1) \( \mathbb{Z} \) is a PID.

2) Suppose \( R \) is a commutative ring, \( I_1 \) and \( I_2 \) are ideals in \( R \), \( P \) is a prime ideal in \( R \), and \( I_1 \cap I_2 \subseteq P \). Show that \( I_1 \subseteq P \) or \( I_2 \subseteq P \).

3) Let \( D \) be an integral domain, \( a, b \in D \). Suppose \( a^n = b^n \) and \( a^m = b^m \) for two relatively prime positive integers \( m \) and \( n \). Prove that \( a = b \).

4) Let \( F \) be the field of real numbers. Prove that \( \mathbb{F}[x]/\langle x^2 + 1 \rangle \) is a field isomorphic to \( \mathbb{C} \).

5) Let \( R \) be an integral domain and \( f(x), g(x) \in R[x] \). Prove that \( \deg(f(x), g(x)) = \deg(f(x)) + \deg(g(x)) \).

6) If \( A \) and \( B \) are ideals in a ring \( R \) such that \( AB = \{0\} \), prove that \( a \in A \), \( b \in B \), \( ab = 0 \).

7) Let \( R \) be a commutative ring and suppose that \( A \) is an ideal of \( R \). Let \( N(A) = \{ x \in R \mid x^n \in A \text{ for some } n \} \). Prove

(a) \( N(A) \) is an ideal of \( R \) which contains \( A \).

(b) \( N(N(A)) = N(A) \).

(c) Let \( N = \{ x \in R \mid x \text{ is nilpotent} \} \). Is \( N \) an ideal? Prove or disprove.
1) Let $G$ be a finite abelian group. Prove that $G$ is isomorphic to the direct product of its Sylow subgroups.

2) Let $G$ be a group of order 12. Then either $G$ has a normal Sylow 3-subgroup or it is isomorphic to $A_4$.

3) There are no simple groups of order 120, 56, 40, 70.

4) A group of order 28 with a normal subgroup of order 4 is abelian.

5) Let $p$ be a prime number, $P$ be a Sylow $p$-subgroup of a finite group $G$ and $Q$ be any $p$-subgroup of $G$. Then $Q \leq N_G(P)$ if $Q \leq P$. 
