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> # Prof. Dr. Serkan Dağ
# ME 451 Introduction to Composite Structures
> # File 7.4.2
# Example on design of a laminate

> restart :
with(LinearAlgebra) :
Digits := 16 :
> # Enter the number of plies
> n := 1 :
> # Define extensional, coupling, and bending stiffness matrices
> A := Matrix(3) :
B := Matrix(3) :
Dm := Matrix(3) :
> # Define laminate stiffness matrix
> QL := Matrix(6) :
> # Define ply surface coordinate vector in inches

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$$h := \begin{bmatrix} -\frac{h0}{2} \\ \frac{h0}{2} \end{bmatrix};$$

$$h := \begin{bmatrix} -\frac{1}{2} h0 \\ \frac{1}{2} h0 \end{bmatrix}$$

(1)

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> # Define ply angle vector in radians
> theta := [ 0 ];

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$$\theta := [0]$$

(2)

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> # Define Qbar array
Qbar := Array(1..3, 1..3, 1..n) :
ArrayNumElems(Qbar);

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9

(3)

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> # Enter properties of the unidirectional lamina
# From Table 2.2 for glass/epoxy (unit = Msi)

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> E1 := 5.6 :
E2 := 1.2 :
nu12 := 0.26 :
G12 := 0.6 :

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> # Calculate elements of the compliance matrix for the
unidirectional lamina

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> S11 :=  $\frac{1}{E1}$  :
S12 :=  $-\frac{\nu12}{E1}$  :

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$$S_{22} := \frac{1}{E_2} :$$

$$S_{66} := \frac{1}{G_{12}} :$$

> # Calculate elements of the reduced stiffness matrix for the unidirectional lamina

$$Q_{11} := \frac{S_{22}}{S_{11} \cdot S_{22} - S_{12}^2} :$$

$$Q_{22} := \frac{S_{11}}{S_{11} \cdot S_{22} - S_{12}^2} :$$

$$Q_{12} := -\frac{S_{12}}{S_{11} \cdot S_{22} - S_{12}^2} :$$

$$Q_{66} := \frac{1}{S_{66}} :$$

> # Calculate elements of transformed reduced stiffness matrix for each angle lamina

Unit = Msi

> for i from 1 by 1 to n
while true do

$$Q_{bar}[1, 1, i] := Q_{11} \cdot (\cos(\theta_{[i, 1]}))^4 + Q_{22} \cdot (\sin(\theta_{[i, 1]}))^4 + 2 \cdot (Q_{12} + 2 \cdot Q_{66}) \cdot (\cos(\theta_{[i, 1]}))^2 \cdot (\sin(\theta_{[i, 1]}))^2 :$$

$$Q_{bar}[1, 2, i] := (Q_{11} + Q_{22} - 4 \cdot Q_{66}) \cdot (\sin(\theta_{[i, 1]}))^2 \cdot (\cos(\theta_{[i, 1]}))^2 + Q_{12} \cdot ((\cos(\theta_{[i, 1]}))^4 + (\sin(\theta_{[i, 1]}))^4) :$$

$$Q_{bar}[1, 3, i] := (Q_{11} - Q_{12} - 2 \cdot Q_{66}) \cdot (\sin(\theta_{[i, 1]})) \cdot (\cos(\theta_{[i, 1]}))^3 - (Q_{22} - Q_{12} - 2 \cdot Q_{66}) \cdot (\sin(\theta_{[i, 1]}))^3 \cdot \cos(\theta_{[i, 1]}) :$$

$$Q_{bar}[2, 2, i] := Q_{11} \cdot (\sin(\theta_{[i, 1]}))^4 + Q_{22} \cdot (\cos(\theta_{[i, 1]}))^4 + 2 \cdot (Q_{12} + 2 \cdot Q_{66}) \cdot (\cos(\theta_{[i, 1]}))^2 \cdot (\sin(\theta_{[i, 1]}))^2 :$$

$$Q_{bar}[2, 3, i] := (Q_{11} - Q_{12} - 2 \cdot Q_{66}) \cdot (\cos(\theta_{[i, 1]})) \cdot (\sin(\theta_{[i, 1]}))^3 - (Q_{22} - Q_{12} - 2 \cdot Q_{66}) \cdot (\cos(\theta_{[i, 1]}))^3 \cdot \sin(\theta_{[i, 1]}) :$$

$$Q_{bar}[3, 3, i] := (Q_{11} + Q_{22} - 2 \cdot Q_{12} - 2 \cdot Q_{66}) \cdot (\cos(\theta_{[i, 1]}))^2 \cdot (\sin(\theta_{[i, 1]}))^2 + Q_{66} \cdot ((\cos(\theta_{[i, 1]}))^4 + (\sin(\theta_{[i, 1]}))^4) :$$

$$Q_{bar}[2, 1, i] := Q_{bar}[1, 2, i] :$$

$$Q_{bar}[3, 1, i] := Q_{bar}[1, 3, i] :$$

$$Q_{bar}[3, 2, i] := Q_{bar}[2, 3, i] :$$

end do:

> # Calculate elements of extensional stiffness matrix [A], coupling stiffness matrix [B], and bending stiffness matrix [Dm]
Units: [A]--> Msi.in; [B]--> Msi.in^2; [Dm]--> Msi.in^3

> for i from 1 by 1 to 3

while true do

for j from 1 by 1 to 3

while true do

$$A[i, j] = 0 :$$

$$B[i, j] := 0 :$$

$$Dm[i, j] := 0 :$$

for k from 1 by 1 to n

while true do

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A[i,j] := A[i,j] + Qbar[i,j,k]·(h[k+1,1] - h[k,1]) :
B[i,j] := B[i,j] +  $\frac{1}{2}$  · Qbar[i,j,k]·(h[k+1,1]2 - h[k,1]2) :
Dm[i,j] := Dm[i,j] +  $\frac{1}{3}$  · Qbar[i,j,k]·(h[k+1,1]3 - h[k,1]3) :

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end do:
end do:
end do:

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> evalf( A );
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$$\begin{bmatrix} 5.682312353231092 h0 & 0.3165859739657323 h0 & 0. \\ 0.3165859739657323 h0 & 1.217638361406663 h0 & 0. \\ 0. & 0. & 0.5999999999999999 h0 \end{bmatrix} \quad (4)$$

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> evalf( B );
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$$\begin{bmatrix} 0. & 0. & 0. \\ 0. & 0. & 0. \\ 0. & 0. & 0. \end{bmatrix} \quad (5)$$

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> evalf( Dm );
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$$\begin{bmatrix} 0.4735260294359243 h0^3 & 0.02638216449714436 h0^3 & 0. \\ 0.02638216449714436 h0^3 & 0.1014698634505553 h0^3 & 0. \\ 0. & 0. & 0.050000000000000000 h0^3 \end{bmatrix} \quad (6)$$

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> # Form laminate stiffness matrix QL
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> for i from 1 by 1 to 3
  while true do
    for j from 1 by 1 to 3
      while true do
        QL[i,j] := A[i,j]:
        QL[i,j+3] := B[i,j]:
        QL[i+3,j] := B[i,j]:
        QL[i+3,j+3] := Dm[i,j]:
      end do:
    end do:
  end do:

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> # Form loading vector N
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$$N := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3250 \cdot (10)^{-6} \\ 0 \\ 0 \end{bmatrix};$$

$$N := \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{13}{4000} \\ 0 \\ 0 \end{bmatrix} \quad (7)$$

> # Find midplane strains and curvatures

> Res := evalf(LinearSolve(QL, N));

$$Res := \begin{bmatrix} 0. \\ 0. \\ 0. \\ \frac{0.006964285714285712}{h0^3} \\ -\frac{0.001810714285714284}{h0^3} \\ 0. \end{bmatrix} \quad (8)$$

> # Find strains

> eps_x := z·Res[4, 1];

eps_y := z·Res[5, 1];

$$\begin{aligned} eps_x &:= \frac{0.006964285714285712 z}{h0^3} \\ eps_y &:= -\frac{0.001810714285714284 z}{h0^3} \end{aligned} \quad (9)$$

> # Find stresses

> sigma_x := Qbar[1, 1, 1]·eps_x + Qbar[1, 2, 1]·eps_y;

sigma_y := Qbar[2, 1, 1]·eps_x + Qbar[2, 2, 1]·eps_y;

sigma_xy := Qbar[3, 1, 1]·eps_x + Qbar[3, 2, 1]·eps_y;

$$\begin{aligned} \sigma_x &:= \frac{0.039000000000000000 z}{h0^3} \\ \sigma_y &:= \frac{1 \cdot 10^{-18} z}{h0^3} \\ \sigma_{xy} &:= 0. \end{aligned} \quad (10)$$

> # Find thickness

> h := sqrt($\frac{3.9e-2}{2} \cdot \frac{1000}{\left(\frac{88.47}{2}\right)}$);

(11)



$h := 0.6639483407000466$

(11)