Problem 1. (14pts) (a) State Euler’s formula for graphs on surfaces. Under which condition on the graph this formula is satisfied?

(b) Give a definition of simplicial complex.

(c) What is a chain complex?

(d) What are the homology groups of a chain complex?

(e) How the boundary map of a simplicial chain complex is defined?

(f) What are the homology groups of surfaces $F_g$ and $N_k$?

(g) Which $\Delta$-triangulation of a surface is a simplicial triangulation?
Problem 2. (5pts) For the link diagram sketched below
(a) find the topological type of its span,

(b) sketch a graph being a deformational retract of this span.

Problem 3. (10pts) Consider surface $F$ glued from a hexagon according to the word $abacb^{-1}c^{-1}$. Determine if the following curves on the hexagonal model are one-sided or two-sided.

(a) Line segment connecting the midpoints of the sides “a”.

(b) Line segment connecting the midpoints of the sides “b”.

(c) The diagonal separating sides $ab$ from $acb^{-1}c^{-1}$.

(d) The side $a$.

(e) The side $b$. 
Problem 4. (4pts) Consider a graph with vertices $A, B, C$ and edges $[AB], [BC], [AC]$ as a simplicial complex, $C$.
(a) For $x = 2[AB] - 3[BC] + [AC]$ find $\partial_1 x$.

(b) Give examples of a cycle in $C_1(C)$ and a boundary in $C_0(C)$.

Problem 5. (12pts) Consider surface $F$ obtained from a hexagon $ABCDEF$ by gluing side $AB$ to $DE$ and $BC$ to $FE$. Divide the hexagon into triangles by diagonals $AC, CE, and CF$.
(a) Is it a simplicial triangulation, or $\Delta$-triangulation of $F$? (Explain.)

(b) What are the generators of the chain groups $C_0, C_1$ and $C_2$?

(c) Find the boundary of the chain $2[ABC] - [CDE]$.

(d) Calculate the homology group $H_2(F)$ using this chain complex.
Problem 6. (15pts) Consider a polygonal cell complex $X$, whose 2-cells are represented by words $abcb^{-1}$, $acda^{-1}$, $bcb$.

(a) Describe its chain groups $C_i$, $i = 0, 1, 2$, and the boundary maps $\partial_2 : C_2 \to C_1$ and $\partial_1 : C_1 \to C_0$.

(b) Find the homology groups $H_i(X)$, $i = 0, 1, 2$. 
