Problem 1. (15 pts) Solve a linear system in $\mathbb{Z}_{13}$.

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\begin{align*}
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Problem 2. (15 pts) Assume that $G$ is a group and $H \subset G$ its subgroup.
(a) Prove that subgroup $H$ is necessarily normal if it has index $[G : H] = 2$. 

(b) Give an example of a subgroup $H$ of index $[G : H] = 3$ which is not normal. Justify that it is not normal.
Problem 3. (15 pts) Consider the ring of upper triangular matrices $T_2 = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} | a, b, c \in \mathbb{Z}$

and let $I = \begin{bmatrix} 0 & x \\ 0 & y \end{bmatrix} | x, y \in \mathbb{Z}$.

(a) Verify that $I$ is a (2-sided) ideal in $T_2$.

(b) Prove that $T_2/I$ is isomorphic to $\mathbb{Z}$. Write explicitly a map $T_2/I \to \mathbb{Z}$ and verify that it is a ring isomorphism.
Problem 4. (15 pts) (a) Present polynomial $f(x) = x^4 + x^3 + x^2 + x$ as a product of irreducible polynomials over $\mathbb{Z}_3$. Explain, why your factors are irreducible.

(b) Show that polynomial $f(x) = 2x^3 + x^2 + x + 1$ is irreducible over $\mathbb{Q}$. 