EXERCISE SET #4

1. Classify the following PDEs defined over $-\infty < x < \infty$, $-\infty < y < \infty$ as elliptic, parabolic or hyperbolic. If the equation is of mixed type, identify the relevant regions and give the classification within each region.

   (a) $u_{xx} + u_{xy} - x^2 u_y = e^{xy}$, (b) $xu_{xx} - u_{xy} + yu_{yy} + 3u_y = 1$, (c) $u_{xy} + u_x - 4u_y = 6u$,

   (d) $xu_{xx} - (\sin^2 y + 1)u_{yy} = x^2 u$, (e) $u_{xx} + u_{xy} + u_{yy} + u_x + u_y + u = 1$, (f) $u_{xx} + (\cos x)u_{yy} = 2xy$,

   (g) $u_{xx} + u_x + u_y = x^3 u$, (h) $u_{xy} - u_{yy} + e^x u = f(x, y)$

2. Classify the following PDEs as linear, non-linear, homogeneous and non-homogeneous.

   (a) $u_{xx} = u_{tt} + u_t + u$, (b) $u_{xx} = 3u_t + u^2 + t^2 x$, (c) $u_{xx} u_{tt} = u_t + u_x$, (d) $u_{xx} + u_{tt} = u_t u_x$,

   (e) $u_{xx} + u_{yy} + u_{zz} = F(x, y, z)u$, (f) $x^2 u_x + xu_{yy} = xy$, (g) $2u_t = xtu_{xx} + e^t u_x + t$,

   (h) $u_{tt} + 2u_{xt} + u_{xx} = u(u_x + u_t)$, (i) $u_{xx} - u_{yy} = 0$, (j) $u_{xx} + u_{yy} + u_x - u_y = 3u$,

   (k) $u_t + uu_x + u_{xxx} = 0$, (l) $u_{xx} + u_{yy} = e^u$, (m) $u_{xxxx} + 2u_{xxyy} + u_{yyyy} = 0$.

3. State whether the following PDEs admit separable solutions $u(x, t) = X(x)T(t)$. If yes, proceed to construct the associated ODEs.

   (a) $u_{xx} = u_t + 3u$, (b) $u_{xx} + 2u_x = u_t$, (c) $u_{xx} + 2u_{xt} = u_t$, (d) $u_{xx} + 2u_{xt} = u_{tt}$

4. Solve the diffusion problem, $\alpha^2 u_{xx} = u_t$, $0 < x < L$, $t > 0$, using separation of variables subject to the following boundary and initial conditions:

   (a) $u(0, t) = 20$, $u_x(\pi, t) = 3$, $u(x, 0) = 0$,

   (b) $u(0, t) = 10$, $u_x(2, t) = -5$, $u(x, 0) = 10$,

   (c) $u(0, t) = 0$, $u_x(2, t) = 0$, $u(x, 0) = 50\sin(\pi x/2)$,

   (d) $u(0, t) = 0$, $u_x(2, t) = 0$, $u(x, 0) = 5\sin(\pi x/4) - 12\sin(5\pi x/4)$,

   (e) $u(0, t) = 25$, $u_x(4, t) = 0$, $u(x, 0) = 25$,

   (f) $u(0, t) = 25$, $u_x(2, t) = 0$, $u(x, 0) = \begin{cases} 0, & 0 < x < 1 \\ 25, & 1 < x < 2 \end{cases}$,

   (g) $u_x(0, t) = 0$, $u_x(\pi, t) = 0$, $u(x, 0) = 300$,

   (h) $u_x(0, t) = 0$, $u_x(3\pi, t) = 0$, $u(x, 0) = \begin{cases} 0, & 0 < x < 2\pi \\ 60, & 2\pi < x < 3\pi \end{cases}$,

   (i) $u_x(0, t) = 5$, $u_x(10, t) = 5$, $u(x, 0) = 45 + 5x$,

   (j) $u(0, t) = 0$, $u(5, t) = 0$, $u(x, 0) = \sin(\pi x) - 37\sin(\pi x/5) + 6\sin(9\pi x/5)$,

   (k) $u(0, t) = 0$, $u(10, t) = 100$, $u(x, 0) = 0$,

   (l) $u_x(0, t) = 2$, $u(6, t) = 12$, $u(x, 0) = 0$,

(m) $u_x(0,t)=0$, $u(6,t)=0$, $u(x,0)=\sin x$,

5. Solve the heat conduction equation $\alpha^2 u_{xx}=u_t$ on a ring, thus, having periodic boundary conditions, $u(0,t)=u(L,t)$ and $u_x(0,t)=u_x(L,t)$ subject to the initial condition $u(x,0)=f(x)$, where $f(x)$ is L-periodic. Hint: Consider periodic Sturm-Liouville problem.

6. Consider the diffusion problem with a constant source F, $\alpha^2 u_{xx}=u_t-F$ subject to boundary $u(0,t)=0$, $u(L,t)=50$ and initial $u(x,0)=f(x)$ conditions.

   (a) Show that the direct separation of variables approach, $u(x,t)=X(x)T(t)$, fails.

   (b) Devise an indirect approach and solve this problem using separation of variables.

7. Solve the Newton cooling problem, $\alpha^2 u_{xx}=u_t+hu$, where h is the convective heat transfer coefficient, subject to boundary $u(0,t)=u(L,t)=50$ and initial $u(x,0)=f(x)$ conditions, by separation of variables technique

   (a) after introducing a steady-state temperature distribution $u_s(x)$ in $u(x,t)=u_s(x)+v(x,t)$.

   (b) directly by applying $u(x,t)=X(x)T(t)$.

   (c) after the change of variables $w(x,t)=e^{ht}u(x,t)$.

8. Solve the heat conduction in an infinite rod: $\alpha^2 u_{xx}=u_t$, $-\infty<x<\infty$, $t>0$ subject to the initial condition $u(x,0)=f(x)$, using

   (a) Fourier transform,

   (b) Laplace transform.

   Hint: Use transform tables and $F^{-1}[e^{-\alpha^2\omega}]=\frac{1}{2\alpha\sqrt{\pi}}e^{-x^2/4\alpha^2}$, if necessary.

9. Solve the problem $\alpha^2 u_{xx}=u_t+Vu_x$, $-\infty<x<\infty$, $t>0$ where V is a constant, subject to the initial condition $u(x,0)=f(x)$, using an appropriate transform technique. Compare your result with (8) for vanishing V. Hint: Use transform tables, if necessary.

10. Solve the problem $\alpha^2 u_{xx}=u_t$, $0<x<\infty$, $t>0$, subject to the initial condition $u(x,0)=f(x)$ and boundary condition $u(0,t)=g(t)$, using an appropriate transform technique. Hint: Use transform tables and $L^{-1}[e^{-\sqrt{s}/\alpha}]=\frac{1}{2\alpha t^{3/2}\sqrt{\pi}}e^{-1/4\alpha^2 t}$, if necessary.

11. Solve the problem $\alpha^2 u_{xx}=u_t$, $0<x<\infty$, $t>0$, subject to the initial condition $u(x,0)=0$, $u_x(0,t)=-Q$, where Q is a prescribed constant, using an appropriate transform technique. Hint: Use transform tables, if necessary.

12. Solve the 2D diffusion problem $\alpha^2(u_{xx}+u_{yy})=u_t$ in a rectangular plate $0<x<a$, $0<y<b$, subject to the boundary $u(0,y)=u(a,y)=u(x,0)=u(x,b)=0$ and initial $u(x,y,0)=100$ conditions, using separation of variables technique.

13. Solve the wave equation, $c^2u_{xx}=u_{tt}$, $0<x<L$, $t>0$, using separation of variables subject to the following boundary and initial conditions:

(a) $u(0,t)=0$, $u(L,t)=0$, $u(x,0)=\begin{cases} 2x/L, & 0<x<L/2 \\ 2(L-x)/L, & L/2<x<L \end{cases}$, $u_t(x,0)=0$,

(b) $u(0,t)=0$, $u(L,t)=0$, $u(x,0)=0$, $u_t(x,0)=50\sin(\pi x/L)$,

(c) $u(0,t)=0$, $u(L,t)=0$, $u(x,0)=0$, $u_t(x,0)=3\sin(\pi x/L)-5\sin(4\pi x/L)$,

(d) $u(0,t)=0$, $u(L,t)=0$, $u(x,0)=0$, $u_t(x,0)=\sin(2\pi x/L)+\sin(3\pi x/L)+4\sin(8\pi x/L)$,

(e) $u(0,t)=0$, $u_x(L,t)=0$, $u(x,0)=f(x)$, $u_t(x,0)=0$,

(e) $u_x(0,t)=0$, $u_x(L,t)=0$, $u(x,0)=0$, $u_t(x,0)=V$, where V is a constant,

(f) $u_x(0,t)=0$, $u(L,t)=0$, $u(x,0)=0$, $u_t(x,0)=g(x)$,

Here $u_x(0,t)=0$ or $u_x(L,t)=0$ correspond to string being looped around a vertical frictionless wire and moving freely.

14. Solve the damped wave equation, $c^2u_{xx}=u_{tt}+au_t$, where the damping force is proportional to the velocity u_t and a is the proportionality constant, subject to the boundary $u(0,t)=0$, $u(L,t)=0$ and initial $u(x,0)=f(x)$, $u_t(x,0)=0$ conditions, using separation of variables.

15. Solve the modified wave equation, $c^2u_{xx}-bu=u_{tt}$, along a string with stiffness b, subject to the boundary $u(0,t)=0$, $u(L,t)=0$ and initial $u(x,0)=f(x)$, $u_t(x,0)=0$ conditions, using separation of variables.

16. Solve the wave equation, $c^2(u_{xx}+u_{yy})=u_{tt}$, governing the vibrating rectangular membrane $0<x<a$, $0<y<b$, subject to the boundary $u(0,y,t)=u(a,y,t)=u(x,0,t)=u(x,b,t)=0$ and initial $u(x,y,0)=f(x,y)$, $u_t(x,y,0)=g(x,y)$ conditions, using separation of variables technique.

17. Solve the wave equation, $c^2(u_{xx}+u_{yy})=u_{tt}$, governing the vibrating rectangular membrane $0<x<\pi$, $0<y<2\pi$, subject to the boundary $u(0,y,t)=u(a,y,t)=u(x,0,t)=u(x,b,t)=0$ and initial $u(x,y,0)=8\sin 2x\sin 2y$, $u_t(x,y,0)=0$ conditions, using separation of variables technique.

18. Consider the infinite string problem, $c^2u_{xx} = u_{tt}$, $-\infty < x < \infty$, $t > 0$, subject to the initial conditions $u(x,0) = f(x)$, $u_t(x,0) = g(x)$. Show that the change of variables, $\xi = x - ct$, $\eta = x + ct$ reduces the PDE to $u_{\xi\eta} = 0$ and leads to the d'Alembert solution

$$u(x,t) = \tfrac{1}{2}\left[f(x-ct) + f(x+ct)\right] + \tfrac{1}{2c}\int_{x-ct}^{x+ct} g(\tau)d\tau .$$

19. Solve Laplace equation, $u_{xx} + u_{yy} = 0$ in the rectangle $0 < x < 3$, $0 < y < 2$ by separation of variables subject to the given boundary conditions:
   (a) $u(0,y) = u(x,2) = u(3,y) = 0$, $u(x,0) = 50\sin(\pi x/3)$,
   (b) $u(0,y) = u(x,0) = u(3,y) = 0$, $u(x,2) = 10\sin(\pi x/3) - 4\sin(\pi x)$,
   (c) $u(x,0) = u(3,y) = u(x,2) = 0$, $u(0,y) = 5\sin(\pi y) + 4\sin(2\pi y) - \sin(3\pi y)$,
   (d) $u(0,y) = u(x,2) = u(3,y) = 0$, $u(x,0) = 50H(x-2)$,
   (e) $u_x(0,y) = u(x,2) = u(3,y) = 0$, $u(x,0) = 50H(x-2)$,
   (f) $u(0,y) = u(x,2) = u_x(3,y) = 0$, $u(x,0) = 50H(x-2)$,
   (g) $u_x(0,y) = u(x,2) = u_x(3,y) = 0$, $u(x,0) = 50H(x-2)$,
   (h) $u_y(x,2) = u(3,y) = u(x,0) = 0$, $u(0,y) = H(y-1)$,
   (i) $u(x,2) = u(3,y) = u(x,0) = 0$, $u_x(0,y) = 5\sin(3\pi y)$,
   (j) $u(x,2) = u(3,y) = u_y(x,0) = 0$, $u_x(0,y) = 20$,
   (k) $u_y(x,2) = u(3,y) = u_y(x,0) = 0$, $u_x(0,y) = 20$,

   Here, $H(x-a) = \begin{cases} 0, & x < a \\ 1, & x > a \end{cases}$ is the Heaviside function.

20. Solve Laplace equation, $u_{xx} + u_{yy} = 0$ in the rectangle $0 < x < a$, $0 < y < b$ by separation of variables subject to the given boundary conditions:
   (a) $u(0,y) = g(y)$, $u(x,b) = u_2$, $u(a,y) = f(y)$, $u(x,0) = u_1$,
   (b) $u(0,y) = u_1$, $u(x,b) = g(x)$, $u(a,y) = u_2$, $u(x,0) = f(x)$,
   (c) $u_x(0,y) = g(y)$, $u(x,b) = u_2$, $u(a,y) = f(y)$, $u(x,0) = u_1$,
   (d) $u_x(0,y) = g(y)$, $u(x,b) = u_2$, $u_x(a,y) = f(y)$, $u(x,0) = u_1$,

   Here, u_1 and u_2 are constants.

21. Solve Laplace equation, $u_{xx} + u_{yy} = 0$ in semi-infinite strip $0 < x < \infty$, $0 < y < 1$ by separation of variables subject to the given boundary conditions and the condition that u is bounded as $x \to \infty$:
   (a) $u(0,y) = 0$, $u(x,0) = 10$, $u_y(x,1) = 0$,
   (b) $u(0,y) = 100$, $u_y(x,0) = u_y(x,1) = 0$,

(c) $u_x(0,y)=5$, $u(x,0)=u(x,1)=0$,

(d) $u(0,y)=0$, $u(x,0)=50$, $u(x,1)=10$,

(e) $u(0,y)=10y$, $u(x,0)=20$, $u(x,1)=50$.

22. Devise a method to solve the Poisson problem, $u_{xx}+u_{yy}=f(x,y)$ in the rectangle $0<x<a$, $0<y<b$ subject to the boundary conditions $u(0,y)=u(x,b)=u(a,y)=u(x,0)=0$ by separation of variables for the case $f(x,y)=f$ is constant.

23. Consider the 3D problem, $u_{xx}+u_{yy}+u_{zz}=0$ in the rectangular prism $0<x<a$, $0<y<b$, $0<z<c$ where $u=0$ on each of the six except for the face $z=c$ on which $u(x,y,c)=f(x,y)$. Use separation of variables to derive the solution in the form

$$u(x,y,z)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_{mn}\sin\tfrac{m\pi x}{a}\sin\tfrac{n\pi y}{b}\sinh(\omega_{mn}z).$$

24. Solve Laplace equation, $u_{xx}+u_{yy}=0$ in the rectangle $0<x<4$, $0<y<3$ by separation of variables subject to the boundary conditions: $u(0,y)=u(x,0)=u(x,3)+5u_y(x,3)=0$, $u(4,y)=100$.

25. Solve the nonhomogeneous diffusion problem, $u_t=\alpha^2u_{xx}+g(x,t)$, $0<x<L$, $t>0$, using separation of variables subject to the following boundary and initial conditions:

(a) $u(0,t)=0$, $u(L,t)=0$, $u(x,0)=10$ with $g(x,t)=e^{-at}$, $a>0$.

(b) $u(0,t)=U_0$, $u_x(L,t)=0$, $u(x,0)=U_0(1-x/L)$ with $g(x,t)=0$ and U_0, a constant.

(c) $u(0,t)=0$, $u(L,t)=100e^{-t}$, $u(x,0)=100$ with $g(x,t)=0$.

(d) $u(0,t)=U_0$, $u(L,t)=U_L$, $u(x,0)=0$ with $g(x,t)=-hu(x,t)$ and U_0,U_L,h, constants. .

(e) $u_x(0,t)=\begin{cases} q, & t<t_0 \\ 0, & t>t_0 \end{cases}$, $u(L,t)=U_0$, $u(x,0)=U_0$ with $g(x,t)=0$ and U_0,q, constants. .

(f) $u_x(0,t)=\begin{cases} q, & t<t_0 \\ 0, & t>t_0 \end{cases}$, $u_x(L,t)=0$, $u(x,0)=U_0$ with $g(x,t)=0$ and U_0,q, constants. .

26. Solve the nonhomogeneous vibration problem, $u_{tt}=c^2u_{xx}+g(x,t)$, $0<x<L$, $t>0$, using separation of variables subject to the following boundary and initial conditions:

(a) $u(0,t)=0$, $u_x(L,t)=F$, $u(x,0)=0$, $u_t(x,0)=0$ with $g(x,t)=0$ and F, constant.

(b) $u_x(0,t)=F$, $u_x(L,t)=0$, $u(x,0)=kx$, $u_t(x,0)=0$ with $g(x,t)=0$ and F, k, constants.

(c) $u(0,t)=0$, $u(L,t)=0$, $u(x,0)=0$, $u_t(x,0)=0$ with $g(x,t)=F_0\sin\omega t$ and F_0,ω constants. Discuss the resonance.

(d) $u(0,t)=0$, $u_x(L,t)=F_0\sin\omega t$, $u(x,0)=0$, $u_t(x,0)=0$ with $g(x,t)=0$ and F_0,ω constants.

(e) $u_x(0,t)=0$, $u_x(L,t)=F_0\sin\omega t$, $u(x,0)=0$, $u_t(x,0)=0$ with $g(x,t)=0$ and F_0,ω constants.

(f) $u(0,t)=0$, $u(L,t)=A_0 \sin \omega t$, $u(x,0)=0$, $u_t(x,0)=0$ with $g(x,t)=0$ and A_0, ω constants.

(g) $u_x(0,t)=0$, $u(L,t)=A_0 \sin \omega t$, $u(x,0)=0$, $u_t(x,0)=0$ with $g(x,t)=0$ and A_0, ω constants.

(h) $u(0,t)=A_0 \sin \omega t$, $u(L,t)=B_0 \sin \phi t$, $u(x,0)=0$, $u_t(x,0)=0$ with $g(x,t)=0$ and A_0, B_0, ω, ϕ constants.

(i) $u_x(0,t)=F_0 \sin \omega t$, $u_x(L,t)=G_0 \sin \phi t$, $u(x,0)=0$, $u_t(x,0)=0$ with $g(x,t)=0$ and F_0, G_0, ω, ϕ constants.

(j) $u(0,t)=0$, $u(L,t)=g(t)$, $u(x,0)=0$, $u_t(x,0)=0$ with $g(x,t)=-\beta u_t(x,t)$ and β constant.

27. Solve the nonhomogeneous problem, $u_{xx}+u_{yy}=g(x,y)$, $0<x<a$, $0<y<b$, using separation of variables subject to the following boundary conditions:

(a) $u(0,y)=u(a,y)=0$, $u(x,0)=u(x,b)=0$ with $g(x,y)=\sigma$ and σ constant.

(b) $u(0,y)=u(a,y)=0$, $u(x,0)=u(x,b)=0$ with $g(x,y)=x$.

(c) $u(0,y)=u(a,y)=0$, $u(x,0)=u(x,b)=0$ with $g(x,y)=xy$.

Recall that the Laplacian operator $\Delta \equiv \nabla^2$ is in the form:

(i) $\Delta u = u_{xx}+u_{yy}+u_{zz}$ in rectangular coordinates (x,y,z),

(ii) $\Delta u = u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}$ in polar coordinates (r,θ),

(iii) $\Delta u = u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}+u_{zz}$ in cylindrical coordinates (r,θ,z),

(iv) $\Delta u = u_{rr}+\frac{2}{r}u_r+\frac{1}{r^2 \sin\phi}\frac{\partial}{\partial\phi}\left(\sin\phi\, u_\phi\right)+\frac{1}{r^2 \sin^2\phi}u_{\theta\theta}$ in spherical coordinates (r,θ,ϕ).

28. Solve the diffusion problem, $u_t=\alpha^2\nabla^2 u$, $0<r<a$, $0<\theta<\theta_0$, $t>0$, in polar coordinates using separation of variables subject to the following boundary and initial conditions:

(a) $u(a,\theta,t)=0$, $u(r,\theta,0)=a^2-r^2$ with $\theta_0=2\pi$.

(b) $u_r(a,\theta,t)=0$, $u(r,\theta,0)=a^2-r^2$ with $\theta_0=2\pi$.

(c) $u(r,0,t)=0$, $u_\theta(r,\pi/2,t)=0$, $u_r(a,\theta,t)=0$, $u(r,\theta,0)=U_0$ with $\theta_0=\pi/2$ and U_0 constant.

(d) $u(r,0,t)=0$, $u(r,\pi,t)=0$, $u_r(a,\theta,t)=0$, $u(r,\theta,0)=U_0$ with $\theta_0=\pi$ and U_0 constant.

29. Solve the diffusion problem, $u_t=\alpha^2\nabla^2 u$, $0<r<a$, $-\pi<\theta\le\pi$, $0<\phi<\pi$, in spherical coordinates using separation of variables subject to the following boundary and initial conditions:

(a) $u(a,\theta,\phi,t)=0$, $u(r,\theta,\phi,0)=U_0$ where U_0 constant.

(b) $u_r(a,\theta,\phi,t)=0$, $u(r,\theta,\phi,0)=U_0$ where U_0 constant.

30. Solve the vibration problem, $u_{tt}=c^2\nabla^2 u$, $0<r<a$, $-\pi<\theta<\pi$, $t>0$, in polar coordinates using separation of variables subject to the following boundary and initial conditions:

(a) $u(a,\theta,t)=0$, $u(r,\theta,0)=a^2-r^2$, $u_t(r,\theta,0)=0$.

(b) $u(a,\theta,t)=0$, $u(r,\theta,0)=0$, $u_t(r,\theta,0)=U_0$ where U_0 constant.

31. (a) Solve the Helmholtz equation $\nabla^2 u + k^2 u = 0$ in polar coordinates $0<r<a$, $-\pi<\theta<\pi$, subject to the boundary condition $u(a,\theta)=1$.

(b) Solve the Laplace equation $\nabla^2 u = 0$ in cylindrical coordinates $0<r<a$, $-\pi<\theta<\pi$, $0<z<L$ subject to the boundary conditions $u_r(a,\theta,z)+u(a,\theta,z)=0$, $u(r,\theta,0)=U_0$. $u(r,\theta,L)=0$ where U_0 constant.

(c) Solve the Laplace equation $\nabla^2 u = 0$ in spherical coordinates $0<r<a$, $-\pi<\theta<\pi$, $0<\phi<\pi$ subject to the boundary conditions at the top half $u(a,\theta,\phi)=U_0$ for $0<\phi<\pi/2$, at the bottom half $u(a,\theta,\phi)=U_1$ for $\pi/2<\phi<\pi$ where U_0, U_1 constants.

32. Consider the wave equation, $c^2 u_{xx} = u_{tt}$, $0<x<L$, $t>0$, subject to the following boundary and initial conditions: $u(0,t)=u(L,t)=0$, $u(x,0)=f(x)\equiv\begin{cases} 2ax/L, & 0<x<L/2 \\ 2a(L-x)/L, & L/2<x<L \end{cases}$, $u_t(x,0)=0$.

(a) Show that separation of variables technique results in the solution

$$u(x,t)=\sum_{k=1}^{\infty} A_k \sin\tfrac{k\pi x}{L}\cos\tfrac{k\pi ct}{L}$$

where $A_k = \frac{8a}{k^2\pi^2}\sin\frac{k\pi}{2}$ such that $u(x,0)=\sum_{k=1}^{\infty} A_k \sin\frac{k\pi x}{L}$.

(b) Verify using trigonometric identities that $u(x,t)$ can also be written as the superposition of the left-going $u^+(x,t)=f(x+ct)$ and right-going $u^-(x,t)=f(x-ct)$, i.e.

$$u(x,t)=\sum_{k=1}^{\infty}\tfrac{1}{2}A_k\left\{\sin\tfrac{k\pi(x-ct)}{L}+\sin\tfrac{k\pi(x+ct)}{L}\right\}=\tfrac{1}{2}\left\{f(x-ct)+f(x+ct)\right\} \quad \text{(see \#18).}$$

(c) Run the following MATLAB script to demonstrate (b) for $c=0.05$, $a=0.5$, $L=1$:

```
% Demonstrating the superposition of right- (um) and
% left-going (up) travelling waves making-up the standing
% wave pattern u as solution to the wave equation
% c^2*u_tt=u_xx,   0<x<1,   t>0,  c=0.05,
% u(0,t)=u(1,t)=0,   t>0
% u(x,0)=f(x),   u_t(x,0)=0,   0<x<1
% where f(x)=x*(0<x<0.5)+(1-x)*(0.5<x<1)

clear all, clf,
N=100; x=linspace(0,1,N); % Actual Domain 0<x<1
M=200; xx=linspace(-.5,1.5,N); % Graph Window -0.5<x<1.5
K=40; c=.05;
set(gca,'NextPlot','replaceChildren');

for t=1:20
xxp=(xx+c*t); xp=(x+c*t);
xxm=(xx-c*t); xm=(x-c*t);
sump=0; summ=0; sum=0;
for k=1:K,
```

```
    coeff=4*sin(k*pi/2)/pi/pi/k/k; % Fourier Coefficients a_k of f(x)
    sump=sump+coeff*sin(k*pi*xxp); % Fourier expansion for up=f(x+ct)
    summ=summ+coeff*sin(k*pi*xxm); % Fourier expansion for um=f(x-ct)
    sum=sum+coeff*(sin(k*pi*xp)+sin(k*pi*xm)); % Fourier expansion for u=0.5*(up+um)
end
u=.5*sum;
plot(xx,sump,'--'), hold on
plot(xx,summ,':'),
plot(x,u,'LineWidth',2),
plot(zeros(1,20),linspace(-1,1,20),'--','LineWidth',2),
plot(ones(1,20),linspace(-1,1,20),'--','LineWidth',2)
legend('f(x+c*t)','f(x-c*t)','u(x,t)')
axis([-.5 1.5 -1 1])
F(t) = getframe; hold off,
end
T=1; v=.5; movie(F,T,v) % Play the movie T times with speed v
```