FALL 2022 Math 461 --- RINGS AND MODULES

Prerequisite: Math 367 or consent of instructor. Credits: (3-0) 3.

Instructor: Semra Öztürk, M 138, Schedule and office hours are at the address http:/www.metu.edu.tr/~sozkap/aa.pdf.

Catalog Contents: Rings, ideals, isomorphism theorems, group rings, localization, factor rings. Modules, submodules, direct products, factor modules. Homomorphisms, classical isomorphism theorems. The endomorphism ring of a module. Free modules, free groups. Tensor product of modules. Finitely generated modules over a principal ideal domain.

Grading will be based on two midterms %40 each, one final exam %50, and attendance/class participation %10.

Exam 1 on Nov 7 Monday, 17:40

Exam 2 on Dec 12 Monday, 17:40

You should take both of the midterm exams, I will omit the lower grade (even if you are happy with your first midterm grade you still have to take the second midterm exam)

Attendance is required, attendance and class participation will be %10 of the course grade.

Description of the course: This course is to provide the background for students who are willing to learn more about rings and modules which are the fundamental mathematical structures occuring everywhere ! It is good for everyone but especially for students who are planning to study any algebra related topics such as algebraic topology, algebraic geometry, even analysis, or even machine learning.
This semester I will spend less than half of the semester on rings and spend more time on modules. Rings will be a more detailed but much faster version of some of the topics you have seen in Math 367, and Math 116. Modules will be new to you. They are generalizations of vector spaces also generalization of abelian groups. (Modules over group algebras are examples of groups acting on vector spaces.)
Thus in module theory linear algebra comes up quite often. You should be comfortable using linear algebra to get more out of this course. We will see the primary decomposition theorem for finitely generated modules over a Euclidean domain.

You can print a tentative syllabus from here.

Textbook : Rather than the textbooks which are listed below and can be used as refence material I will use several lecture notes available online in this semester.

Except for the topics listed below we will use the notes by Neil P. Strickland

- **Localization/ring of fractions**
- **Ring of endomorphisms of a module Musil`s or Dummit and Foote`s book below can be used.**
- **Tensor products: Tensor Products Demystified, Tensor product of abelian groups, Tensor Product of vector spaces, Notes on Tensor products by Rich Schwartz**

Some textbooks which are used in earlier semesters can be useful:

1) Introduction to Rings and Modules, Second Revised Edition, by C. Musili, Narosa Publishing House, 1994, only *Chapters 1--5 (the part for rings is too long and the part for modules is too short)*

2) Abstract Algebra by David S. Dummit and Richard M. Foote, Third Edition, only the Preliminaries and Chapter 7 .

3) A First Course in Module Theory by M. E. Keating, Imperial Clollege Press, only the Chapters 2--9

Further Inspiring Reading:

The number of homomorphisms from Zn to Zm.pdf

Classification of Finite-Finite rings.pdf

A nice set of lecture notes by Mike Prest which covers more topics than our course content advanced students may like it,.(Chapters 0,1,2,6 are usefull for this course, at first reading omit Chapters 3, 4, 5)

Supplementary solved exercises

The Arithmetic of Gaussian Integers

Updated

Tentative Syllabus		2022 Fall - 461	
	Monday	Strickland's Notes from the website	Additional notes from the website
Week 1 Oct 3		1. Introduction 2. Rings and Fields	
Week 2 Oct 10		3. Modules	
Week 3 Oct 17		4. Modules over polynomial rings	
Week 4 Oct 24		5. General module theory	
Week 5 Oct 31		6. Homomorphisms	
Week 6 Nov 7	Exam 1	7. Factor modules	
Week 7 Nov 14			Localization/ring of fractions
Week 8 Nov 21		8. Ideals and factor rings	
Week 9 Nov 28		9. Euclidean domains (ED)	
Week 10 Dec 5		10.Factorisation in ED	
Week 11 Dec 12	Exam 2	11. Finite free modules over ED	
Week 12 Dec 19		13. Primary decomposition	
Week 13 Dec 26			Tensor product
Week 14 Jan 2			Endomorphism ring of a module