In your solutions, provide details such as formulation, implementation and results together with relevant figures, tables, computer codes written.

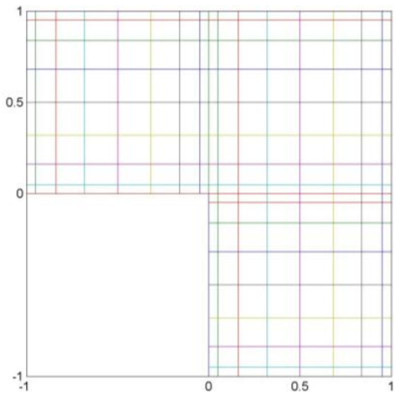
1. Solve Poisson equation on the L-shaped rectangular geometry

$$-\nabla^2 u = 1 \quad \text{in}\,\Omega$$

subject to

$$u = 0 \quad \text{on}\ \partial\Omega$$

using Spectral Element Method with three elements (see **p16_sem.m**). Assess the accuracy of the resulting solution by comparing it with a higher resolution solution



2. Solve Poisson equation on the quarter annular geometry

$$-\nabla^2 u = f \quad \text{in}\,\Omega$$

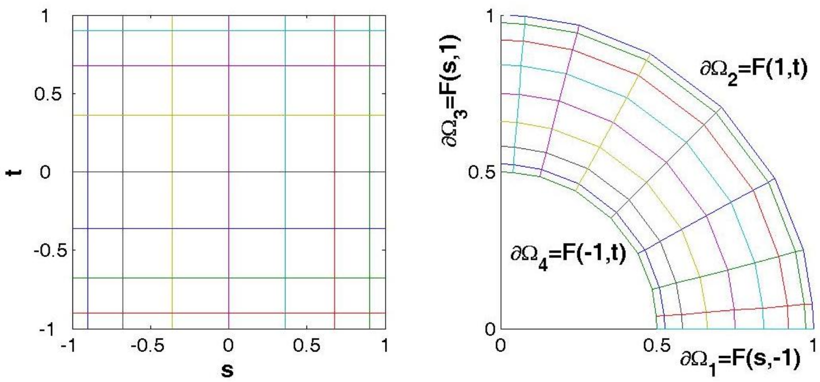
subject to

$$u_y = 0 \quad \text{on}\ \partial\Omega_1$$
$$u = 0 \quad \text{on the rest of}\ \partial\Omega$$

using Spectral Element Method with single element and compare with the exact solution

$$u(x, y) = 10xy^2(1 - x^2 - y^2)(0.25 - x^2 - y^2)\,.$$

(see **p29_sem.m**)



<u>Notes</u>

(1) Determine the forcing function $f(x, y)$ so that given $u(x, y)$ is the exact solution (may use Matlab Symbolic Math Toolbox).

(2) Generate the map $(x, y) = F(s, t)$ by using Gordon & Hall procedure

$$\begin{aligned} F(s,t) = {} & F(s,-1)L_1(t) + F(s,1)L_2(t) \\ & + F(-1,t)L_1(s) + F(1,t)L_2(s) \\ & - F(-1,-1)L_1(s)L_1(t) - F(1,-1)L_2(s)L_1(t) \\ & - F(-1,1)L_1(s)L_2(t) - F(1,1)L_2(s)L_2(t) \end{aligned}$$

where L_1, L_2 are linear cardinal functions. So, modify **map2.m** for testing.

3. Bonus Problem: Solve Poisson equation on the unit circular geometry

$$-\nabla^2 u = r^2 \sin^4(\theta/2) - \sin(6\theta)\cos^2(\theta/2) \quad \text{in} \quad \Omega$$

subject to

$$u = 0 \quad \text{on} \quad \partial\Omega$$

using Spectral Element Method with the five-element configuration given in the figure. Assess the accuracy of the resulting solution by comparing it with **p29**.