -\lambda a_{21} & 1-\lambda a_{22} & \cdots & -\lambda a_{2n}\\ \vdots & & & \\ -\lambda a_{n1} & -\lambda a_{n2} & \cdots & 1-\lambda a_{nn}\end{vmatrix}$$

with $a_{ik}=\int b_i(t)a_k(t)dt$.