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> # Prof. Dr. Serkan Dağ
  # ME 451 Introduction to Composite Structures
> # File 7.4.1
  # Example on design of a laminate
> restart :
  with(LinearAlgebra) :
  Digits := 16 :
> # Enter the number of plies
> n := 1 :
> # Define extensional, coupling, and bending stiffness matrices
> A := Matrix(3) :
  B := Matrix(3) :
  Dm := Matrix(3) :
> # Define laminate stiffness matrix
> QL := Matrix(6) :
> # Define ply surface coordinate vector in inches
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> $h := \begin{bmatrix} -\frac{h0}{2} \\ \frac{h0}{2} \end{bmatrix}$;

$$h := \begin{bmatrix} -\frac{1}{2}\,h0 \\ \frac{1}{2}\,h0 \end{bmatrix} \qquad (1)$$

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> # Define ply angle vector in radians
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> theta := $[\ 0\ ]$;

$$\theta := [\ 0\ ] \qquad (2)$$

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> # Define Qbar array
  Qbar := Array(1 ..3, 1 ..3, 1 ..n) :
  ArrayNumElems(Qbar);
```

$$9 \qquad (3)$$

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> # Enter properties of the unidirectional lamina
  # From Table 3.4 for aluminum (unit = Msi)
> E1 := 10.3 :
  E2 := 10.3 :
  nu12 := 0.30 :
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$G12 := \frac{E1}{2\cdot(1+nu12)}$:

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> # Calculate elements of the compliance matrix for the
  unidirectional lamina
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> $S11 := \frac{1}{E1}$:

$S12 := -\frac{nu12}{E1}$:

$S22 := \frac{1}{E2}$:

$S66 := \frac{1}{G12}$:

> **# Calculate elements of the reduced stiffness matrix for the unidirectional lamina**

> $Q11 := \frac{S22}{S11 \cdot S22 - S12^2}$:

$Q22 := \frac{S11}{S11 \cdot S22 - S12^2}$:

$Q12 := -\frac{S12}{S11 \cdot S22 - S12^2}$:

$Q66 := \frac{1}{S66}$:

> **# Calculate elements of transformed reduced stiffness matrix for each angle lamina**
> **# Unit = Msi**

> **for** *i* **from** 1 **by** 1 **to** n
while *true* **do**

$Qbar[1, 1, i] := Q11 \cdot (\cos(\text{theta}[i, 1]))^4 + Q22 \cdot (\sin(\text{theta}[i, 1]))^4 + 2 \cdot (Q12 + 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2$:

$Qbar[1, 2, i] := (Q11 + Q22 - 4 \cdot Q66) \cdot (\sin(\text{theta}[i, 1]))^2 \cdot (\cos(\text{theta}[i, 1]))^2 + Q12 \cdot ((\cos(\text{theta}[i, 1]))^4 + (\sin(\text{theta}[i, 1]))^4)$:

$Qbar[1, 3, i] := (Q11 - Q12 - 2 \cdot Q66) \cdot (\sin(\text{theta}[i, 1])) \cdot (\cos(\text{theta}[i, 1]))^3 - (Q22 - Q12 - 2 \cdot Q66) \cdot (\sin(\text{theta}[i, 1]))^3 \cdot \cos(\text{theta}[i, 1])$:

$Qbar[2, 2, i] := Q11 \cdot (\sin(\text{theta}[i, 1]))^4 + Q22 \cdot (\cos(\text{theta}[i, 1]))^4 + 2 \cdot (Q12 + 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2$:

$Qbar[2, 3, i] := (Q11 - Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1])) \cdot (\sin(\text{theta}[i, 1]))^3 - (Q22 - Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^3 \cdot \sin(\text{theta}[i, 1])$:

$Qbar[3, 3, i] := (Q11 + Q22 - 2 \cdot Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2 + Q66 \cdot ((\cos(\text{theta}[i, 1]))^4 + (\sin(\text{theta}[i, 1]))^4)$:

$Qbar[2, 1, i] := Qbar[1, 2, i]$:
$Qbar[3, 1, i] := Qbar[1, 3, i]$:
$Qbar[3, 2, i] := Qbar[2, 3, i]$:
end do:

> **# Calculate elements of extensional stiffness matrix [A], coupling stiffness matrix [B], and bending stiffness matrix [Dm]**
> **# Units: [A]--> Msi.in; [B]--> Msi.in^2; [Dm]--> Msi.in^3**

> **for** *i* **from** 1 **by** 1 **to** 3
while *true* **do**
for *j* **from** 1 **by** 1 **to** 3
while *true* **do**
$A[i, j] = 0$:
$B[i, j] := 0$:
$Dm[i, j] := 0$:
for k **from** 1 **by** 1 **to** n
while *true* **do**

$A[i,j] := A[i,j] + Qbar[i,j,k]\cdot(h[k+1,1] - h[k,1])$:

$B[i,j] := B[i,j] + \frac{1}{2}\cdot Qbar[i,j,k]\cdot\left(h[k+1,1]^2 - h[k,1]^2\right)$:

$Dm[i,j] := Dm[i,j] + \frac{1}{3}\cdot Qbar[i,j,k]\cdot\left(h[k+1,1]^3 - h[k,1]^3\right)$:

end do:
end do:
end do:

> *evalf*(*A*);

$$\begin{bmatrix} 11.31868131868132\, h0 & 3.395604395604396\, h0 & 0. \\ 3.395604395604396\, h0 & 11.31868131868132\, h0 & 0. \\ 0. & 0. & 3.961538461538462\, h0 \end{bmatrix} \quad \mathbf{(4)}$$

> evalf(B);

$$\begin{bmatrix} 0. & 0. & 0. \\ 0. & 0. & 0. \\ 0. & 0. & 0. \end{bmatrix} \quad \mathbf{(5)}$$

> evalf(Dm);

$$\begin{bmatrix} 0.9432234432234433\, h0^3 & 0.2829670329670330\, h0^3 & 0. \\ 0.2829670329670330\, h0^3 & 0.9432234432234433\, h0^3 & 0. \\ 0. & 0. & 0.3301282051282052\, h0^3 \end{bmatrix} \quad \mathbf{(6)}$$

```
> # Form laminate stiffness matrix QL
```

> **for** *i* **from** 1 **by** 1 **to** 3
while *true* **do**
for *j* **from** 1 **by** 1 **to** 3
while *true* **do**
$QL[i,j] := A[i,j]$:
$QL[i,j+3] := B[i,j]$:
$QL[i+3,j] := B[i,j]$:
$QL[i+3,j+3] := Dm[i,j]$:
end do:
end do:

```
> # Form loading vector N
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> $N := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3250\cdot(10)^{-6} \\ 0 \\ 0 \end{bmatrix}$;

$$N := \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{13}{4000} \\ 0 \\ 0 \end{bmatrix} \quad (7)$$

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> # Find midplane strains and curvatures
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> $Res := evalf(\,LinearSolve(QL, N)\,);$

$$Res := \begin{bmatrix} 0. \\ 0. \\ 0. \\ \frac{0.003786407766990290}{h0^3} \\ -\frac{0.001135922330097087}{h0^3} \\ 0. \end{bmatrix} \quad (8)$$

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> # Find strains
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> $eps_x := z \cdot Res[4, 1];$
> $eps_y := z \cdot Res[5, 1];$

$$eps_x := \frac{0.003786407766990290\, z}{h0^3}$$
$$eps_y := -\frac{0.001135922330097087\, z}{h0^3} \quad (9)$$

```
> # Find stresses
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> $\sigma_x := Qbar[1, 1, 1] \cdot eps_x + Qbar[1, 2, 1] \cdot eps_y;$
> $\sigma_y := Qbar[2, 1, 1] \cdot eps_x + Qbar[2, 2, 1] \cdot eps_y;$
> $\sigma_{xy} := Qbar[3, 1, 1] \cdot eps_x + Qbar[3, 2, 1] \cdot eps_y;$

$$\sigma_x := \frac{0.03899999999999999\, z}{h0^3}$$
$$\sigma_y := 0.$$
$$\sigma_{xy} := 0. \quad (10)$$

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> # Find thickness
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> $h := \mathrm{sqrt}\left(\frac{3.9e-2}{2} \cdot \frac{1000}{\left(\frac{40.02}{2}\right)} \right);$

$$h := 0.9871741202180018 \quad (11)$$

> $\frac{h}{2}$;

$$0.4935870601090009 \quad \textbf{(12)}$$

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