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> # Prof. Dr. Serkan Dağ
  # ME 451 Introduction to Composite Structures
> # File 7.2
  # Example on failure loads of a laminate (cont'd)
> restart :
  with(LinearAlgebra) :
> # Enter the number of plies
> n := 3 :
> # Define extensional, coupling, and bending stiffness matrices
> A := Matrix(3) :
  B := Matrix(3) :
  Dm := Matrix(3) :
> # Define ply surface coordinate vector in meters
```

$$h := \begin{bmatrix} -\frac{7.5}{1000} \\ -\frac{2.5}{1000} \\ \frac{2.5}{1000} \\ \frac{7.5}{1000} \end{bmatrix} :$$

```
> # Define ply angle vector in radians
```

$$\text{theta} := \begin{bmatrix} 0 \\ \frac{\text{Pi}}{2} \\ 0 \end{bmatrix} :$$

```
> # Define Qbar array
  Qbar := Array(1..3, 1..3, 1..n) :
  ArrayNumElems(Qbar);
```

27 **(1)**

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> # Enter properties of the unidirectional lamina
  # From Table 2.1 for graphite/epoxy (unit = MPa)
> E1 := 181000 :
  E2 := 10300 :
  nu12 := 0.28 :
  G12 := 7170 :
> # Calculate elements of the compliance matrix for the
  unidirectional lamina
```

$$S11 := \frac{1}{E1} :$$

$$S12 := -\frac{nu12}{E1} :$$

$$S22 := \frac{1}{E2} :$$

$S66 := \frac{1}{G12}$:

```
> # Calculate elements of the reduced stiffness matrix for the unidirectional lamina
```

$> Q11 := \frac{S22}{S11 \cdot S22 - S12^2}$:

$Q22 := \frac{S11}{S11 \cdot S22 - S12^2}$:

$Q12 := -\frac{S12}{S11 \cdot S22 - S12^2}$:

$Q66 := \frac{1}{S66}$:

```
> # Calculate elements of transformed reduced stiffness matrix for each angle lamina
# Unit = MPa
```

> **for** *i* **from** 1 **by** 1 **to** n
while *true* **do**

$Qbar[1, 1, i] := Q11 \cdot (\cos(\text{theta}[i, 1]))^4 + Q22 \cdot (\sin(\text{theta}[i, 1]))^4 + 2 \cdot (Q12 + 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2$:

$Qbar[1, 2, i] := (Q11 + Q22 - 4 \cdot Q66) \cdot (\sin(\text{theta}[i, 1]))^2 \cdot (\cos(\text{theta}[i, 1]))^2 + Q12 \cdot \left((\cos(\text{theta}[i, 1]))^4 + (\sin(\text{theta}[i, 1]))^4 \right)$:

$Qbar[1, 3, i] := (Q11 - Q12 - 2 \cdot Q66) \cdot (\sin(\text{theta}[i, 1])) \cdot (\cos(\text{theta}[i, 1]))^3 - (Q22 - Q12 - 2 \cdot Q66) \cdot (\sin(\text{theta}[i, 1]))^3 \cdot \cos(\text{theta}[i, 1])$:

$Qbar[2, 2, i] := Q11 \cdot (\sin(\text{theta}[i, 1]))^4 + Q22 \cdot (\cos(\text{theta}[i, 1]))^4 + 2 \cdot (Q12 + 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2$:

$Qbar[2, 3, i] := (Q11 - Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1])) \cdot (\sin(\text{theta}[i, 1]))^3 - (Q22 - Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^3 \cdot \sin(\text{theta}[i, 1])$:

$Qbar[3, 3, i] := (Q11 + Q22 - 2 \cdot Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2 + Q66 \cdot \left((\cos(\text{theta}[i, 1]))^4 + (\sin(\text{theta}[i, 1]))^4 \right)$:

$Qbar[2, 1, i] := Qbar[1, 2, i]$:
$Qbar[3, 1, i] := Qbar[1, 3, i]$:
$Qbar[3, 2, i] := Qbar[2, 3, i]$:
end do:

```
> # Calculate elements of extensional stiffness matrix [A], coupling stiffness matrix [B], and bending stiffness matrix [Dm]
# Units: [A]--> MPa.m; [B]--> MPa.m^2; [Dm]--> MPa.m^3
> # Do not consider contributions from the 90-degree ply
```

> **for** *i* **from** 1 **by** 1 **to** 3
while *true* **do**
for *j* **from** 1 **by** 1 **to** 3
while *true* **do**
$A[i, j] = 0$:
$B[i, j] := 0$:
$Dm[i, j] := 0$:
for k **from** 1 **by** 1 **to** n
while *true* **do**
if $k = 2$ **then**

```
        next
        end if:
        A[i, j] := A[i, j] + Qbar[i, j, k]·(h[k + 1, 1] − h[k, 1]) :
        B[i, j] := B[i, j] + 1/2·Qbar[i, j, k]·(h[k + 1, 1]^2 − h[k, 1]^2) :
      Dm[i, j] := Dm[i, j] + 1/3·Qbar[i, j, k]·(h[k + 1, 1]^3 − h[k, 1]^3) :
      end do:
    end do:
    end do:
> evalf( A );
```

$$\begin{bmatrix} 1818.111388 & 28.96924444 & 0. \\ 28.96924444 & 103.4615873 & 0. \\ 0. & 0. & 71.70000000 \end{bmatrix} \quad (2)$$

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> evalf( B );
```

$$\begin{bmatrix} 0. & 0. & 0. \\ 0. & 0. & 0. \\ 0. & 0. & 0. \end{bmatrix} \quad (3)$$

```
> evalf( Dm );
```

$$\begin{bmatrix} 0.04924051676 & 0.0007845837034 & 0. \\ 0.0007845837034 & 0.002802084656 & 0. \\ 0. & 0. & 0.001941875000 \end{bmatrix} \quad (4)$$

```
> # Compute Astar matrix in 1/(MPa.m)
> Astar := MatrixInverse( A );
```

$$Astar := \begin{bmatrix} 5.525 \times 10^{-4} & -1.547 \times 10^{-4} & 0.000 \times 10^{0} \\ -1.547 \times 10^{-4} & 9.709 \times 10^{-3} & 0.000 \times 10^{0} \\ 0.000 \times 10^{0} & 0.000 \times 10^{0} & 1.395 \times 10^{-2} \end{bmatrix} \quad (5)$$

```
> # Define load vector
```

$$> N := \begin{bmatrix} Nx \\ 0 \\ 0 \end{bmatrix};$$

$$N := \begin{bmatrix} Nx \\ 0 \\ 0 \end{bmatrix} \quad (6)$$

```
> # Find strains (Note that strains in this case are equal to mid-
plane strains)
> Eps := Multiply( Astar, N );
```

$$Eps := \begin{bmatrix} 5.5249 \times 10^{-4}\, Nx \\ -1.5470 \times 10^{-4}\, Nx \\ 0.0000 \times 10^{0} \end{bmatrix} \quad (7)$$

```
> # Calculate global stresses in the 0-degree ply (n=1)
```

$sigmax1 := Qbar[1,1,1] \cdot Eps[1,1] + Qbar[1,2,1] \cdot Eps[2,1] + Qbar[1,3,1] \cdot Eps[3,1];$
$sigmay1 := Qbar[2,1,1] \cdot Eps[1,1] + Qbar[2,2,1] \cdot Eps[2,1] + Qbar[2,3,1] \cdot Eps[3,1];$
$tauxy1 := Qbar[3,1,1] \cdot Eps[1,1] + Qbar[3,2,1] \cdot Eps[2,1] + Qbar[3,3,1] \cdot Eps[3,1];$

$$sigmax1 := 99.99999999\, Nx$$
$$sigmay1 := 0.$$
$$tauxy1 := 0. \quad (8)$$

```
> # Calculate global stresses in the 0-degree ply (n=3)
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$sigmax3 := Qbar[1,1,3] \cdot Eps[1,1] + Qbar[1,2,3] \cdot Eps[2,1] + Qbar[1,3,3] \cdot Eps[3,1];$
$sigmay3 := Qbar[2,1,3] \cdot Eps[1,1] + Qbar[2,2,3] \cdot Eps[2,1] + Qbar[2,3,3] \cdot Eps[3,1];$
$tauxy3 := Qbar[3,1,3] \cdot Eps[1,1] + Qbar[3,2,3] \cdot Eps[2,1] + Qbar[3,3,3] \cdot Eps[3,1];$

$$sigmax3 := 99.99999999\, Nx$$
$$sigmay3 := 0.$$
$$tauxy3 := 0. \quad (9)$$

```
> # Calculate local stresses in the 0-degree ply (n=1)
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$sigma11 := (\cos(\theta[1,1]))^2 \cdot sigmax1 + (\sin(\theta[1,1]))^2 \cdot sigmay1 + 2 \cdot \sin(\theta[1,1]) \cdot \cos(\theta[1,1]) \cdot tauxy1;$

$sigma21 := (\sin(\theta[1,1]))^2 \cdot sigmax1 + (\cos(\theta[1,1]))^2 \cdot \mathrm{sigmay1} - 2 \cdot \sin(\theta[1,1]) \cdot \cos(\theta[1,1]) \cdot tauxy1;$

$tau121 := -\sin(\theta[1,1]) \cdot \cos(\theta[1,1]) \cdot \mathrm{sigmax1} + \sin(\theta[1,1]) \cdot \cos(\theta[1,1]) \cdot \mathrm{sigmay1} + \left((\cos(\theta[1,1]))^2 - (\sin(\theta[1,1]))^2\right) \cdot \cos(\theta[1,1]) \cdot tauxy1;$

$$\sigma 11 := 99.99999999\, Nx$$
$$\sigma 21 := 0.$$
$$\tau 121 := 0. \quad (10)$$

```
> # Calculate local stresses in the 0-degree ply (n=3)
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$sigma13 := (\cos(\theta[3,1]))^2 \cdot sigmax3 + (\sin(\theta[3,1]))^2 \cdot sigmay3 + 2 \cdot \sin(\theta[3,1]) \cdot \cos(\theta[3,1]) \cdot tauxy3;$

$sigma23 := (\sin(\theta[3,1]))^2 \cdot sigmax3 + (\cos(\theta[3,1]))^2 \cdot \mathrm{sigmay3} - 2 \cdot \sin(\theta[3,1]) \cdot \cos(\theta[3,1]) \cdot tauxy3;$

$tau123 := -\sin(\theta[3,1]) \cdot \cos(\theta[3,1]) \cdot \mathrm{sigmax3} + \sin(\theta[3,1]) \cdot \cos(\theta[3,1]) \cdot \mathrm{sigmay3} + \left((\cos(\theta[3,1]))^2 - (\sin(\theta[3,1]))^2\right) \cdot \cos(\theta[3,1]) \cdot tauxy3;$

$$\sigma 13 := 99.99999999\, Nx$$
$$\sigma 23 := 0.$$
$$\tau 123 := 0. \quad (11)$$

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> # Input strength values for graphite/epoxy lamina
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$sigma1Tult := 1500 :$
$sigma1Cult := 1500 :$
$sigma2Tult := 40 :$
$sigma2Cult := 246 :$
$tau12ult := 68 :$

```
> # Determine coefficients of Tsai-Wu failure criterion
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$$H1 := \frac{1}{sigma1Tult} - \frac{1}{sigma1Cult}:$$

$$H2 := \frac{1}{sigma2Tult} - \frac{1}{sigma2Cult}:$$

$$H11 := \frac{1}{sigma1Tult \cdot sigma1Cult}:$$

$$H22 := \frac{1}{sigma2Tult \cdot sigma2Cult}:$$

$$H12 := -\frac{1}{2} \cdot \mathrm{sqrt}\left(\frac{1}{sigma1Tult \cdot sigma1Cult \cdot sigma2Tult \cdot sigma2Cult}\right):$$

```
> # Apply Tsai-Wu failure criterion for the 0-degree ply (n=1)
```

$$eq1 := H1 \cdot sigma11 + H2 \cdot sigma21 + H11 \cdot \sigma 11^2 + H22 \cdot \sigma 21^2 + H66 \cdot \tau 121^2 + 2 \cdot H12 \cdot sigma11 \cdot sigma21;$$

$$solve(eq1 = 1, Nx);$$

$$eq1 := 0.004444444444\ Nx^2$$

$$15.00000000,\ -15.00000000 \tag{12}$$

```
> # Calculate mid-plane strain at this level of loading
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$$Nx := 15.00000000:$$

$$epsx0 := Eps[1, 1];$$

$$epsx0 := 0.008287292820 \tag{13}$$

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