Applied Statistics and Probability for Engineers

Third Edition

Douglas C. Montgomery
Arizona State University

George C. Runger
Arizona State University

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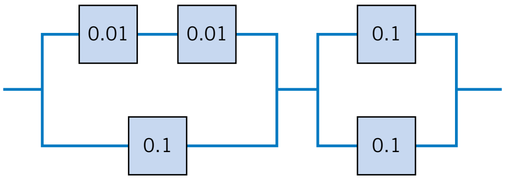
device is as shown. What is the probability that the circuit operates?



2-109. The probability of getting through by telephone to buy concert tickets is 0.92. For the same event, the probability of accessing the vendor's Web site is 0.95. Assume that these two ways to buy tickets are independent. What is the probability that someone who tries to buy tickets through the Internet and by phone will obtain tickets?

2-110. The British government has stepped up its information campaign regarding foot and mouth disease by mailing brochures to farmers around the country. It is estimated that 99% of Scottish farmers who receive the brochure possess enough information to deal with an outbreak of the disease, but only 90% of those without the brochure can deal with an outbreak. After the first three months of mailing, 95% of the farmers in Scotland received the informative brochure. Compute the probability that a randomly selected farmer will have enough information to deal effectively with an outbreak of the disease.

2-111. In an automated filling operation, the probability of an incorrect fill when the process is operated at a low speed is 0.001. When the process is operated at a high speed, the probability of an incorrect fill is 0.01. Assume that 30% of the containers are filled when the process is operated at a high speed and the remainder are filled when the process is operated at a low speed.

(a) What is the probability of an incorrectly filled container?

(b) If an incorrectly filled container is found, what is the probability that it was filled during the high-speed operation?

2-112. An encryption-decryption system consists of three elements: encode, transmit, and decode. A faulty encode occurs in 0.5% of the messages processed, transmission errors occur in 1% of the messages, and a decode error occurs in 0.1% of the messages. Assume the errors are independent.

(a) What is the probability of a completely defect-free message?

(b) What is the probability of a message that has either an encode or a decode error?

2-113. It is known that two defective copies of a commercial software program were erroneously sent to a shipping lot that has now a total of 75 copies of the program. A sample of copies will be selected from the lot without replacement.

(a) If three copies of the software are inspected, determine the probability that exactly one of the defective copies will be found.

(b) If three copies of the software are inspected, determine the probability that both defective copies will be found.

(c) If 73 copies are inspected, determine the probability that both copies will be found. Hint: Work with the copies that remain in the lot.

2-114. A robotic insertion tool contains 10 primary components. The probability that any component fails during the warranty period is 0.01. Assume that the components fail independently and that the tool fails if any component fails. What is the probability that the tool fails during the warranty period?

2-115. An e-mail message can travel through one of two server routes. The probability of transmission error in each of the servers and the proportion of messages that travel each route are shown in the following table. Assume that the servers are independent.

		probability of error			
	percentage of messages	server 1	server 2	server 3	server 4
route 1	30	0.01	0.015		
route 2	70			0.02	0.003

(a) What is the probability that a message will arrive without error?

(b) If a message arrives in error, what is the probability it was sent through route 1?

2-116. A machine tool is idle 15% of the time. You request immediate use of the tool on five different occasions during the year. Assume that your requests represent independent events.

(a) What is the probability that the tool is idle at the time of all of your requests?

(b) What is the probability that the machine is idle at the time of exactly four of your requests?

(c) What is the probability that the tool is idle at the time of at least three of your requests?

2-117. A lot of 50 spacing washers contains 30 washers that are thicker than the target dimension. Suppose that three washers are selected at random, without replacement, from the lot.

(a) What is the probability that all three washers are thicker than the target?

(b) What is the probability that the third washer selected is thicker than the target if the first two washers selected are thinner than the target?

(c) What is the probability that the third washer selected is thicker than the target?

2-118. Continuation of Exercise 2-117. Washers are selected from the lot at random, without replacement.

(a) What is the minimum number of washers that need to be selected so that the probability that all the washers are thinner than the target is less than 0.10?

(b) What is the minimum number of washers that need to be selected so that the probability that one or more washers are thicker than the target is at least 0.90?

2-119. The following table lists the history of 940 orders for features in an entry-level computer product.

		extra memory	
		no	yes
optional high-	no	514	68
speed processor	yes	112	246

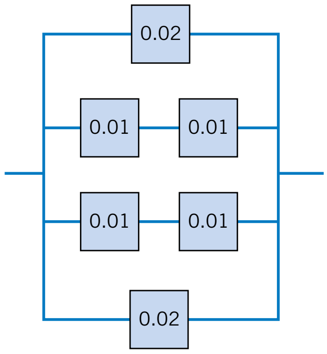
Let A be the event that an order requests the optional high-speed processor, and let B be the event that an order requests extra memory. Determine the following probabilities:

(a) $P(A \cup B)$ (b) $P(A \cap B)$
(c) $P(A' \cup B)$ (d) $P(A' \cap B')$
(e) What is the probability that an order requests an optional high-speed processor given that the order requests extra memory?
(f) What is the probability that an order requests extra memory given that the order requests an optional high-speed processor?

2-120. The alignment between the magnetic tape and head in a magnetic tape storage system affects the performance of the system. Suppose that 10% of the read operations are degraded by skewed alignments, 5% of the read operations are degraded by off-center alignments, and the remaining read operations are properly aligned. The probability of a read error is 0.01 from a skewed alignment, 0.02 from an off-center alignment, and 0.001 from a proper alignment.

(a) What is the probability of a read error?
(b) If a read error occurs, what is the probability that it is due to a skewed alignment?

2-121. The following circuit operates if and only if there is a path of functional devices from left to right. Assume that devices fail independently and that the probability of *failure* of each device is as shown. What is the probability that the circuit does not operate?



2-122. A company that tracks the use of its web site determined that the more pages a visitor views, the more likely the visitor is to provide contact information. Use the following tables to answer the questions:

Number of pages viewed:	1	2	3	4 or more
Percentage of visitors:	40	30	20	10
Percentage of visitors in each page-view catgory that provide contact information:	10	10	20	40

(a) What is the probability that a visitor to the web site provides contact information?
(b) If a visitor provides contact information, what is the probability that the visitor viewed four or more pages?

EXAMPLE 3-14 As in Example 3-1, let the random variable X denote the number of the 48 voice lines that are in use at a particular time. Assume that X is a discrete uniform random variable with a range of 0 to 48. Then,

$$E(X) = (48 + 0)/2 = 24$$

and

$$\sigma = \{[(48 - 0 + 1)^2 - 1]/12\}^{1/2} = 14.14$$

Equation 3-6 is more useful than it might first appear. If all the values in the range of a random variable X are multiplied by a constant (without changing any probabilities), the mean and standard deviation of X are multiplied by the constant. You are asked to verify this result in an exercise. Because the variance of a random variable is the square of the standard deviation, the variance of X is multiplied by the constant squared. More general results of this type are discussed in Chapter 5.

EXAMPLE 3-15 Let the random variable Y denote the proportion of the 48 voice lines that are in use at a particular time, and X denotes the number of lines that are in use at a particular time. Then, $Y = X/48$. Therefore,

$$E(Y) = E(X)/48 = 0.5$$

and

$$V(Y) = V(X)/48^2 = 0.087$$

EXERCISES FOR SECTION 3-5

3-46. Let the random variable X have a discrete uniform distribution on the integers $0 \le x \le 100$. Determine the mean and variance of X.

3-47. Let the random variable X have a discrete uniform distribution on the integers $1 \le x \le 3$. Determine the mean and variance of X.

3-48. Let the random variable X be equally likely to assume any of the values $1/8$, $1/4$, or $3/8$. Determine the mean and variance of X.

3-49. Thickness measurements of a coating process are made to the nearest hundredth of a millimeter. The thickness measurements are uniformly distributed with values 0.15, 0.16, 0.17, 0.18, and 0.19. Determine the mean and variance of the coating thickness for this process.

3-50. Product codes of 2, 3, or 4 letters are equally likely. What is the mean and standard deviation of the number of letters in 100 codes?

3-51. The lengths of plate glass parts are measured to the nearest tenth of a millimeter. The lengths are uniformly distributed, with values at every tenth of a millimeter starting at 590.0 and continuing through 590.9. Determine the mean and variance of lengths.

3-52. Suppose that X has a discrete uniform distribution on the integers 0 through 9. Determine the mean, variance, and standard deviation of the random variable $Y = 5X$ and compare to the corresponding results for X.

3-53. Show that for a discrete uniform random variable X, if each of the values in the range of X is multiplied by the constant c, the effect is to multiply the mean of X by c and the variance of X by c^2. That is, show that $E(cX) = cE(X)$ and $V(cX) = c^2V(X)$.

3-54. The probability of an operator entering alphanumeric data incorrectly into a field in a database is equally likely. The random variable X is the number of fields on a data entry form with an error. The data entry form has 28 fields. Is X a discrete uniform random variable? Why or why not.

(c) Four identical electronic components are wired to a controller that can switch from a failed component to one of the remaining spares. Let X denote the number of components that have failed after a specified period of operation.
(d) Let X denote the number of accidents that occur along the federal highways in Arizona during a one-month period.
(e) Let X denote the number of correct answers by a student taking a multiple choice exam in which a student can eliminate some of the choices as being incorrect in some questions and all of the incorrect choices in other questions.
(f) Defects occur randomly over the surface of a semiconductor chip. However, only 80% of defects can be found by testing. A sample of 40 chips with one defect each is tested. Let X denote the number of chips in which the test finds a defect.
(g) Reconsider the situation in part (f). Now, suppose the sample of 40 chips consists of chips with 1 and with 0 defects.
(h) A filling operation attempts to fill detergent packages to the advertised weight. Let X denote the number of detergent packages that are underfilled.
(i) Errors in a digital communication channel occur in bursts that affect several consecutive bits. Let X denote the number of bits in error in a transmission of 100,000 bits.
(j) Let X denote the number of surface flaws in a large coil of galvanized steel.

3-56. The random variable X has a binomial distribution with $n = 10$ and $p = 0.5$. Sketch the probability mass function of X.
(a) What value of X is most likely?
(b) What value(s) of X is(are) least likely?

3-57. The random variable X has a binomial distribution with $n = 10$ and $p = 0.5$. Determine the following probabilities:
(a) $P(X = 5)$ (b) $P(X \leq 2)$
(c) $P(X \geq 9)$ (d) $P(3 \leq X < 5)$

3-58. Sketch the probability mass function of a binomial distribution with $n = 10$ and $p = 0.01$ and comment on the shape of the distribution.
(a) What value of X is most likely?
(b) What value of X is least likely?

3-59. The random variable X has a binomial distribution with $n = 10$ and $p = 0.01$. Determine the following probabilities.
(a) $P(X = 5)$ (b) $P(X \leq 2)$
(c) $P(X \geq 9)$ (d) $P(3 \leq X < 5)$

3-60. Determine the cumulative distribution function of a binomial random variable with $n = 3$ and $p = 1/2$.

3-61. Determine the cumulative distribution function of a binomial random variable with $n = 3$ and $p = 1/4$.

3-62. An electronic product contains 40 integrated circuits. The probability that any integrated circuit is defective is 0.01, and the integrated circuits are independent. The product operates only if there are no defective integrated circuits. What is the probability that the product operates?

3-63. Let X denote the number of bits received in error in a digital communication channel, and assume that X is a binomial random variable with $p = 0.001$. If 1000 bits are transmitted, determine the following:
(a) $P(X = 1)$ (b) $P(X \geq 1)$
(c) $P(X \leq 2)$ (d) mean and variance of X

3-64. The phone lines to an airline reservation system are occupied 40% of the time. Assume that the events that the lines are occupied on successive calls are independent. Assume that 10 calls are placed to the airline.
(a) What is the probability that for exactly three calls the lines are occupied?
(b) What is the probability that for at least one call the lines are not occupied?
(c) What is the expected number of calls in which the lines are all occupied?

3-65. Batches that consist of 50 coil springs from a production process are checked for conformance to customer requirements. The mean number of nonconforming coil springs in a batch is 5. Assume that the number of nonconforming springs in a batch, denoted as X, is a binomial random variable.
(a) What are n and p?
(b) What is $P(X \leq 2)$?
(c) What is $P(X \geq 49)$?

3-66. **A statistical process control chart example.** Samples of 20 parts from a metal punching process are selected every hour. Typically, 1% of the parts require rework. Let X denote the number of parts in the sample of 20 that require rework. A process problem is suspected if X exceeds its mean by more than three standard deviations.
(a) If the percentage of parts that require rework remains at 1%, what is the probability that X exceeds its mean by more than three standard deviations?
(b) If the rework percentage increases to 4%, what is the probability that X exceeds 1?
(c) If the rework percentage increases to 4%, what is the probability that X exceeds 1 in at least one of the next five hours of samples?

3-67. Because not all airline passengers show up for their reserved seat, an airline sells 125 tickets for a flight that holds only 120 passengers. The probability that a passenger does not show up is 0.10, and the passengers behave independently.
(a) What is the probability that every passenger who shows up can take the flight?
(b) What is the probability that the flight departs with empty seats?

3-68. This exercise illustrates that poor quality can affect schedules and costs. A manufacturing process has 100 customer orders to fill. Each order requires one component part that is purchased from a supplier. However, typically, 2% of the components are identified as defective, and the components can be assumed to be independent.
(a) If the manufacturer stocks 100 components, what is the probability that the 100 orders can be filled without reordering components?

3-73. The probability of a successful optical alignment in the assembly of an optical data storage product is 0.8. Assume the trials are independent.
(a) What is the probability that the first successful alignment requires exactly four trials?
(b) What is the probability that the first successful alignment requires at most four trials?
(c) What is the probability that the first successful alignment requires at least four trials?

3-74. In a clinical study, volunteers are tested for a gene that has been found to increase the risk for a disease. The probability that a person carries the gene is 0.1.
(a) What is the probability 4 or more people will have to be tested before 2 with the gene are detected?
(b) How many people are expected to be tested before 2 with the gene are detected?

3-75. Assume that each of your calls to a popular radio station has a probability of 0.02 of connecting, that is, of not obtaining a busy signal. Assume that your calls are independent.
(a) What is the probability that your first call that connects is your tenth call?
(b) What is the probability that it requires more than five calls for you to connect?
(c) What is the mean number of calls needed to connect?

3-76. In Exercise 3-70, recall that a particularly long traffic light on your morning commute is green 20% of the time that you approach it. Assume that each morning represents an independent trial.
(a) What is the probability that the first morning that the light is green is the fourth morning that you approach it?
(b) What is the probability that the light is not green for 10 consecutive mornings?

3-77. A trading company has eight computers that it uses to trade on the New York Stock Exchange (NYSE). The probability of a computer failing in a day is 0.005, and the computers fail independently. Computers are repaired in the evening and each day is an independent trial.
(a) What is the probability that all eight computers fail in a day?
(b) What is the mean number of days until a specific computer fails?
(c) What is the mean number of days until all eight computers fail in the same day?

3-78. In Exercise 3-66, recall that 20 parts are checked each hour and that X denotes the number of parts in the sample of 20 that require rework.
(a) If the percentage of parts that require rework remains at 1%, what is the probability that hour 10 is the first sample at which X exceeds 1?
(b) If the rework percentage increases to 4%, what is the probability that hour 10 is the first sample at which X exceeds 1?
(c) If the rework percentage increases to 4%, what is the expected number of hours until X exceeds 1?

3-79. Consider a sequence of independent Bernoulli trials with $p = 0.2$.
(a) What is the expected number of trials to obtain the first success?
(b) After the eighth success occurs, what is the expected number of trials to obtain the ninth success?

3-80. Show that the probability density function of a negative binomial random variable equals the probability density function of a geometric random variable when $r = 1$. Show that the formulas for the mean and variance of a negative binomial random variable equal the corresponding results for geometric random variable when $r = 1$.

3-81. Suppose that X is a negative binomial random variable with $p = 0.2$ and $r = 4$. Determine the following:
(a) $E(X)$ (b) $P(X = 20)$
(c) $P(X = 19)$ (d) $P(X = 21)$
(e) The most likely value for X

3-82. The probability is 0.6 that a calibration of a transducer in an electronic instrument conforms to specifications for the measurement system. Assume the calibration attempts are independent. What is the probability that at most three calibration attempts are required to meet the specifications for the measurement system?

3-83. An electronic scale in an automated filling operation stops the manufacturing line after three underweight packages are detected. Suppose that the probability of an underweight package is 0.001 and each fill is independent.
(a) What is the mean number of fills before the line is stopped?
(b) What is the standard deviation of the number of fills before the line is stopped?

3-84. A fault-tolerant system that processes transactions for a financial services firm uses three separate computers. If the operating computer fails, one of the two spares can be immediately switched online. After the second computer fails, the last computer can be immediately switched online. Assume that the probability of a failure during any transaction is 10^{-8} and that the transactions can be considered to be independent events.
(a) What is the mean number of transactions before all computers have failed?
(b) What is the variance of the number of transactions before all computers have failed?

3-85. Derive the expressions for the mean and variance of a geometric random variable with parameter p. (Formulas for infinite series are required.)

sharpness. If any dull blade is found, the assembly is replaced with a newly sharpened set of blades.
(a) If 10 of the blades in an assembly are dull, what is the probability that the assembly is replaced the first day it is evaluated?
(b) If 10 of the blades in an assembly are dull, what is the probability that the assembly is not replaced until the third day of evaluation? [*Hint:* Assume the daily decisions are independent, and use the geometric distribution.]
(c) Suppose on the first day of evaluation, two of the blades are dull, on the second day of evaluation six are dull, and on the third day of evaluation, ten are dull. What is the probability that the assembly is not replaced until the third day of evaluation? [*Hint:* Assume the daily decisions are independent. However, the probability of replacement changes every day.]

3-94. A state runs a lottery in which 6 numbers are randomly selected from 40, without replacement. A player chooses 6 numbers before the state's sample is selected.
(a) What is the probability that the 6 numbers chosen by a player match all 6 numbers in the state's sample?
(b) What is the probability that 5 of the 6 numbers chosen by a player appear in the state's sample?
(c) What is the probability that 4 of the 6 numbers chosen by a player appear in the state's sample?
(d) If a player enters one lottery each week, what is the expected number of weeks until a player matches all 6 numbers in the state's sample?

3-95. Continuation of Exercises 3-86 and 3-87.
(a) Calculate the finite population corrections for Exercises 3-86 and 3-87. For which exercise should the binomial approximation to the distribution of X be better?
(b) For Exercise 3-86, calculate $P(X = 1)$ and $P(X = 4)$ assuming that X has a binomial distribution and compare these results to results derived from the hypergeometric distribution.
(c) For Exercise 3-87, calculate $P(X = 1)$ and $P(X = 4)$ assuming that X has a binomial distribution and compare these results to the results derived from the hypergeometric distribution.

3-96. Use the binomial approximation to the hypergeometric distribution to approximate the probabilities in Exercise 3-92. What is the finite population correction in this exercise?

3-9 POISSON DISTRIBUTION

We introduce the Poisson distribution with an example.

EXAMPLE 3-30

Consider the transmission of n bits over a digital communication channel. Let the random variable X equal the number of bits in error. When the probability that a bit is in error is constant and the transmissions are independent, X has a binomial distribution. Let p denote the probability that a bit is in error. Let $\lambda = pn$. Then, $E(x) = pn = \lambda$ and

$$P(X = x) = \binom{n}{x} P^x(1 - p)^{n-x} = \binom{n}{x}\left(\frac{\lambda}{n}\right)^x\left(1 - \frac{\lambda}{n}\right)^{n-x}$$

Now, suppose that the number of bits transmitted increases and the probability of an error decreases exactly enough that pn remains equal to a constant. That is, n increases and p decreases accordingly, such that $E(X) = \lambda$ remains constant. Then, with some work, it can be shown that

$$\lim_{n\to\infty} P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}, \qquad x = 0, 1, 2, \ldots$$

Also, because the number of bits transmitted tends to infinity, the number of errors can equal any nonnegative integer. Therefore, the range of X is the integers from zero to infinity.

The distribution obtained as the limit in the above example is more useful than the derivation above implies. The following example illustrates the broader applicability.

Because this sum is tedious to compute, many computer programs calculate cumulative Poisson probabilities. From one such program, $P(X \leq 12) = 0.791$.

The derivation of the mean and variance of a Poisson random variable is left as an exercise. The results are as follows.

If X is a Poisson random variable with parameter λ, then

$$\mu = E(X) = \lambda \quad \text{and} \quad \sigma^2 = V(X) = \lambda \tag{3-16}$$

The mean and variance of a Poisson random variable are equal. For example, if particle counts follow a Poisson distribution with a mean of 25 particles per square centimeter, the variance is also 25 and the standard deviation of the counts is 5 per square centimeter. Consequently, information on the variability is very easily obtained. Conversely, if the variance of count data is much greater than the mean of the same data, the Poisson distribution is not a good model for the distribution of the random variable.

EXERCISES FOR SECTION 3-9

3-97. Suppose X has a Poisson distribution with a mean of 4. Determine the following probabilities:
(a) $P(X = 0)$ (b) $P(X \leq 2)$
(c) $P(X = 4)$ (d) $P(X = 8)$

3-98. Suppose X has a Poisson distribution with a mean of 0.4. Determine the following probabilities:
(a) $P(X = 0)$ (b) $P(X \leq 2)$
(c) $P(X = 4)$ (d) $P(X = 8)$

3-99. Suppose that the number of customers that enter a bank in an hour is a Poisson random variable, and suppose that $P(X = 0) = 0.05$. Determine the mean and variance of X.

3-100. The number of telephone calls that arrive at a phone exchange is often modeled as a Poisson random variable. Assume that on the average there are 10 calls per hour.
(a) What is the probability that there are exactly 5 calls in one hour?
(b) What is the probability that there are 3 or less calls in one hour?
(c) What is the probability that there are exactly 15 calls in two hours?
(d) What is the probability that there are exactly 5 calls in 30 minutes?

3-101. The number of flaws in bolts of cloth in textile manufacturing is assumed to be Poisson distributed with a mean of 0.1 flaw per square meter.
(a) What is the probability that there are two flaws in 1 square meter of cloth?
(b) What is the probability that there is one flaw in 10 square meters of cloth?
(c) What is the probability that there are no flaws in 20 square meters of cloth?
(d) What is the probability that there are at least two flaws in 10 square meters of cloth?

3-102. When a computer disk manufacturer tests a disk, it writes to the disk and then tests it using a certifier. The certifier counts the number of missing pulses or errors. The number of errors on a test area on a disk has a Poisson distribution with $\lambda = 0.2$.
(a) What is the expected number of errors per test area?
(b) What percentage of test areas have two or fewer errors?

3-103. The number of cracks in a section of interstate highway that are significant enough to require repair is assumed to follow a Poisson distribution with a mean of two cracks per mile.
(a) What is the probability that there are no cracks that require repair in 5 miles of highway?
(b) What is the probability that at least one crack requires repair in 1/2 mile of highway?
(c) If the number of cracks is related to the vehicle load on the highway and some sections of the highway have a heavy load of vehicles whereas other sections carry a light load, how do you feel about the assumption of a Poisson distribution for the number of cracks that require repair?

3-104. The number of failures for a cytogenics machine from contamination in biological samples is a Poisson random variable with a mean of 0.01 per 100 samples.
(a) If the lab usually processes 500 samples per day, what is the expected number of failures per day?

Mean and Variance of the Normal Distribution (CD Only)

EXERCISES FOR SECTION 4-6

4-39. Use Appendix Table II to determine the following probabilities for the standard normal random variable Z:
(a) $P(Z < 1.32)$ (b) $P(Z < 3.0)$
(c) $P(Z > 1.45)$ (d) $P(Z > -2.15)$
(e) $P(-2.34 < Z < 1.76)$

4-40. Use Appendix Table II to determine the following probabilities for the standard normal random variable Z:
(a) $P(-1 < Z < 1)$ (b) $P(-2 < Z < 2)$
(c) $P(-3 < Z < 3)$ (d) $P(Z > 3)$
(e) $P(0 < Z < 1)$

4-41. Assume Z has a standard normal distribution. Use Appendix Table II to determine the value for z that solves each of the following:
(a) $P(Z < z) = 0.9$ (b) $P(Z < z) = 0.5$
(c) $P(Z > z) = 0.1$ (d) $P(Z > z) = 0.9$
(e) $P(-1.24 < Z < z) = 0.8$

4-42. Assume Z has a standard normal distribution. Use Appendix Table II to determine the value for z that solves each of the following:
(a) $P(-z < Z < z) = 0.95$ (b) $P(-z < Z < z) = 0.99$
(c) $P(-z < Z < z) = 0.68$ (d) $P(-z < Z < z) = 0.9973$

4-43. Assume X is normally distributed with a mean of 10 and a standard deviation of 2. Determine the following:
(a) $P(X < 13)$ (b) $P(X > 9)$
(c) $P(6 < X < 14)$ (d) $P(2 < X < 4)$
(e) $P(-2 < X < 8)$

4-44. Assume X is normally distributed with a mean of 10 and a standard deviation of 2. Determine the value for x that solves each of the following:
(a) $P(X > x) = 0.5$
(b) $P(X > x) = 0.95$
(c) $P(x < X < 10) = 0.2$
(d) $P(-x < X - 10 < x) = 0.95$
(e) $P(-x < X - 10 < x) = 0.99$

4-45. Assume X is normally distributed with a mean of 5 and a standard deviation of 4. Determine the following:
(a) $P(X < 11)$ (b) $P(X > 0)$
(c) $P(3 < X < 7)$ (d) $P(-2 < X < 9)$
(e) $P(2 < X < 8)$

4-46. Assume X is normally distributed with a mean of 5 and a standard deviation of 4. Determine the value for x that solves each of the following:
(a) $P(X > x) = 0.5$ (b) $P(X > x) = 0.95$
(c) $P(x < X < 9) = 0.2$ (d) $P(3 < X < x) = 0.95$
(e) $P(-x < X < x) = 0.99$

4-47. The compressive strength of samples of cement can be modeled by a normal distribution with a mean of 6000 kilograms per square centimeter and a standard deviation of 100 kilograms per square centimeter.
(a) What is the probability that a sample's strength is less than 6250 Kg/cm^2?
(b) What is the probability that a sample's strength is between 5800 and 5900 Kg/cm^2?
(c) What strength is exceeded by 95% of the samples?

4-48. The tensile strength of paper is modeled by a normal distribution with a mean of 35 pounds per square inch and a standard deviation of 2 pounds per square inch.
(a) What is the probability that the strength of a sample is less than 40 lb/in^2?
(b) If the specifications require the tensile strength to exceed 30 lb/in^2, what proportion of the samples is scrapped?

4-49. The line width of for semiconductor manufacturing is assumed to be normally distributed with a mean of 0.5 micrometer and a standard deviation of 0.05 micrometer.
(a) What is the probability that a line width is greater than 0.62 micrometer?
(b) What is the probability that a line width is between 0.47 and 0.63 micrometer?
(c) The line width of 90% of samples is below what value?

4-50. The fill volume of an automated filling machine used for filling cans of carbonated beverage is normally distributed with a mean of 12.4 fluid ounces and a standard deviation of 0.1 fluid ounce.
(a) What is the probability a fill volume is less than 12 fluid ounces?
(b) If all cans less than 12.1 or greater than 12.6 ounces are scrapped, what proportion of cans is scrapped?
(c) Determine specifications that are symmetric about the mean that include 99% of all cans.

4-51. The time it takes a cell to divide (called mitosis) is normally distributed with an average time of one hour and a standard deviation of 5 minutes.
(a) What is the probability that a cell divides in less than 45 minutes?
(b) What is the probability that it takes a cell more than 65 minutes to divide?
(c) What is the time that it takes approximately 99% of all cells to complete mitosis?

4-52. In the previous exercise, suppose that the mean of the filling operation can be adjusted easily, but the standard deviation remains at 0.1 ounce.
(a) At what value should the mean be set so that 99.9% of all cans exceed 12 ounces?
(b) At what value should the mean be set so that 99.9% of all cans exceed 12 ounces if the standard deviation can be reduced to 0.05 fluid ounce?

4-63. The manufacturing of semiconductor chips produces 2% defective chips. Assume the chips are independent and that a lot contains 1000 chips.
(a) Approximate the probability that more than 25 chips are defective.
(b) Approximate the probability that between 20 and 30 chips are defective.

4-64. A supplier ships a lot of 1000 electrical connectors. A sample of 25 is selected at random, without replacement. Assume the lot contains 100 defective connectors.
(a) Using a binomial approximation, what is the probability that there are no defective connectors in the sample?
(b) Use the normal approximation to answer the result in part (a). Is the approximation satisfactory?
(c) Redo parts (a) and (b) assuming the lot size is 500. Is the normal approximation to the probability that there are no defective connectors in the sample satisfactory in this case?

4-65. An electronic office product contains 5000 electronic components. Assume that the probability that each component operates without failure during the useful life of the product is 0.999, and assume that the components fail independently. Approximate the probability that 10 or more of the original 5000 components fail during the useful life of the product.

4-66. Suppose that the number of asbestos particles in a sample of 1 squared centimeter of dust is a Poisson random variable with a mean of 1000. What is the probability that 10 squared centimeters of dust contains more than 10,000 particles?

4-67. A corporate Web site contains errors on 50 of 1000 pages. If 100 pages are sampled randomly, without replacement, approximate the probability that at least 1 of the pages in error are in the sample.

4-68. Hits to a high-volume Web site are assumed to follow a Poisson distribution with a mean of 10,000 per day. Approximate each of the following:
(a) The probability of more than 20,000 hits in a day
(b) The probability of less than 9900 hits in a day
(c) The value such that the probability that the number of hits in a day exceed the value is 0.01

4-69. Continuation of Exercise 4-68.
(a) Approximate the expected number of days in a year (365 days) that exceed 10,200 hits.
(b) Approximate the probability that over a year (365 days) more than 15 days each have more than 10,200 hits.

4-70. The percentage of people exposed to a bacteria who become ill is 20%. Assume that people are independent. Assume that 1000 people are exposed to the bacteria. Approximate each of the following:
(a) The probability that more than 225 become ill
(b) The probability that between 175 and 225 become ill
(c) The value such that the probability that the number of people that become ill exceeds the value is 0.01

4-71. A high-volume printer produces minor print-quality errors on a test pattern of 1000 pages of text according to a Poisson distribution with a mean of 0.4 per page.
(a) Why are the number of errors on each page independent random variables?
(b) What is the mean number of pages with errors (one or more)?
(c) Approximate the probability that more than 350 pages contain errors (one or more).

4-8 CONTINUITY CORRECTION TO IMPROVE THE APPROXIMATION (CD ONLY)

4-9 EXPONENTIAL DISTRIBUTION

The discussion of the Poisson distribution defined a random variable to be the number of flaws along a length of copper wire. The distance between flaws is another random variable that is often of interest. Let the random variable X denote the length from any starting point on the wire until a flaw is detected.

As you might expect, the distribution of X can be obtained from knowledge of the distribution of the number of flaws. The key to the relationship is the following concept. The distance to the first flaw exceeds 3 millimeters if and only if there are no flaws within a length of 3 millimeters—simple, but sufficient for an analysis of the distribution of X.

In general, let the random variable N denote the number of flaws in x millimeters of wire. If the mean number of flaws is λ per millimeter, N has a Poisson distribution with mean λx. We assume that the wire is longer than the value of x. Now,

$$P(X > x) = P(N = 0) = \frac{e^{-\lambda x}(\lambda x)^0}{0!} = e^{-\lambda x}$$

(b) Determine x such that the probability that you wait less than x minutes is 0.90.
(c) Determine x such that the probability that you wait less than x minutes is 0.50.

4-83. The distance between major cracks in a highway follows an exponential distribution with a mean of 5 miles.
(a) What is the probability that there are no major cracks in a 10-mile stretch of the highway?
(b) What is the probability that there are two major cracks in a 10-mile stretch of the highway?
(c) What is the standard deviation of the distance between major cracks?

4-84. Continuation of Exercise 4-83.
(a) What is the probability that the first major crack occurs between 12 and 15 miles of the start of inspection?
(b) What is the probability that there are no major cracks in two separate 5-mile stretches of the highway?
(c) Given that there are no cracks in the first 5 miles inspected, what is the probability that there are no major cracks in the next 10 miles inspected?

4-85. The lifetime of a mechanical assembly in a vibration test is exponentially distributed with a mean of 400 hours.
(a) What is the probability that an assembly on test fails in less than 100 hours?
(b) What is the probability that an assembly operates for more than 500 hours before failure?
(c) If an assembly has been on test for 400 hours without a failure, what is the probability of a failure in the next 100 hours?

4-86. Continuation of Exercise 4-85.
(a) If 10 assemblies are tested, what is the probability that at least one fails in less than 100 hours? Assume that the assemblies fail independently.
(b) If 10 assemblies are tested, what is the probability that all have failed by 800 hours? Assume the assemblies fail independently.

4-87. When a bus service reduces fares, a particular trip from New York City to Albany, New York, is very popular. A small bus can carry four passengers. The time between calls for tickets is exponentially distributed with a mean of 30 minutes. Assume that each call orders one ticket. What is the probability that the bus is filled in less than 3 hours from the time of the fare reduction?

4-88. The time between arrivals of small aircraft at a county airport is exponentially distributed with a mean of one hour. What is the probability that more than three aircraft arrive within an hour?

4-89. Continuation of Exercise 4-88.
(a) If 30 separate one-hour intervals are chosen, what is the probability that no interval contains more than three arrivals?
(b) Determine the length of an interval of time (in hours) such that the probability that no arrivals occur during the interval is 0.10.

4-90. The time between calls to a corporate office is exponentially distributed with a mean of 10 minutes.
(a) What is the probability that there are more than three calls in one-half hour?
(b) What is the probability that there are no calls within one-half hour?
(c) Determine x such that the probability that there are no calls within x hours is 0.01.

4-91. Continuation of Exercise 4-90.
(a) What is the probability that there are no calls within a two-hour interval?
(b) If four nonoverlapping one-half hour intervals are selected, what is the probability that none of these intervals contains any call?
(c) Explain the relationship between the results in part (a) and (b).

4-92. If the random variable X has an exponential distribution with mean θ, determine the following:
(a) $P(X > \theta)$ (b) $P(X > 2\theta)$
(c) $P(X > 3\theta)$
(d) How do the results depend on θ?

4-93. Assume that the flaws along a magnetic tape follow a Poisson distribution with a mean of 0.2 flaw per meter. Let X denote the distance between two successive flaws.
(a) What is the mean of X?
(b) What is the probability that there are no flaws in 10 consecutive meters of tape?
(c) Does your answer to part (b) change if the 10 meters are not consecutive?
(d) How many meters of tape need to be inspected so that the probability that at least one flaw is found is 90%?

4-94. Continuation of Exercise 4-93. (More difficult questions.)
(a) What is the probability that the first time the distance between two flaws exceeds 8 meters is at the fifth flaw?
(b) What is the mean number of flaws before a distance between two flaws exceeds 8 meters?

4-95. Derive the formula for the mean and variance of an exponential random variable.

4-10 ERLANG AND GAMMA DISTRIBUTIONS

4-10.1 Erlang Distribution

An exponential random variable describes the length until the first count is obtained in a Poisson process. A generalization of the exponential distribution is the length until r counts

4-145. A square inch of carpeting contains 50 carpet fibers. The probability of a damaged fiber is 0.0001. Assume the damaged fibers occur independently.

(a) Approximate the probability of one or more damaged fibers in 1 square yard of carpeting.

(b) Approximate the probability of four or more damaged fibers in 1 square yard of carpeting.

4-146. An airline makes 200 reservations for a flight that holds 185 passengers. The probability that a passenger arrives for the flight is 0.9 and the passengers are assumed to be independent.

(a) Approximate the probability that all the passengers that arrive can be seated.

(b) Approximate the probability that there are empty seats.

(c) Approximate the number of reservations that the airline should make so that the probability that everyone who arrives can be seated is 0.95. [*Hint*: Successively try values for the number of reservations.]

MIND-EXPANDING EXERCISES

4-147. The steps in this exercise lead to the probability density function of an Erlang random variable X with parameters λ and r, $f(x) = \lambda^r x^{r-1} e^{-\lambda x}/(r-1)!$, $x > 0$, $r = 1, 2, \ldots$.

(a) Use the Poisson distribution to express $P(X > x)$.

(b) Use the result from part (a) to determine the cumulative distribution function of X.

(c) Differentiate the cumulative distribution function in part (b) and simplify to obtain the probability density function of X.

4-148. A bearing assembly contains 10 bearings. The bearing diameters are assumed to be independent and normally distributed with a mean of 1.5 millimeters and a standard deviation of 0.025 millimeter. What is the probability that the maximum diameter bearing in the assembly exceeds 1.6 millimeters?

4-149. Let the random variable X denote a measurement from a manufactured product. Suppose the target value for the measurement is m. For example, X could denote a dimensional length, and the target might be 10 millimeters. The **quality loss** of the process producing the product is defined to be the expected value of $\$k(X - m)^2$, where k is a constant that relates a deviation from target to a loss measured in dollars.

(a) Suppose X is a continuous random variable with $E(X) = m$ and $V(X) = \sigma^2$. What is the quality loss of the process?

(b) Suppose X is a continuous random variable with $E(X) = \mu$ and $V(X) = \sigma^2$. What is the quality loss of the process?

4-150. The lifetime of an electronic amplifier is modeled as an exponential random variable. If 10% of the amplifiers have a mean of 20,000 hours and the remaining amplifiers have a mean of 50,000 hours, what proportion of the amplifiers fail before 60,000 hours?

4-151. Lack of Memory Property. Show that for an exponential random variable X, $P(X < t_1 + t_2 \mid X > t_1) = P(X < t_2)$

4-152. A process is said to be of **six-sigma quality** if the process mean is at least six standard deviations from the nearest specification. Assume a normally distributed measurement.

(a) If a process mean is centered between the upper and lower specifications at a distance of six standard deviations from each, what is the probability that a product does not meet specifications? Using the result that 0.000001 equals one part per million, express the answer in parts per million.

(b) Because it is difficult to maintain a process mean centered between the specifications, the probability of a product not meeting specifications is often calculated after assuming the process shifts. If the process mean positioned as in part (a) shifts upward by 1.5 standard deviations, what is the probability that a product does not meet specifications? Express the answer in parts per million.

(c) Rework part (a). Assume that the process mean is at a distance of three standard deviations.

(d) Rework part (b). Assume that the process mean is at a distance of three standard deviations and then shifts upward by 1.5 standard deviations.

(e) Compare the results in parts (b) and (d) and comment.

and the binomial distributions for X and Y can be used to determine these probabilities as $P(X \le 1) = 0.9639$ and $P(Y \le 1) = 0.9831$. Therefore, $P(X \le 1, Y \le 1) = 0.948$.

Consequently, the probability that the shipment is accepted for use in manufacturing is 0.948 even if 1% of the parts do not conform to specifications. If the supplier and the purchaser change their policies so that the shipment is acceptable only if zero nonconforming parts are found in the sample, the probability that the shipment is accepted for production is still quite high. That is,

$$P(X = 0, Y = 0) = P(X = 0)P(Y = 0) = 0.605$$

This example shows that inspection is not an effective means of achieving quality.

EXERCISES FOR SECTION 5-1

5-1. Show that the following function satisfies the properties of a joint probability mass function.

x	y	$f_{XY}(x, y)$
1	1	1/4
1.5	2	1/8
1.5	3	1/4
2.5	4	1/4
3	5	1/8

5-2. Continuation of Exercise 5-1. Determine the following probabilities:
(a) $P(X < 2.5, Y < 3)$ (b) $P(X < 2.5)$
(c) $P(Y < 3)$ (d) $P(X > 1.8, Y > 4.7)$

5-3. Continuation of Exercise 5-1. Determine $E(X)$ and $E(Y)$.

5-4. Continuation of Exercise 5-1. Determine
(a) The marginal probability distribution of the random variable X.
(b) The conditional probability distribution of Y given that $X = 1.5$.
(c) The conditional probability distribution of X given that $Y = 2$.
(d) $E(Y|X = 1.5)$
(e) Are X and Y independent?

5-5. Determine the value of c that makes the function $f(x, y) = c(x + y)$ a joint probability mass function over the nine points with $x = 1, 2, 3$ and $y = 1, 2, 3$.

5-6. Continuation of Exercise 5-5. Determine the following probabilities:
(a) $P(X = 1, Y < 4)$ (b) $P(X = 1)$
(c) $P(Y = 2)$ (d) $P(X < 2, Y < 2)$

5-7. Continuation of Exercise 5-5. Determine $E(X)$, $E(Y)$, $V(X)$, and $V(Y)$.

5-8. Continuation of Exercise 5-5. Determine
(a) The marginal probability distribution of the random variable X.
(b) The conditional probability distribution of Y given that $X = 1$.
(c) The conditional probability distribution of X given that $Y = 2$.
(d) $E(Y|X = 1)$
(e) Are X and Y independent?

5-9. Show that the following function satisfies the properties of a joint probability mass function.

x	y	$f_{XY}(x, y)$
-1	-2	1/8
-0.5	-1	1/4
0.5	1	1/2
1	2	1/8

5-10. Continuation of Exercise 5-9. Determine the following probabilities:
(a) $P(X < 0.5, Y < 1.5)$ (b) $P(X < 0.5)$
(c) $P(Y < 1.5)$ (d) $P(X > 0.25, Y < 4.5)$

5-11. Continuation of Exercise 5-9. Determine $E(X)$ and $E(Y)$.

5-12. Continuation of Exercise 5-9. Determine
(a) The marginal probability distribution of the random variable X.
(b) The conditional probability distribution of Y given that $X = 1$.
(c) The conditional probability distribution of X given that $Y = 1$.
(d) $E(X|y = 1)$
(e) Are X and Y independent?

5-13. Four electronic printers are selected from a large lot of damaged printers. Each printer is inspected and classified as containing either a major or a minor defect. Let the random variables X and Y denote the number of printers with major and minor defects, respectively. Determine the range of the joint probability distribution of X and Y.

5-36. Continuation of Exercise 5-34. Determine the following:
(a) Marginal probability distribution of the random variable X
(b) Conditional probability distribution of Y given that $X = 1.5$
(c) $E(Y|X) = 1.5)$
(d) $P(Y < 2|X = 1.5)$
(e) Conditional probability distribution of X given that $Y = 2$

5-37. Determine the value of c that makes the function $f(x, y) = c(x + y)$ a joint probability density function over the range $0 < x < 3$ and $x < y < x + 2$.

5-38. Continuation of Exercise 5-37. Determine the following:
(a) $P(X < 1, Y < 2)$ (b) $P(1 < X < 2)$
(c) $P(Y > 1)$ (d) $P(X < 2, Y < 2)$
(e) $E(X)$

5-39. Continuation of Exercise 5-37. Determine the following:
(a) Marginal probability distribution of X
(b) Conditional probability distribution of Y given that $X = 1$
(c) $E(Y|X = 1)$
(d) $P(Y > 2|X = 1)$
(e) Conditional probability distribution of X given that $Y = 2$

5-40. Determine the value of c that makes the function $f(x, y) = cxy$ a joint probability density function over the range $0 < x < 3$ and $0 < y < x$.

5-41. Continuation of Exercise 5-40. Determine the following:
(a) $P(X < 1, Y < 2)$ (b) $P(1 < X < 2)$
(c) $P(Y > 1)$ (d) $P(X < 2, Y < 2)$
(e) $E(X)$ (f) $E(Y)$

5-42. Continuation of Exercise 5-40. Determine the following:
(a) Marginal probability distribution of X
(b) Conditional probability distribution of Y given $X = 1$
(c) $E(Y|X = 1)$
(d) $P(Y > 2|X = 1)$
(e) Conditional probability distribution of X given $Y = 2$

5-43. Determine the value of c that makes the function $f(x, y) = ce^{-2x-3y}$ a joint probability density function over the range $0 < x$ and $0 < y < x$.

5-44. Continuation of Exercise 5-43. Determine the following:
(a) $P(X < 1, Y < 2)$ (b) $P(1 < X < 2)$
(c) $P(Y > 3)$ (d) $P(X < 2, Y < 2)$
(e) $E(X)$ (f) $E(Y)$

5-45. Continuation of Exercise 5-43. Determine the following:
(a) Marginal probability distribution of X
(b) Conditional probability distribution of Y given $X = 1$
(c) $E(Y|X = 1)$
(d) Conditional probability distribution of X given $Y = 2$

5-46. Determine the value of c that makes the function $f(x, y) = ce^{-2x-3y}$ a joint probability density function over the range $0 < x$ and $x < y$.

5-47. Continuation of Exercise 5-46. Determine the following:
(a) $P(X < 1, Y < 2)$ (b) $P(1 < X < 2)$
(c) $P(Y > 3)$ (d) $P(X < 2, Y < 2)$
(e) $E(X)$ (f) $E(Y)$

5-48. Continuation of Exercise 5-46. Determine the following:
(a) Marginal probability distribution of X
(b) Conditional probability distribution of Y given $X = 1$
(c) $E(Y|X = 1)$
(d) $P(Y < 2|X = 1)$
(e) Conditional probability distribution of X given $Y = 2$

5-49. Two methods of measuring surface smoothness are used to evaluate a paper product. The measurements are recorded as deviations from the nominal surface smoothness in coded units. The joint probability distribution of the two measurements is a uniform distribution over the region $0 < x < 4$, $0 < y$, and $x - 1 < y < x + 1$. That is, $f_{XY}(x, y) = c$ for x and y in the region. Determine the value for c such that $f_{XY}(x, y)$ is a joint probability density function.

5-50. Continuation of Exercise 5-49. Determine the following:
(a) $P(X < 0.5, Y < 0.5)$ (b) $P(X < 0.5)$
(c) $E(X)$ (d) $E(Y)$

5-51. Continuation of Exercise 5-49. Determine the following:
(a) Marginal probability distribution of X
(b) Conditional probability distribution of Y given $X = 1$
(c) $E(Y|X = 1)$
(d) $P(Y < 0.5|X = 1)$

5-52. The time between surface finish problems in a galvanizing process is exponentially distributed with a mean of 40 hours. A single plant operates three galvanizing lines that are assumed to operate independently.
(a) What is the probability that none of the lines experiences a surface finish problem in 40 hours of operation?
(b) What is the probability that all three lines experience a surface finish problem between 20 and 40 hours of operation?
(c) Why is the joint probability density function not needed to answer the previous questions?

5-53. A popular clothing manufacturer receives Internet orders via two different routing systems. The time between orders for each routing system in a typical day is known to be exponentially distributed with a mean of 3.2 minutes. Both systems operate independently.
(a) What is the probability that no orders will be received in a 5 minute period? In a 10 minute period?
(b) What is the probability that both systems receive two orders between 10 and 15 minutes after the site is officially open for business?

Thus,

$$E(XY) - E(X)E(Y) = 32/9 - (4/3)(8/3) = 0$$

It can be shown that these two random variables are independent. You can check that $f_{XY}(x, y) = f_X(x)f_Y(y)$ for all x and y.

However, if the correlation between two random variables is zero, we *cannot* immediately conclude that the random variables are independent. Figure 5-13(d) provides an example.

EXERCISES FOR SECTION 5-5

5-67. Determine the covariance and correlation for the following joint probability distribution:

x	1	1	2	4
y	3	4	5	6
$f_{XY}(x, y)$	1/8	1/4	1/2	1/8

5-68. Determine the covariance and correlation for the following joint probability distribution:

x	-1	-0.5	0.5	1
y	-2	-1	1	2
$f_{XY}(x, y)$	1/8	1/4	1/2	1/8

5-69. Determine the value for c and the covariance and correlation for the joint probability mass function $f_{XY}(x, y) = c(x + y)$ for $x = 1, 2, 3$ and $y = 1, 2, 3$.

5-70. Determine the covariance and correlation for the joint probability distribution shown in Fig. 5-4(a) and described in Example 5-8.

5-71. Determine the covariance and correlation for X_1 and X_2 in the joint distribution of the multinomial random variables X_1, X_2 and X_3 in with $p_1 = p_2 = p_3 = 1/3$ and $n = 3$. What can you conclude about the sign of the correlation between two random variables in a multinomial distribution?

5-72. Determine the value for c and the covariance and correlation for the joint probability density function $f_{XY}(x, y) = cxy$ over the range $0 < x < 3$ and $0 < y < x$.

5-73. Determine the value for c and the covariance and correlation for the joint probability density function $f_{XY}(x, y) = c$ over the range $0 < x < 5$, $0 < y$, and $x - 1 < y < x + 1$.

5-74. Determine the covariance and correlation for the joint probability density function $f_{XY}(x, y) = 6 \times 10^{-6}e^{-0.001x-0.002y}$ over the range $0 < x$ and $x < y$ from Example 5-15.

5-75. Determine the covariance and correlation for the joint probability density function $f_{XY}(x, y) = e^{-x-y}$ over the range $0 < x$ and $0 < y$.

5-76. Suppose that the correlation between X and Y is ρ. For constants a, b, c, and d, what is the correlation between the random variables $U = aX + b$ and $V = cY + d$?

5-77. The joint probability distribution is

x	-1	0	0	1
y	0	-1	1	0
$f_{XY}(x, y)$	1/4	1/4	1/4	1/4

Show that the correlation between X and Y is zero, but X and Y are not independent.

5-78. Suppose X and Y are independent continuous random variables. Show that $\sigma_{XY} = 0$.

5-6 BIVARIATE NORMAL DISTRIBUTION

An extension of a normal distribution to two random variables is an important bivariate probability distribution.

EXAMPLE 5-32

At the start of this chapter, the length of different dimensions of an injection-molded part was presented as an example of two random variables. Each length might be modeled by a normal distribution. However, because the measurements are from the same part, the random variables are typically not independent. A probability distribution for two normal random variables that are not independent is important in many applications. As stated at the start of the

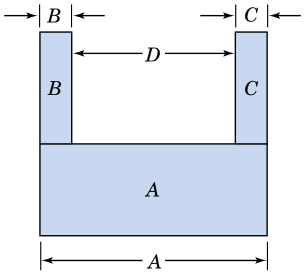


Figure 5-20 Figure for the U-shaped component.

(b) What is the probability that the width of the casing minus the width of the door exceeds 1/4 inch?
(c) What is the probability that the door does not fit in the casing?

5-92. A U-shaped component is to be formed from the three parts A, B, and C. The picture is shown in Fig. 5-20. The length of A is normally distributed with a mean of 10 millimeters and a standard deviation of 0.1 millimeter. The thickness of parts B and C is normally distributed with a mean of 2 millimeters and a standard deviation of 0.05 millimeter. Assume all dimensions are independent.
(a) Determine the mean and standard deviation of the length of the gap D.
(b) What is the probability that the gap D is less than 5.9 millimeters?

5-93. Soft-drink cans are filled by an automated filling machine and the standard deviation is 0.5 fluid ounce. Assume that the fill volumes of the cans are independent, normal random variables.
(a) What is the standard deviation of the average fill volume of 100 cans?
(b) If the mean fill volume is 12.1 ounces, what is the probability that the average fill volume of the 100 cans is below 12 fluid ounces?
(c) What should the mean fill volume equal so that the probability that the average of 100 cans is below 12 fluid ounces is 0.005?
(d) If the mean fill volume is 12.1 fluid ounces, what should the standard deviation of fill volume equal so that the probability that the average of 100 cans is below 12 fluid ounces is 0.005?
(e) Determine the number of cans that need to be measured such that the probability that the average fill volume is less than 12 fluid ounces is 0.01.

5-94. The photoresist thickness in semiconductor manufacturing has a mean of 10 micrometers and a standard deviation of 1 micrometer. Assume that the thickness is normally distributed and that the thicknesses of different wafers are independent.
(a) Determine the probability that the average thickness of 10 wafers is either greater than 11 or less than 9 micrometers.
(b) Determine the number of wafers that needs to be measured such that the probability that the average thickness exceeds 11 micrometers is 0.01.
(c) If the mean thickness is 10 micrometers, what should the standard deviation of thickness equal so that the probability that the average of 10 wafers is either greater than 11 or less than 9 micrometers is 0.001?

5-95. Assume that the weights of individuals are independent and normally distributed with a mean of 160 pounds and a standard deviation of 30 pounds. Suppose that 25 people squeeze into an elevator that is designed to hold 4300 pounds.
(a) What is the probability that the load (total weight) exceeds the design limit?
(b) What design limit is exceeded by 25 occupants with probability 0.0001?

5-8 FUNCTIONS OF RANDOM VARIABLES (CD ONLY)

5-9 MOMENT GENERATING FUNCTION (CD ONLY)

5-10 CHEBYSHEV'S INEQUALITY (CD ONLY)

Supplemental Exercises

5-96. Show that the following function satisfies the properties of a joint probability mass function:

x	y	$f(x, y)$
0	0	1/4
0	1	1/8
1	0	1/8
1	1	1/4
2	2	1/4

5-97. Continuation of Exercise 5-96. Determine the following probabilities:
(a) $P(X < 0.5, Y < 1.5)$ (b) $P(X \le 1)$
(c) $P(X < 1.5)$ (d) $P(X > 0.5, Y < 1.5)$
(e) Determine $E(X)$, $E(Y)$, $V(X)$, and $V(Y)$.

5-98. Continuation of Exercise 5-96. Determine the following:
(a) Marginal probability distribution of the random variable X
(b) Conditional probability distribution of Y given that $X = 1$
(c) $E(Y|X = 1)$
(d) Are X and Y independent? Why or why not?
(e) Calculate the correlation between X and Y.

5-99. The percentage of people given an antirheumatoid medication who suffer severe, moderate, or minor side effects chi-squared distribution with one degree of freedom. Let $Z = (X - \mu)/\sigma$, and $Y = Z^2$. The probability distribution of Z is the standard normal; that is,

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}, \qquad -\infty < z < \infty$$

The inverse solutions of $y = z^2$ are $z = \pm\sqrt{y}$, so the transformation is not one to one. Define $z_1 = -\sqrt{y}$ and $z_2 = +\sqrt{y}$ so that $J_1 = -(1/2)/\sqrt{y}$ and $J_2 = (1/2)/\sqrt{y}$. Then by Equation S5-6, the probability distribution of Y is

$$\begin{aligned} f_Y(y) &= \frac{1}{\sqrt{2\pi}} e^{-y/2} \left| \frac{-1}{2\sqrt{y}} \right| + \frac{1}{\sqrt{2\pi}} e^{-y/2} \left| \frac{1}{2\sqrt{y}} \right| \\ &= \frac{1}{2^{1/2}\sqrt{\pi}} y^{1/2-1} e^{-y/2}, \qquad y > 0 \end{aligned}$$

Now it can be shown that $\sqrt{\pi} = \Gamma(1/2)$, so we may write $f(y)$ as

$$f_Y(y) = \frac{1}{2^{1/2}\,\Gamma\left(\frac{1}{2}\right)} y^{1/2-1} e^{-y/2}, \qquad y > 0$$

which is the chi-squared distribution with 1 degree of freedom.

EXERCISES FOR SECTION 5-8

S5-1. Suppose that X is a random variable with probability distribution

$$f_X(x) = 1/4, \qquad x = 1, 2, 3, 4$$

Find the probability distribution of the random $Y = 2X + 1$.

S5-2. Let X be a binomial random variable with $p = 0.25$ and $n = 3$. Find the probability distribution of the random variable $Y = X^2$.

S5-3. Suppose that X is a continuous random variable with probability distribution

$$f_X(x) = \frac{x}{18}, \qquad 0 \le x \le 6$$

(a) Find the probability distribution of the random variable $Y = 2X + 10$.
(b) Find the expected value of Y.

S5-4. Suppose that X has a uniform probability distribution

$$f_X(x) = 1, \qquad 0 \le x \le 1$$

Show that the probability distribution of the random variable $Y = -2 \ln X$ is chi-squared with two degrees of freedom.

S5-5. A current of I amperes flows through a resistance of R ohms according to the probability distribution

$$f_I(i) = 2i, \qquad 0 \le i \le 1$$

Suppose that the resistance is also a random variable with probability distribution

$$f_R(r) = 1, \qquad 0 \le r \le 1$$

Assume that I and R are independent.
(a) Find the probability distribution for the power (in watts) $P = I^2R$.
(b) Find $E(P)$.

S5-6. A random variable X has the following probability distribution:

$$f_X(x) = e^{-x}, \qquad x \ge 0$$

(a) Find the probability distribution for $Y = X^2$.
(b) Find the probability distribution for $Y = X^{1/2}$.
(c) Find the probability distribution for $Y = \ln X$.

S5-7. The velocity of a particle in a gas is a random variable V with probability distribution

$$f_V(v) = av^2 e^{-bv} \qquad v > 0$$

where b is a constant that depends on the temperature of the gas and the mass of the particle.
(a) Find the value of the constant a.
(b) The kinetic energy of the particle is $W = mV^2/2$. Find the probability distribution of W.

S5-8. Suppose that X has the probability distribution

$$f_X(x) = 1, \qquad 1 \le x \le 2$$

Find the probability distribution of the random variable $Y = e^X$.

S5-9. Prove that Equation S5-3 holds when $y = h(x)$ is a decreasing function of x.

S5-10. The random variable X has the probability distribution

$$f_X(x) = \frac{x}{8}, \qquad 0 \le x \le 4$$

Find the probability distribution of $Y = (X - 2)^2$.

S5-11. Consider a rectangle with sides of length S_1 and S_2, where S_1 and S_2 are independent random variables. The probability distributions of S_1 and S_2 are

$$f_{S_1}(s_1) = 2s_1, \qquad 0 \le s_1 \le 1$$

and

$$f_{S_2}(s_2) = \frac{s_2}{8}, \qquad 0 \le s_2 \le 4$$

(a) Find the joint distribution of the area of the rectangle $A = S_1 S_2$ and the random variable $Y = S_1$.
(b) Find the probability distribution of the area A of the rectangle.

S5-12. Suppose we have a simple electrical circuit in which Ohm's law $V = IR$ holds. We are interested in the probability distribution of the resistance R given that V and I are independent random variables with the following distributions:

$$f_V(v) = e^{-v}, \qquad v \ge 0$$

and

$$f_I(i) = 1, \qquad 1 \le i \le 2$$

Find the probability distribution of R.

5-9 MOMENT GENERATING FUNCTIONS (CD ONLY)

Suppose that X is a random variable with mean μ. Throughout this book we have used the idea of the expected value of the random variable X, and in fact $E(X) = \mu$. Now suppose that we are interested in the expected value of a particular function of X, say, $g(X) = X^r$. The expected value of this function, or $E[g(X)] = E(X^r)$, is called the rth moment about the origin of the random variable X, which we will denote by μ'_r.

Definition

The rth **moment about the origin** of the random variable X is

$$\mu'_r = E(X^r) = \begin{cases} \displaystyle\sum_x x^r f(x), & X \text{ discrete} \\ \displaystyle\int_{-\infty}^{\infty} x^r f(x)\,dx, & X \text{ continuous} \end{cases} \qquad \text{(S5-7)}$$

Notice that the first moment about the origin is just the mean, that is, $E(X) = \mu'_1 = \mu$. Furthermore, since the second moment about the origin is $E(X^2) = \mu'_2$, we can write the variance of a random variable in terms of origin moments as follows:

$$\sigma^2 = E(X^2) - [E(X)]^2 = \mu'_2 - \mu^2$$

(a) Show that the moment generating function is

$$M_X(t) = \frac{pe^t}{1 - (1 - p)e^t}$$

(b) Use $M_X(t)$ to find the mean and variance of X.

S5-16. The chi-squared random variable with k degrees of freedom has moment generating function $M_X(t) = (1 - 2t)^{-k/2}$. Suppose that X_1 and X_2 are independent chi-squared random variables with k_1 and k_2 degrees of freedom, respectively. What is the distribution of $Y = X_1 + X_2$?

S5-17. A continuous random variable X has the following probability distribution:

$$f(x) = 4xe^{-2x}, \qquad x > 0$$

(a) Find the moment generating function for X.
(b) Find the mean and variance of X.

S5-18. The continuous uniform random variable X has density function

$$f(x) = \frac{1}{\beta - \alpha}, \qquad \alpha \le x \le \beta$$

(a) Show that the moment generating function is

$$M_X(t) = \frac{e^{t\beta} - e^{t\alpha}}{t(\beta - \alpha)}$$

(b) Use $M_X(t)$ to find the mean and variance of X.

S5-19. A random variable X has the exponential distribution

$$f(x) = \lambda e^{-\lambda x}, \qquad x > 0$$

(a) Show that the moment generating function of X is

$$M_X(t) = \left(1 - \frac{t}{\lambda}\right)^{-1}$$

(b) Find the mean and variance of X.

S5-20. A random variable X has the gamma distribution

$$f(x) = \frac{\lambda}{\Gamma(r)} (\lambda x)^{r-1} e^{-\lambda x}, \qquad x > 0$$

(a) Show that the moment generating function of X is

$$M_X(t) = \left(1 - \frac{t}{\lambda}\right)^{-r}$$

(b) Find the mean and variance of X.

S5-21. Let $X_1, X_2, \ldots, X_r$ be independent exponential random variables with parameter λ.
(a) Find the moment generating function of $Y = X_1 + X_2 + \cdots + X_r$.
(b) What is the distribution of the random variable Y?
[*Hint:* Use the results of Exercise S5-20].

S5-22. Suppose that X_i has a normal distribution with mean μ_i and variance $\sigma_i^2, i = 1, 2$. Let X_1 and X_2 be independent.
(a) Find the moment generating function of $Y = X_1 + X_2$.
(b) What is the distribution of the random variable Y?

S5-23. Show that the moment generating function of the chi-squared random variable with k degrees of freedom is $M_X(t) = (1 - 2t)^{-k/2}$. Show that the mean and variance of this random variable are k and $2k$, respectively.

S5-24. Continuation of Exercise S5-20.
(a) Show that by expanding e^{tX} in a power series and taking expectations term by term we may write the moment generating function as

$$\begin{aligned} M_X(t) &= E(e^{tX}) \\ &= 1 + \mu'_1 t + \mu'_2 \frac{t^2}{2!} + \cdots \\ &\quad + \mu'_r \frac{t^r}{r!} + \cdots \end{aligned}$$

Thus, the coefficient of $t^r/r!$ in this expansion is μ'_r, the rth origin moment.
(b) Continuation of Exercise S5-20. Write the power series expansion for $M_X(t)$, the gamma random variable.
(c) Continuation of Exercise S5-20. Find μ'_1 and μ'_2 using the results of parts (a) and (b). Does this approach give the same answers that you found for the mean and variance of the gamma random variable in Exercise S5-20?

5-10 CHEBYSHEV'S INEQUALITY (CD ONLY)

In Chapter 3 we showed that if X is a normal random variable with mean μ and standard deviation σ, $P(\mu - 1.96\sigma < X < \mu + 1.96\sigma) = 0.95$. This result relates the probability of a normal random variable to the magnitude of the standard deviation. An interesting, similar result that applies to any discrete or continuous random variable was developed by the mathematician Chebyshev in 1867.

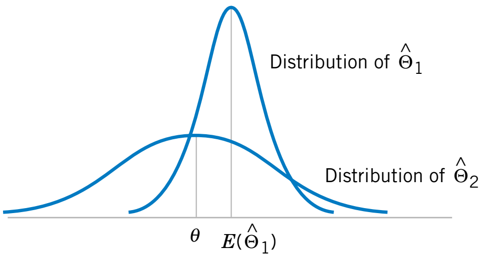


Figure 7-2 A biased estimator $\hat{\Theta}_1$ that has smaller variance than the unbiased estimator $\hat{\Theta}_2$.

That is, the mean square error of $\hat{\Theta}$ is equal to the variance of the estimator plus the squared bias. If $\hat{\Theta}$ is an unbiased estimator of θ, the mean square error of $\hat{\Theta}$ is equal to the variance of $\hat{\Theta}$.

The mean square error is an important criterion for comparing two estimators. Let $\hat{\Theta}_1$ and $\hat{\Theta}_2$ be two estimators of the parameter θ, and let MSE $(\hat{\Theta}_1)$ and MSE $(\hat{\Theta}_2)$ be the mean square errors of $\hat{\Theta}_1$ and $\hat{\Theta}_2$. Then the **relative efficiency** of $\hat{\Theta}_2$ to $\hat{\Theta}_1$ is defined as

$$\frac{\text{MSE}(\hat{\Theta}_1)}{\text{MSE}(\hat{\Theta}_2)} \tag{7-4}$$

If this relative efficiency is less than 1, we would conclude that $\hat{\Theta}_1$ is a more efficient estimator of θ than $\hat{\Theta}_2$, in the sense that it has a smaller mean square error.

Sometimes we find that biased estimators are preferable to unbiased estimators because they have smaller mean square error. That is, we may be able to reduce the variance of the estimator considerably by introducing a relatively small amount of bias. As long as the reduction in variance is greater than the squared bias, an improved estimator from a mean square error viewpoint will result. For example, Fig. 7-2 shows the probability distribution of a biased estimator $\hat{\Theta}_1$ that has a smaller variance than the unbiased estimator $\hat{\Theta}_2$. An estimate based on $\hat{\Theta}_1$ would more likely be close to the true value of θ than would an estimate based on $\hat{\Theta}_2$. Linear regression analysis (Chapters 11 and 12) is an area in which biased estimators are occasionally used.

An estimator $\hat{\Theta}$ that has a mean square error that is less than or equal to the mean square error of any other estimator, for all values of the parameter θ, is called an **optimal** estimator of θ. Optimal estimators rarely exist.

EXERCISES FOR SECTION 7-2

7-1. Suppose we have a random sample of size $2n$ from a population denoted by X, and $E(X) = \mu$ and $V(X) = \sigma^2$. Let

$$\overline{X}_1 = \frac{1}{2n}\sum_{i=1}^{2n} X_i \quad \text{and} \quad \overline{X}_2 = \frac{1}{n}\sum_{i=1}^{n} X_i$$

be two estimators of μ. Which is the better estimator of μ? Explain your choice.

7-2. Let $X_1, X_2, \ldots, X_7$ denote a random sample from a population having mean μ and variance σ^2. Consider the following estimators of μ:

$$\hat{\Theta}_1 = \frac{X_1 + X_2 + \cdots + X_7}{7}$$

$$\hat{\Theta}_2 = \frac{2X_1 - X_6 + X_4}{2}$$

(a) Is either estimator unbiased?
(b) Which estimator is best? In what sense is it best?

7-3. Suppose that $\hat{\Theta}_1$ and $\hat{\Theta}_2$ are unbiased estimators of the parameter θ. We know that $V(\hat{\Theta}_1) = 10$ and $V(\hat{\Theta}_2) = 4$. Which estimator is best and in what sense is it best?

7-4. Calculate the relative efficiency of the two estimators in Exercise 7-2.

7-5. Calculate the relative efficiency of the two estimators in Exercise 7-3.

7-6. Suppose that $\hat{\Theta}_1$ and $\hat{\Theta}_2$ are estimators of the parameter θ. We know that $E(\hat{\Theta}_1) = \theta$, $E(\hat{\Theta}_2) = \theta/2$, $V(\hat{\Theta}_1) = 10$, $V(\hat{\Theta}_2) = 4$. Which estimator is best? In what sense is it best?

7-7. Suppose that $\hat{\Theta}_1$, $\hat{\Theta}_2$, and $\hat{\Theta}_3$ are estimators of θ. We know that $E(\hat{\Theta}_1) = E(\hat{\Theta}_2) = \theta$, $E(\hat{\Theta}_3) \neq \theta$, $V(\hat{\Theta}_1) = 12$, $V(\hat{\Theta}_2) = 10$, and $E(\hat{\Theta}_3 - \theta)^2 = 6$. Compare these three estimators. Which do you prefer? Why?

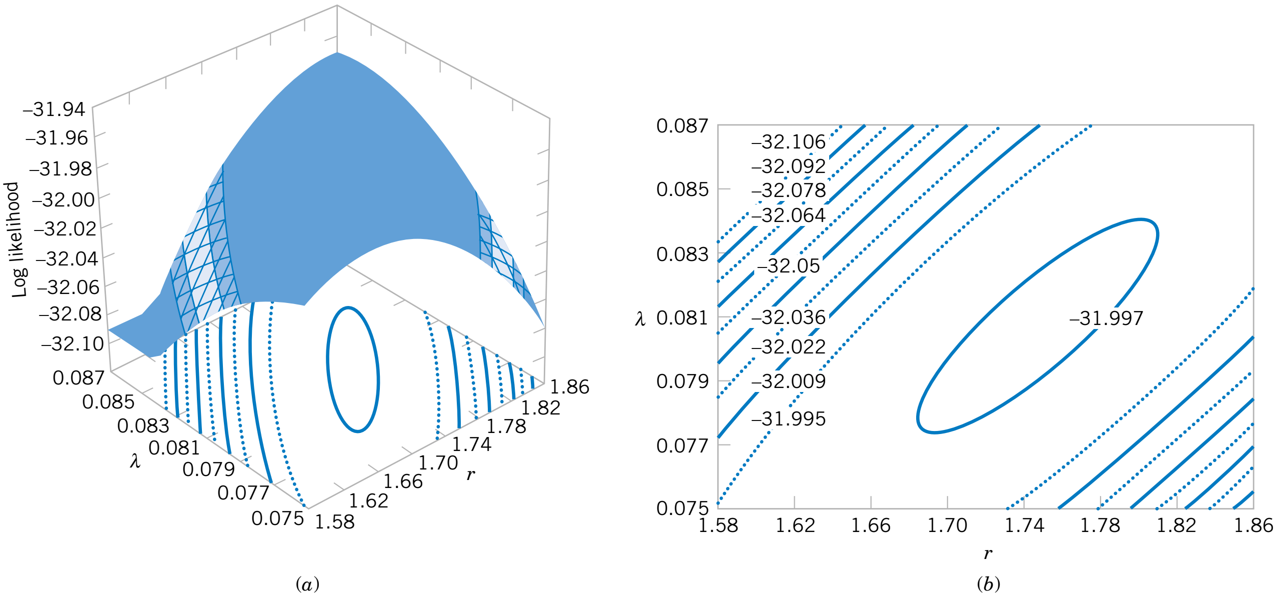


Figure 7-5 Log likelihood for the gamma distribution using the failure time data. (a) Log likelihood surface. (b) Contour plot.

surface as a function of r and λ, and Figure 7-5(b) is a **contour plot.** These plots reveal that the log likelihood is maximized at approximately $\hat{r} = 1.75$ and $\hat{\lambda} = 0.08$. Many statistics computer programs use numerical techniques to solve for the maximum likelihood estimates when no simple solution exists.

7-3.3 Bayesian Estimation of Parameters (CD Only)

EXERCISES FOR SECTION 7-3

7-19. Consider the Poisson distribution

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \qquad x = 0, 1, 2, \ldots$$

Find the maximum likelihood estimator of λ, based on a random sample of size n.

7-20. Consider the shifted exponential distribution

$$f(x) = \lambda e^{-\lambda(x-\theta)}, \qquad x \geq \theta$$

When $\theta = 0$, this density reduces to the usual exponential distribution. When $\theta > 0$, there is only positive probability to the right of θ.

(a) Find the maximum likelihood estimator of λ and θ, based on a random sample of size n.

(b) Describe a practical situation in which one would suspect that the shifted exponential distribution is a plausible model.

7-21. Let X be a geometric random variable with parameter p. Find the maximum likelihood estimator of p, based on a random sample of size n.

7-22. Let X be a random variable with the following probability distribution:

$$f(x) = \begin{cases} (\theta + 1)x^{\theta}, & 0 \leq x \leq 1 \\ 0 \quad , & \text{otherwise} \end{cases}$$

Find the maximum likelihood estimator of θ, based on a random sample of size n.

7-23. Consider the Weibull distribution

$$f(x) = \begin{cases} \dfrac{\beta}{\delta}\left(\dfrac{x}{\delta}\right)^{\beta-1} e^{-\left(\frac{x}{\delta}\right)^{\beta}}, & 0 < x \\ 0 \quad , & \text{otherwise} \end{cases}$$

(a) Find the likelihood function based on a random sample of size n. Find the log likelihood.

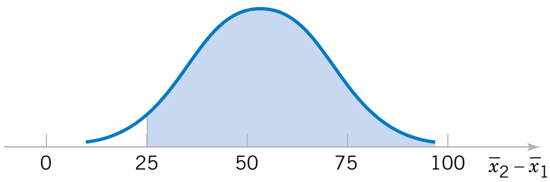


Figure 7-9 The sampling distribution of $\bar{X}_2 - \bar{X}_1$ in Example 7-15.

Corresponding to the value $\bar{x}_2 - \bar{x}_1 = 25$ in Fig. 7-9, we find that

$$z = \frac{25 - 50}{\sqrt{136}} = -2.14$$

and we find that

$$P(\bar{X}_2 - \bar{X}_1 \geq 25) = P(Z \geq -2.14) = 0.9838$$

EXERCISES FOR SECTION 7-5

7-33. PVC pipe is manufactured with a mean diameter of 1.01 inch and a standard deviation of 0.003 inch. Find the probability that a random sample of $n = 9$ sections of pipe will have a sample mean diameter greater than 1.009 inch and less than 1.012 inch.

7-34. Suppose that samples of size $n = 25$ are selected at random from a normal population with mean 100 and standard deviation 10. What is the probability that the sample mean falls in the interval from $\mu_{\bar{X}} - 1.8\sigma_{\bar{X}}$ to $\mu_{\bar{X}} + 1.0\sigma_{\bar{X}}$?

7-35. A synthetic fiber used in manufacturing carpet has tensile strength that is normally distributed with mean 75.5 psi and standard deviation 3.5 psi. Find the probability that a random sample of $n = 6$ fiber specimens will have sample mean tensile strength that exceeds 75.75 psi.

7-36. Consider the synthetic fiber in the previous exercise. How is the standard deviation of the sample mean changed when the sample size is increased from $n = 6$ to $n = 49$?

7-37. The compressive strength of concrete is normally distributed with $\mu = 2500$ psi and $\sigma = 50$ psi. Find the probability that a random sample of $n = 5$ specimens will have a sample mean diameter that falls in the interval from 2499 psi to 2510 psi.

7-38. Consider the concrete specimens in the previous example. What is the standard error of the sample mean?

7-39. A normal population has mean 100 and variance 25. How large must the random sample be if we want the standard error of the sample average to be 1.5?

7-40. Suppose that the random variable X has the continuous uniform distribution

$$f(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Suppose that a random sample of $n = 12$ observations is selected from this distribution. What is the probability distribution of $\bar{X} - 6$? Find the mean and variance of this quantity.

7-41. Suppose that X has a discrete uniform distribution

$$f(x) = \begin{cases} 1/3, & x = 1, 2, 3 \\ 0, & \text{otherwise} \end{cases}$$

A random sample of $n = 36$ is selected from this population. Find the probability that the sample mean is greater than 2.1 but less than 2.5, assuming that the sample mean would be measured to the nearest tenth.

7-42. The amount of time that a customer spends waiting at an airport check-in counter is a random variable with mean 8.2 minutes and standard deviation 1.5 minutes. Suppose that a random sample of $n = 49$ customers is observed. Find the probability that the average time waiting in line for these customers is

(a) Less than 10 minutes
(b) Between 5 and 10 minutes
(c) Less than 6 minutes

7-43. A random sample of size $n_1 = 16$ is selected from a normal population with a mean of 75 and a standard deviation of 8. A second random sample of size $n_2 = 9$ is taken from another normal population with mean 70 and standard deviation 12. Let $\bar{X}_1$ and $\bar{X}_2$ be the two sample means. Find

(a) The probability that $\bar{X}_1 - \bar{X}_2$ exceeds 4
(b) The probability that $3.5 \leq \bar{X}_1 - \bar{X}_2 \leq 5.5$

7-44. A consumer electronics company is comparing the brightness of two different types of picture tubes for use in its television sets. Tube type A has mean brightness of 100 and standard deviation of 16, while tube type B has unknown

3. H_1: The form of the distribution is nonnormal.

4. $\alpha = 0.05$

5. The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(o_i - E_i)^2}{E_i}$$

6. Since two parameters in the normal distribution have been estimated, the chi-square statistic above will have $k - p - 1 = 8 - 2 - 1 = 5$ degrees of freedom. Therefore, we will reject H_0 if $\chi_0^2 > \chi_{0.05,5}^2 = 11.07$.

7. Computations:

$$\chi_0^2 = \sum_{i=1}^{8} \frac{(o_i - E_i)^2}{E_i} = \frac{(12 - 12.5)^2}{12.5} + \frac{(14 - 12.5)^2}{12.5} + \cdots + \frac{(14 - 12.5)^2}{12.5} = 0.64$$

8. Conclusions: Since $\chi_0^2 = 0.64 < \chi_{0.05,5}^2 = 11.07$, we are unable to reject H_0, and there is no strong evidence to indicate that output voltage is not normally distributed. The P-value for the chi-square statistic $\chi_0^2 = 0.64$ is $P = 0.9861$.

EXERCISES FOR SECTION 9-7

9-59. Consider the following frequency table of observations on the random variable X.

Values	0	1	2	3	4
Observed Frequency	24	30	31	11	4

(a) Based on these 100 observations, is a Poisson distribution with a mean of 1.2 an appropriate model? Perform a goodness-of-fit procedure with $\alpha = 0.05$.
(b) Calculate the P-value for this test.

9-60. Let X denote the number of flaws observed on a large coil of galvanized steel. Seventy-five coils are inspected and the following data were observed for the values of X:

Values	1	2	3	4	5	6	7	8
Observed Frequency	1	11	8	13	11	12	10	9

(a) Does the assumption of the Poisson distribution seem appropriate as a probability model for this data? Use $\alpha = 0.01$.
(b) Calculate the P-value for this test.

9-61. The number of calls arriving at a switchboard from noon to 1 PM during the business days Monday through Friday is monitored for six weeks (i.e., 30 days). Let X be defined as the number of calls during that one-hour period. The relative frequency of calls was recorded and reported as

Value	5	6	8	9	10
Relative Frequency	0.067	0.067	0.100	0.133	0.200
Value	11	12	13	14	15
Relative Frequency	0.133	0.133	0.067	0.033	0.067

(a) Does the assumption of a Poisson distribution seem appropriate as a probability model for this data? Use $\alpha = 0.05$.
(b) Calculate the P-value for this test.

9-62. Consider the following frequency table of observations on the random variable X:

Values	0	1	2	3	4
Frequency	4	21	10	13	2

(a) Based on these 50 observations, is a binomial distribution with $n = 6$ and $p = 0.25$ an appropriate model? Perform a goodness-of-fit procedure with $\alpha = 0.05$.
(b) Calculate the P-value for this test.