Problem 1. (5 pts) Justify that $h = (x - 1)^4 + (y - 1)^4 - 1$ belongs to the ideal $(f_0, f_1)$, where $f_0 = xy$ and $f_1 = (x - 1)^2 + (y - 1)^2 - 1$, by applying the fundamental Noether’s theorem.
Problem 2. (5 pts) Two quintics $A$ and $B$ have both a cusp at a point $P$. The other intersection points, $P_1, \ldots, P_n$, are non-singular. What can be the values of $n$?

Problem 3. (5 pts) Consider points $P_{\pm} = (-1, \pm \sqrt{3})$ on the curve $A = \{y^2 = x^3 - 4x\}$. Find a family of functions $f \in L(D)$ (including not only constants), where $D = P_+ + P_-$. Conclude that $\ell(D) \geq 2$. 
**Problem 4. (10 pts)** Let $A$ be the normalization of a quartic curve with one cuspidal singularity. Suppose that the canonical class divisors on $A$ contains a multiple of some point, $mP$.

(a) Find $m$.

(b) Find all possible sequences $\ell(P), \ell(2P), \ell(3P), \ell(4P), \ell(5P), \ell(6P), \ldots$.

(c) Show that $A$ is hyperelliptic by considering the projection $f : A \to \mathbb{P}^1$ from the cusp of $A$.

(d) How many branch points of $f$ are there?

(e) Does $f$ have a branch point at the cusp?