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> # Prof. Dr. Serkan Dağ
  # ME 451 Introduction to Composite Structures
> # File 7.4.3
  # Example on design of a laminate
> restart :
  with(LinearAlgebra) :
  Digits := 16 :
> # Enter the number of plies
> n := 1 :
> # Define extensional, coupling, and bending stiffness matrices
> A := Matrix(3) :
  B := Matrix(3) :
  Dm := Matrix(3) :
> # Define laminate stiffness matrix
> QL := Matrix(6) :
> # Define ply surface coordinate vector in inches
```

> $h := \begin{bmatrix} -\frac{h0}{2} \\ \frac{h0}{2} \end{bmatrix};$

$$h := \begin{bmatrix} -\frac{1}{2}\, h0 \\ \frac{1}{2}\, h0 \end{bmatrix} \quad (1)$$

```
> # Define ply angle vector in radians
> theta := [ 0 ];
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$$\theta := \left[\ 0\ \right] \quad (2)$$

```
> # Define Qbar array
  Qbar := Array(1..3, 1..3, 1..n) :
  ArrayNumElems(Qbar);
```

$$9 \quad (3)$$

```
> # Enter properties of the unidirectional lamina
  # From Table 2.2 for graphite/epoxy (unit = Msi)
> E1 := 26.25 :
  E2 := 1.49 :
  nu12 := 0.28 :
  G12 := 1.040 :
> # Calculate elements of the compliance matrix for the
  unidirectional lamina
```

> $S11 := \frac{1}{E1}:$

$S12 := -\frac{nu12}{E1}:$

```
S22 := 1/E2 :
S66 := 1/G12 :
> # Calculate elements of the reduced stiffness matrix for the
unidirectional lamina
> Q11 := S22/(S11·S22 − S12^2) :
  Q22 := S11/(S11·S22 − S12^2) :
  Q12 := - S12/(S11·S22 − S12^2) :
  Q66 := 1/S66 :
> # Calculate elements of transformed reduced stiffness matrix for
each angle lamina
# Unit = Msi
> for i from 1 by 1 to n
while true do
Qbar[1, 1, i] := Q11·(cos(theta[i, 1]))^4 + Q22·(sin(theta[i, 1]))^4 + 2·(Q12 + 2·Q66)
    ·(cos(theta[i, 1]))^2·(sin(theta[i, 1]))^2 :
 Qbar[1, 2, i] := (Q11 + Q22 − 4·Q66)·(sin(theta[i, 1]))^2·(cos(theta[i, 1]))^2 + Q12
    ·((cos(theta[i, 1]))^4 + (sin(theta[i, 1]))^4) :
 Qbar[1, 3, i] := (Q11 − Q12 − 2·Q66)·(sin(theta[i, 1]))·(cos(theta[i, 1]))^3 − (Q22 − Q12
    − 2·Q66)·(sin(theta[i, 1]))^3·cos(theta[i, 1]) :
 Qbar[2, 2, i] := Q11·(sin(theta[i, 1]))^4 + Q22·(cos(theta[i, 1]))^4 + 2·(Q12 + 2·Q66)
    ·(cos(theta[i, 1]))^2·(sin(theta[i, 1]))^2 :
 Qbar[2, 3, i] := (Q11 − Q12 − 2·Q66)·(cos(theta[i, 1]))·(sin(theta[i, 1]))^3 − (Q22 − Q12
    − 2·Q66)·(cos(theta[i, 1]))^3·sin(theta[i, 1]) :
 Qbar[3, 3, i] := (Q11 + Q22 − 2·Q12 − 2·Q66)·(cos(theta[i, 1]))^2·(sin(theta[i, 1]))^2
    + Q66·((cos(theta[i, 1]))^4 + (sin(theta[i, 1]))^4) :
 Qbar[2, 1, i] := Qbar[1, 2, i] :
 Qbar[3, 1, i] := Qbar[1, 3, i] :
 Qbar[3, 2, i] := Qbar[2, 3, i] :
 end do:
> # Calculate elements of extensional stiffness matrix [A],
coupling stiffness matrix [B], and bending stiffness matrix [Dm]
# Units: [A]--> Msi.in; [B]--> Msi.in^2; [Dm]--> Msi.in^3
> for i from 1 by 1 to 3
 while true do
for j from 1 by 1 to 3
 while true do
  A[i,j]=0 :
  B[i,j] := 0 :
  Dm[i,j] := 0 :
  for k from 1 by 1 to n
  while true do
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A[i,j] := A[i,j] + Qbar[i,j,k]·(h[k+1,1] - h[k,1]) :
B[i,j] := B[i,j] + 1/2·Qbar[i,j,k]·(h[k+1,1]^2 - h[k,1]^2) :
Dm[i,j] := Dm[i,j] + 1/3·Qbar[i,j,k]·(h[k+1,1]^3 - h[k,1]^3) :
 end do:
end do:
end do:
```

> evalf(A);

$$\begin{bmatrix} 26.36733817050383\, h0 & 0.4190648946565411\, h0 & 0. \\ 0.4190648946565411\, h0 & 1.496660338059075\, h0 & 0. \\ 0. & 0. & 1.040000000000000\, h0 \end{bmatrix} \quad \textbf{(4)}$$

> evalf(B);

$$\begin{bmatrix} 0. & 0. & 0. \\ 0. & 0. & 0. \\ 0. & 0. & 0. \end{bmatrix} \quad \textbf{(5)}$$

> evalf(Dm);

$$\begin{bmatrix} 2.197278180875319\, h0^3 & 0.03492207455471177\, h0^3 & 0. \\ 0.03492207455471177\, h0^3 & 0.1247216948382563\, h0^3 & 0. \\ 0. & 0. & 0.08666666666666667\, h0^3 \end{bmatrix} \quad \textbf{(6)}$$

```
> # Form laminate stiffness matrix QL
> for i from 1 by 1 to 3
 while true do
 for j from 1 by 1 to 3
 while true do
 QL[i,j] := A[i,j] :
 QL[i,j+3] := B[i,j] :
 QL[i+3,j] := B[i,j] :
 QL[i+3,j+3] := Dm[i,j] :
 end do:
 end do:
> # Form loading vector N
```

> $N := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3250\cdot(10)^{-6} \\ 0 \\ 0 \end{bmatrix};$

$$N := \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{13}{4000} \\ 0 \\ 0 \end{bmatrix} \quad (7)$$

```
> # Find midplane strains and curvatures
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> $Res := evalf(\ LinearSolve(QL, N)\ );$

$$Res := \begin{bmatrix} 0. \\ 0. \\ 0. \\ \frac{0.001485714285714286}{h0^3} \\ -\frac{0.0004160000000000003}{h0^3} \\ 0. \end{bmatrix} \quad (8)$$

```
> # Find strains
```

> $eps_x := z \cdot Res[4, 1];$
> $eps_y := z \cdot Res[5, 1];$

$$eps_x := \frac{0.001485714285714286\, z}{h0^3}$$

$$eps_y := -\frac{0.0004160000000000003\, z}{h0^3} \quad (9)$$

```
> # Find stresses
```

> $\sigma_x := Qbar[1, 1, 1] \cdot eps_x + Qbar[1, 2, 1] \cdot eps_y;$
> $\sigma_y := Qbar[2, 1, 1] \cdot eps_x + Qbar[2, 2, 1] \cdot eps_y;$
> $\sigma_{xy} := Qbar[3, 1, 1] \cdot eps_x + Qbar[3, 2, 1] \cdot eps_y;$

$$\sigma_x := \frac{0.03900000000000001\, z}{h0^3}$$

$$\sigma_y := -\frac{1.\ 10^{-19}\, z}{h0^3}$$

$$\sigma_{xy} := 0. \quad (10)$$

```
> # Find thickness
```

> $h := \mathrm{sqrt}\left(\frac{3.9e-2}{2} \cdot \frac{1000}{\left(\frac{217.56}{2}\right)} \right);$

(11)

$$h := 0.4233921274027279 \quad (11)$$

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> # Find number of plies
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$$> N := \frac{h}{0.0049213};$$

$$N := 86.03257826239570 \quad (12)$$

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