```
> # Prof. Dr. Serkan Dağ
  # ME 451 Introduction to Composite Structures
> # File 6.5
  # Example on thermal expansion coefficients of a laminate
> restart :
  with(LinearAlgebra) :
> # Enter the number of plies
> n := 3 :
> # Define extensional stiffness matrix
> A := Matrix(3) :
> # Define fictitious thermal force [NT] vector
> NT := Matrix(3, 1) :
> # Define ply surface coordinate vector in meters
```

$$h := \begin{bmatrix} -\frac{7.5}{1000} \\ -\frac{2.5}{1000} \\ \frac{2.5}{1000} \\ \frac{7.5}{1000} \end{bmatrix};$$

$$h := \begin{bmatrix} -0.007500000000 \\ -0.002500000000 \\ 0.002500000000 \\ 0.007500000000 \end{bmatrix} \quad (1)$$

```
> # Define ply angle vector in radians
```

$$\text{theta} := \begin{bmatrix} 0 \\ \frac{Pi}{2} \\ 0 \end{bmatrix};$$

$$\theta := \begin{bmatrix} 0 \\ \frac{1}{2}\pi \\ 0 \end{bmatrix} \quad (2)$$

```
> # Enter uniform temperature change delta_T in degrees celsius
> delta_T := dT :
> # Define Qbar array
  Qbar := Array(1..3, 1..3, 1..n) :
  ArrayNumElems(Qbar);
```

27 (3)

```
> # Define thermal expansion coefficient array
```

> alpha := *Array*(1..3, 1..1, 1..n) :
> *ArrayNumElems*(alpha);

$$9 \quad \textbf{(4)}$$

```
> # Enter mechanical properties of the unidirectional
graphite/epoxy lamina
# From Table 2.1 for graphite/epoxy (unit = MPa)
```

> $E1 := 181000$:
> $E2 := 10300$:
> $nu12 := 0.28$:
> $G12 := 7170$:

```
> # Enter thermal expansion coefficients of the unidirectional
graphite/epoxy lamina
# From Table 2.1 for graphite/epoxy (unit = 1/ (degrees celsius)
)
```

> $alpha1 := 0.02 \cdot (10)^{-6}$:
> $alpha2 := 22.5 \cdot (10)^{-6}$:

```
> # Calculate elements of the compliance matrix for the
unidirectional lamina
```

> $S11 := \frac{1}{E1}$:
> $S12 := -\frac{nu12}{E1}$:
> $S22 := \frac{1}{E2}$:
> $S66 := \frac{1}{G12}$:

```
> # Calculate elements of the reduced stiffness matrix for the
unidirectional lamina
```

> $Q11 := \frac{S22}{S11 \cdot S22 - S12^2}$:
> $Q22 := \frac{S11}{S11 \cdot S22 - S12^2}$:
> $Q12 := -\frac{S12}{S11 \cdot S22 - S12^2}$:
> $Q66 := \frac{1}{S66}$:

```
> # Calculate elements of transformed reduced stiffness matrix for
each angle lamina
# Unit = MPa
```

> **for** *i* **from** 1 **by** 1 **to** n
> **while** *true* **do**
> $Qbar[1, 1, i] := Q11 \cdot (\cos(\text{theta}[i, 1]))^4 + Q22 \cdot (\sin(\text{theta}[i, 1]))^4 + 2 \cdot (Q12 + 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2$:
> $Qbar[1, 2, i] := (Q11 + Q22 - 4 \cdot Q66) \cdot (\sin(\text{theta}[i, 1]))^2 \cdot (\cos(\text{theta}[i, 1]))^2 + Q12 \cdot ((\cos(\text{theta}[i, 1]))^4 + (\sin(\text{theta}[i, 1]))^4)$:
> $Qbar[1, 3, i] := (Q11 - Q12 - 2 \cdot Q66) \cdot (\sin(\text{theta}[i, 1])) \cdot (\cos(\text{theta}[i, 1]))^3 - (Q22 - Q12 - 2 \cdot Q66) \cdot (\sin(\text{theta}[i, 1]))^3 \cdot \cos(\text{theta}[i, 1])$:

```
Qbar[2, 2, i] := Q11·(sin(theta[i, 1]))^4 + Q22·(cos(theta[i, 1]))^4 + 2·(Q12 + 2·Q66)
    ·(cos(theta[i, 1]))^2·(sin(theta[i, 1]))^2 :
Qbar[2, 3, i] := (Q11 − Q12 − 2·Q66)·(cos(theta[i, 1]))·(sin(theta[i, 1]))^3 − (Q22 − Q12
    − 2·Q66)·(cos(theta[i, 1]))^3·sin(theta[i, 1]) :
Qbar[3, 3, i] := (Q11 + Q22 − 2·Q12 − 2·Q66)·(cos(theta[i, 1]))^2·(sin(theta[i, 1]))^2
    + Q66·((cos(theta[i, 1]))^4 + (sin(theta[i, 1]))^4) :
Qbar[2, 1, i] := Qbar[1, 2, i] :
Qbar[3, 1, i] := Qbar[1, 3, i] :
Qbar[3, 2, i] := Qbar[2, 3, i] :
end do:
```

```
> # Calculate elements of thermal expansion coefficient vector for
  each angle lamina
  # Unit = degrees celsius
```

```
> for i from 1 by 1 to n
  while true do
  alpha[1, 1, i] := alpha1·(cos(theta[i, 1]))^2 + alpha2·(sin(theta[i, 1]))^2 :
  alpha[2, 1, i] := alpha1·(sin(theta[i, 1]))^2 + alpha2·(cos(theta[i, 1]))^2 :
  alpha[3, 1, i] := 2·(alpha1 − alpha2)·sin(theta[i, 1])·cos(theta[i, 1]) :
  end do:
```

```
> # Calculate elements of extensional stiffness matrix [A]
  # Unit: [A]--> MPa.m
```

```
> for i from 1 by 1 to 3
  while true do
  for j from 1 by 1 to 3
  while true do
   A[i, j] = 0 :
   B[i, j] := 0 :
   Dm[i, j] := 0 :
   for k from 1 by 1 to n
   while true do
   A[i, j] := A[i, j] + Qbar[i, j, k]·(h[k + 1, 1] − h[k, 1]) :
   B[i, j] := B[i, j] + 1/2·Qbar[i, j, k]·(h[k + 1, 1]^2 − h[k, 1]^2) :
  Dm[i, j] := Dm[i, j] + 1/3·Qbar[i, j, k]·(h[k + 1, 1]^3 − h[k, 1]^3) :
   end do:
  end do:
  end do:
```

```
> evalf( A );
```

$$\begin{bmatrix} 1869.842182 & 43.45386666 & 0. \\ 43.45386666 & 1012.517281 & 0. \\ 0. & 0. & 107.5500000 \end{bmatrix} \qquad (5)$$

```
> # Form fictitious thermal force [NT] vector
  # [NT] in MPa.m
```

```
> for i from 1 by 1 to 3
  while true do
   NT[i, 1] = 0 :
   for k from 1 by 1 to n
```

```
    while true do
  NT[i, 1] := NT[i, 1] + (Qbar[i, 1, k]·alpha[1, 1, k] + Qbar[i, 2, k]·alpha[2, 1, k] + Qbar[i,
      3, k]·alpha[3, 1, k]) · (h[k + 1, 1] − h[k, 1]) ·delta_T :
   end do:
  end do:
> NT;
```

$$\begin{bmatrix} 0.001852402778\ dT \\ 0.002672550214\ dT \\ 0. \end{bmatrix} \quad (6)$$

```
> # Extensional compliance matrix
> Astar := MatrixInverse( A );
```

Astar := (7)

[[0.0005353384149899040 98, -0.0000229749403190156898, -0.],
[-0.0000229749403190156898, 0.0009886234721885639 64, -0.],
[0., 0., 0.009298000929800009354]]

```
> # Strain vector
> Res := Multiply( Astar, NT );
```

$$Res := \begin{bmatrix} 9.302606854\ 10^{-7}\ dT \\ 0.000002599587029\ dT \\ 0. \end{bmatrix} \quad (8)$$

```
>
```