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> # Prof. Dr. Serkan Dağ
  # ME 451 Introduction to Composite Structures
> # File 7.3
  # Example on first ply failure load of a thermomechanically
  loaded laminate
> restart :
  with(LinearAlgebra) :
> # Enter the number of plies
> n := 3 :
> # Define extensional, coupling, and bending stiffness matrices
> A := Matrix(3) :
  B := Matrix(3) :
  Dm := Matrix(3) :
> # Define fictitious thermal force [NT] and thermal moment [MT]
  vectors
> NT := Matrix(3, 1) :
  MT := Matrix(3, 1) :
> # Define ply surface coordinate vector in meters
```

$$> h := \begin{bmatrix} -\frac{7.5}{1000} \\ -\frac{2.5}{1000} \\ \frac{2.5}{1000} \\ \frac{7.5}{1000} \end{bmatrix};$$

$$h := \begin{bmatrix} -0.007500000000 \\ -0.002500000000 \\ 0.002500000000 \\ 0.007500000000 \end{bmatrix} \qquad \textbf{(1)}$$

```
> # Define ply angle vector in radians
```

$$> \text{theta} := \begin{bmatrix} 0 \\ \frac{\text{Pi}}{2} \\ 0 \end{bmatrix}:$$

```
> # Enter uniform temperature change delta_T in degrees celsius
> delta_T := -75 :
> # Define Qbar array
  Qbar := Array(1..3, 1..3, 1..n) :
  ArrayNumElems(Qbar);
```

$$27 \qquad \textbf{(2)}$$

```
> # Define thermal expansion coefficient array
```

```
> alpha := Array(1..3, 1..1, 1..n) :
  ArrayNumElems(alpha);
```

$$9 \quad (3)$$

```
> # Enter mechanical properties of the unidirectional graphite/epoxy lamina
  # From Table 2.1 for graphite/epoxy (unit = MPa)
```

> $E1 := 181000 :$
> $E2 := 10300 :$
> $nu12 := 0.28 :$
> $G12 := 7170 :$

```
> # Enter thermal expansion coefficients of the unidirectional graphite/epoxy lamina
  # From Table 2.1 for graphite/epoxy (unit = 1/ (degrees celsius) )
```

> $alpha1 := 0.02 \cdot (10)^{-6} :$
> $alpha2 := 22.5 \cdot (10)^{-6} :$

```
> # Calculate elements of the compliance matrix for the unidirectional lamina
```

> $S11 := \frac{1}{E1} :$
> $S12 := -\frac{nu12}{E1} :$
> $S22 := \frac{1}{E2} :$
> $S66 := \frac{1}{G12} :$

```
> # Calculate elements of the reduced stiffness matrix for the unidirectional lamina
```

> $Q11 := \frac{S22}{S11 \cdot S22 - S12^2} :$
> $Q22 := \frac{S11}{S11 \cdot S22 - S12^2} :$
> $Q12 := -\frac{S12}{S11 \cdot S22 - S12^2} :$
> $Q66 := \frac{1}{S66} :$

```
> # Calculate elements of transformed reduced stiffness matrix for each angle lamina
  # Unit = MPa
```

> **for** *i* **from** 1 **by** 1 **to** n
> **while** *true* **do**
> $Qbar[1, 1, i] := Q11 \cdot (\cos(\text{theta}[i, 1]))^4 + Q22 \cdot (\sin(\text{theta}[i, 1]))^4 + 2 \cdot (Q12 + 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2 :$
> $Qbar[1, 2, i] := (Q11 + Q22 - 4 \cdot Q66) \cdot (\sin(\text{theta}[i, 1]))^2 \cdot (\cos(\text{theta}[i, 1]))^2 + Q12 \cdot ((\cos(\text{theta}[i, 1]))^4 + (\sin(\text{theta}[i, 1]))^4) :$
> $Qbar[1, 3, i] := (Q11 - Q12 - 2 \cdot Q66) \cdot (\sin(\text{theta}[i, 1])) \cdot (\cos(\text{theta}[i, 1]))^3 - (Q22 - Q12 - 2 \cdot Q66) \cdot (\sin(\text{theta}[i, 1]))^3 \cdot \cos(\text{theta}[i, 1]) :$

$Qbar[2,2,i] := Q11\cdot(\sin(\text{theta}[i,1]))^4 + Q22\cdot(\cos(\text{theta}[i,1]))^4 + 2\cdot(Q12+2\cdot Q66)\cdot(\cos(\text{theta}[i,1]))^2\cdot(\sin(\text{theta}[i,1]))^2 :$

$Qbar[2,3,i] := (Q11-Q12-2\cdot Q66)\cdot(\cos(\text{theta}[i,1]))\cdot(\sin(\text{theta}[i,1]))^3 - (Q22-Q12-2\cdot Q66)\cdot(\cos(\text{theta}[i,1]))^3\cdot\sin(\text{theta}[i,1]) :$

$Qbar[3,3,i] := (Q11+Q22-2\cdot Q12-2\cdot Q66)\cdot(\cos(\text{theta}[i,1]))^2\cdot(\sin(\text{theta}[i,1]))^2 + Q66\cdot\left((\cos(\text{theta}[i,1]))^4 + (\sin(\text{theta}[i,1]))^4\right) :$

$Qbar[2,1,i] := Qbar[1,2,i] :$

$Qbar[3,1,i] := Qbar[1,3,i] :$

$Qbar[3,2,i] := Qbar[2,3,i] :$

end do:

```
> # Calculate elements of thermal expansion coefficient vector for each angle lamina
  # Unit = degrees celsius
```

> **for** *i* **from** 1 **by** 1 **to** *n*
> **while** *true* **do**

$\text{alpha}[1,1,i] := alpha1\cdot(\cos(\text{theta}[i,1]))^2 + alpha2\cdot(\sin(\text{theta}[i,1]))^2 :$

$\text{alpha}[2,1,i] := alpha1\cdot(\sin(\text{theta}[i,1]))^2 + alpha2\cdot(\cos(\text{theta}[i,1]))^2 :$

$\text{alpha}[3,1,i] := 2\cdot(alpha1-alpha2)\cdot\sin(\text{theta}[i,1])\cdot\cos(\text{theta}[i,1]) :$

end do:

```
> # Calculate elements of extensional stiffness matrix [A], coupling stiffness matrix [B], and bending stiffness matrix [Dm]
  # Units: [A]--> MPa.m; [B]--> MPa.m^2; [Dm]--> MPa.m^3
```

> **for** *i* **from** 1 **by** 1 **to** 3
> **while** *true* **do**
> **for** *j* **from** 1 **by** 1 **to** 3
> **while** *true* **do**

$A[i,j]=0 :$

$B[i,j] := 0 :$

$Dm[i,j] := 0 :$

for k **from** 1 **by** 1 **to** n
while *true* **do**

$A[i,j] := A[i,j] + Qbar[i,j,k]\cdot(h[k+1,1]-h[k,1]) :$

$B[i,j] := B[i,j] + \frac{1}{2}\cdot Qbar[i,j,k]\cdot\left(h[k+1,1]^2 - h[k,1]^2\right) :$

$Dm[i,j] := Dm[i,j] + \frac{1}{3}\cdot Qbar[i,j,k]\cdot\left(h[k+1,1]^3 - h[k,1]^3\right) :$

end do:
end do:
end do:

> *evalf*(*A*);

$$\begin{bmatrix} 1869.842182 & 43.45386666 & 0. \\ 43.45386666 & 1012.517281 & 0. \\ 0. & 0. & 107.5500000 \end{bmatrix} \quad (4)$$

> evalf(B);

(5)

$$\begin{bmatrix} 0. & 0. & 0. \\ 0. & 0. & 0. \\ 0. & 0. & 0. \end{bmatrix} \tag{5}$$

```
> evalf( Dm );
```

$$\begin{bmatrix} 0.04934828925 & 0.0008147599997 & 0. \\ 0.0008147599997 & 0.004695950685 & 0. \\ 0. & 0. & 0.002016562500 \end{bmatrix} \tag{6}$$

```
> # Form fictitious thermal force [NT] and moment [MT]vectors
  # [NT] in MPa.m; [MT] in MPa.m^2
> for i from 1 by 1 to 3
   while true do
    NT[i, 1] = 0 :
    MT[i, 1] := 0 :
    for k from 1 by 1 to n
    while true do
   NT[i, 1] := NT[i, 1] + (Qbar[i, 1, k]·alpha[1, 1, k] + Qbar[i, 2, k]·alpha[2, 1, k] + Qbar[i,
       3, k]·alpha[3, 1, k]) · (h[k + 1, 1] − h[k, 1]) ·delta_T :
    MT[i, 1] := MT[i, 1] + 1/2 · (Qbar[i, 1, k]·alpha[1, 1, k] + Qbar[i, 2, k]·alpha[2, 1, k]
        + Qbar[i, 3, k]·alpha[3, 1, k]) · ((h[k + 1, 1])^2 − (h[k, 1])^2) ·delta_T :
   end do:
   end do:
> NT;
```

$$\begin{bmatrix} -0.1389302083 \\ -0.2004412660 \\ 0. \end{bmatrix} \tag{7}$$

```
> MT;
```

$$\begin{bmatrix} 0. \\ 0. \\ 0. \end{bmatrix} \tag{8}$$

```
> # Compute strains
> Eps := Multiply( MatrixInverse(A), NT );
```

$$Eps := \begin{bmatrix} -0.0000697695513817212622 \\ -0.000194969027118590628 \\ 0. \end{bmatrix} \tag{9}$$

```
> # Global stresses in the 0-degree ply (n=1)
> sigmax1 := Qbar[1, 1, 1]·(Eps[1, 1] − alpha[1, 1, 1]·delta_T) + Qbar[1, 2, 1]·(Eps[2, 1]
      − alpha[2, 1, 1]·delta_T) + Qbar[1, 3, 1]·(Eps[3, 1] − alpha[3, 1, 1]·delta_T);
  sigmay1 := Qbar[2, 1, 1]·(Eps[1, 1] − alpha[1, 1, 1]·delta_T) + Qbar[2, 2, 1]·(Eps[2, 1]
      − alpha[2, 1, 1]·delta_T) + Qbar[2, 3, 1]·(Eps[3, 1] − alpha[3, 1, 1]·delta_T);
  tauxy1 := Qbar[3, 1, 1]·(Eps[1, 1] − alpha[1, 1, 1]·delta_T) + Qbar[3, 2, 1]·(Eps[2, 1]
```

− alpha[2, 1, 1]·delta_T) + Qbar[3, 3, 1]·(Eps[3, 1] − alpha[3, 1, 1]·delta_T);

$$sigmax1 := -8.088415421$$
$$sigmay1 := 15.24419063$$
$$tauxy1 := 0. \quad \textbf{(10)}$$

```
> # Global stresses in the 90-degree ply (n=2)
```

> sigmax2 := Qbar[1, 1, 2]·(Eps[1, 1] − alpha[1, 1, 2]·delta_T) + Qbar[1, 2, 2]·(Eps[2, 1] − alpha[2, 1, 2]·delta_T) + Qbar[1, 3, 2]·(Eps[3, 1] − alpha[3, 1, 2]·delta_T);
sigmay2 := Qbar[2, 1, 2]·(Eps[1, 1] − alpha[1, 1, 2]·delta_T) + Qbar[2, 2, 2]·(Eps[2, 1] − alpha[2, 1, 2]·delta_T) + Qbar[2, 3, 2]·(Eps[3, 1] − alpha[3, 1, 2]·delta_T);
tauxy2 := Qbar[3, 1, 2]·(Eps[1, 1] − alpha[1, 1, 2]·delta_T) + Qbar[3, 2, 2]·(Eps[2, 1] − alpha[2, 1, 2]·delta_T) + Qbar[3, 3, 2]·(Eps[3, 1] − alpha[3, 1, 2]·delta_T);

$$sigmax2 := 16.17683086$$
$$sigmay2 := -30.48838126$$
$$tauxy2 := 0. \quad \textbf{(11)}$$

```
> # Global stresses in the 0-degree ply (n=3)
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> sigmax3 := Qbar[1, 1, 3]·(Eps[1, 1] − alpha[1, 1, 3]·delta_T) + Qbar[1, 2, 3]·(Eps[2, 1] − alpha[2, 1, 3]·delta_T) + Qbar[1, 3, 3]·(Eps[3, 1] − alpha[3, 1, 3]·delta_T);
sigmay3 := Qbar[2, 1, 3]·(Eps[1, 1] − alpha[1, 1, 3]·delta_T) + Qbar[2, 2, 3]·(Eps[2, 1] − alpha[2, 1, 3]·delta_T) + Qbar[2, 3, 3]·(Eps[3, 1] − alpha[3, 1, 3]·delta_T);
tauxy3 := Qbar[3, 1, 3]·(Eps[1, 1] − alpha[1, 1, 3]·delta_T) + Qbar[3, 2, 3]·(Eps[2, 1] − alpha[2, 1, 3]·delta_T) + Qbar[3, 3, 3]·(Eps[3, 1] − alpha[3, 1, 3]·delta_T);

$$sigmax3 := -8.088415421$$
$$sigmay3 := 15.24419063$$
$$tauxy3 := 0. \quad \textbf{(12)}$$

```
> # Local stresses in the 0-degree ply (n=1)
```

> sigma11 := (cos(theta[1, 1]))^2·sigmax1 + (sin(theta[1, 1]))^2·sigmay1 + 2·sin(theta[1, 1])·cos(theta[1, 1])·tauxy1;
sigma21 := (sin(theta[1, 1]))^2·sigmax1 + (cos(theta[1, 1]))^2·sigmay1-2·sin(theta[1, 1])·cos(theta[1, 1])·tauxy1;
tau121 := -sin(theta[1, 1])·cos(theta[1, 1])·sigmax1 + sin(theta[1, 1])·cos(theta[1, 1])·sigmay1 + ((cos(theta[1, 1]))^2 − (sin(theta[1, 1]))^2)·cos(theta[1, 1])·tauxy1;

$$\sigma 11 := -8.088415421$$
$$\sigma 21 := 15.24419063$$
$$\tau 121 := 0. \quad \textbf{(13)}$$

```
> # Local stresses in the 90-degree ply (n=2)
```

> sigma12 := (cos(theta[2, 1]))^2·sigmax2 + (sin(theta[2, 1]))^2·sigmay2 + 2·sin(theta[2, 1])·cos(theta[2, 1])·tauxy2;
sigma22 := (sin(theta[2, 1]))^2·sigmax2 + (cos(theta[2, 1]))^2·sigmay2 − 2·sin(theta[2, 1])·cos(theta[2, 1])·tauxy2;
tau122 := -sin(theta[2, 1])·cos(theta[2, 1])·sigmax2 + sin(theta[2, 1])·cos(theta[2, 1])·sigmay2 + ((cos(theta[2, 1]))^2 − (sin(theta[2, 1]))^2)·cos(theta[2, 1])·tauxy2;

$$\sigma 12 := -30.48838126$$

$$\sigma 22 := 16.17683086$$

$$\tau 122 := 0. \tag{14}$$

```
> # Local stresses in the 0-degree ply (n=3)
```

> $sigma13 := (\cos(\mathrm{theta}[3, 1]))^2 \cdot sigmax3 + (\sin(\mathrm{theta}[3, 1]))^2 \cdot sigmay3 + 2 \cdot \sin(\mathrm{theta}[3, 1]) \cdot \cos(\mathrm{theta}[3, 1]) \cdot tauxy3;$
> $sigma23 := (\sin(\mathrm{theta}[3, 1]))^2 \cdot sigmax3 + (\cos(\mathrm{theta}[3, 1]))^2 \cdot \mathrm{sigmay3} - 2 \cdot \sin(\mathrm{theta}[3, 1]) \cdot \cos(\mathrm{theta}[3, 1]) \cdot tauxy3;$
> $tau123 := -\sin(\mathrm{theta}[3, 1]) \cdot \cos(\mathrm{theta}[3, 1]) \cdot \mathrm{sigmax3} + \sin(\mathrm{theta}[3, 1]) \cdot \cos(\mathrm{theta}[3, 1]) \cdot \mathrm{sigmay3} + \left((\cos(\mathrm{theta}[3, 1]))^2 - (\sin(\mathrm{theta}[3, 1]))^2\right) \cdot \cos(\mathrm{theta}[3, 1]) \cdot tauxy3;$

$$\sigma 13 := -8.088415421$$

$$\sigma 23 := 15.24419063$$

$$\tau 123 := 0. \tag{15}$$

```
> # Update local stresses considering mechanical loading
> # Local stresses in the 0-degree ply (n=1)
```

> $sigma11 := -8.088415421 + 97.26393020\ Nx;$
> $sigma21 := 15.24419063 + 1.313132561\ Nx;$
> $tau121 := 0;$

$$\sigma 11 := -8.088415421 + 97.26393020\ Nx$$

$$\sigma 21 := 15.24419063 + 1.313132561\ Nx$$

$$\tau 121 := 0 \tag{16}$$

```
> # Local stresses in the 90-degree ply (n=2)
```

> $sigma12 := -30.48838126 - 2.626265123\ Nx;$
> $sigma22 := 16.17683086 + 5.472139550\ Nx;$
> $tau122 := 0;$

$$\sigma 12 := -30.48838126 - 2.626265123\ Nx$$

$$\sigma 22 := 16.17683086 + 5.472139550\ Nx$$

$$\tau 122 := 0 \tag{17}$$

```
> # Input strength values for graphite/epoxy lamina
```

> $sigma1Tult := 1500 :$
> $sigma1Cult := 1500 :$
> $sigma2Tult := 40 :$
> $sigma2Cult := 246 :$
> $tau12ult := 68 :$

```
> # Determine coefficients of Tsai-Wu failure criterion
```

> $H1 := \frac{1}{sigma1Tult} - \frac{1}{sigma1Cult} :$
> $H2 := \frac{1}{sigma2Tult} - \frac{1}{sigma2Cult} :$
> $H11 := \frac{1}{sigma1Tult \cdot sigma1Cult} :$
> $H22 := \frac{1}{sigma2Tult \cdot sigma2Cult} :$

$$H12 := -\frac{1}{2}\cdot \mathrm{sqrt}\left(\frac{1}{sigma1Tult\cdot sigma1Cult\cdot sigma2Tult\cdot sigma2Cult}\right):$$

```
> # Apply Tsai-Wu failure criterion for the 0-degree ply (n=1)
```

$$> eq1 := H1\cdot sigma11 + H2\cdot sigma21 + H11\cdot \sigma 11^2 + H22\cdot \sigma 21^2 + H66\cdot \tau 121^2 + 2\cdot H12\cdot sigma11 \cdot sigma21;$$

$solve(eq1 = 1, Nx);$

$$eq1 := 0.3191365112 + 0.02749037679\, Nx + \frac{1}{2250000}\,(-8.088415421 + 97.26393020\, Nx)^2 + \frac{1}{9840}\,(15.24419063 + 1.313132561\, Nx)^2 - \frac{1}{3690000}\sqrt{615}\,(-8.088415421 + 97.26393020\, Nx)\,(15.24419063 + 1.313132561\, Nx)$$

$$10.99660719,\ -16.95051615 \qquad \textbf{(18)}$$

```
> # Apply Tsai-Wu failure criterion for the 90-degree ply (n=1)
```

$$> eq2 := H1\cdot \sigma 12 + H2\cdot \sigma 22 + H11\cdot \sigma 12^2 + H22\cdot \sigma 22^2 + H66\cdot \tau 122^2 + 2\cdot H12\cdot \sigma 12\cdot \sigma 22;$$

$solve(eq2 = 1, Nx);$

$$eq2 := 0.3386612965 + 0.1145590190\, Nx + \frac{1}{2250000}\,(-30.48838126 - 2.626265123\, Nx)^2 + \frac{1}{9840}\,(16.17683086 + 5.472139550\, Nx)^2 - \frac{1}{3690000}\sqrt{615}\,(-30.48838126 - 2.626265123\, Nx)\,(16.17683086 + 5.472139550\, Nx)$$

$$4.278762324,\ -46.92559321 \qquad \textbf{(19)}$$

>