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> # Prof. Dr. Serkan Dağ
  # ME 451 Introduction to Composite Structures
> # File 6.3
  # Example on computation of engineering constants of a symmetric
  laminate
> restart :
  with(LinearAlgebra) :
> # Enter the number of plies
> n := 3 :
> # Define extensional, coupling, and bending stiffness matrices
> A := Matrix(3) :
  B := Matrix(3) :
  Dm := Matrix(3) :
> # Define ply surface coordinate vector in meters
```

$$> h := \begin{bmatrix} -\frac{7.5}{1000} \\ -\frac{2.5}{1000} \\ \frac{2.5}{1000} \\ \frac{7.5}{1000} \end{bmatrix} :$$

```
> # Define ply angle vector in radians
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$$> \text{theta} := \begin{bmatrix} 0 \\ \frac{\text{Pi}}{2} \\ 0 \end{bmatrix} :$$

```
> # Define Qbar array
  Qbar := Array(1..3, 1..3, 1..n) :
  ArrayNumElems(Qbar);
```

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> # Enter properties of the unidirectional lamina
  # From Table 2.1 for graphite/epoxy (unit = MPa)
> E1 := 181000 :
  E2 := 10300 :
  nu12 := 0.28 :
  G12 := 7170 :
> # Calculate elements of the compliance matrix for the
  unidirectional lamina
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$$> S11 := \frac{1}{E1} :$$

$$S12 := -\frac{nu12}{E1} :$$

$S22 := \frac{1}{E2}$:

$S66 := \frac{1}{G12}$:

> # Calculate elements of the reduced stiffness matrix for the unidirectional lamina

> $Q11 := \frac{S22}{S11 \cdot S22 - S12^2}$:

$Q22 := \frac{S11}{S11 \cdot S22 - S12^2}$:

$Q12 := -\frac{S12}{S11 \cdot S22 - S12^2}$:

$Q66 := \frac{1}{S66}$:

> # Calculate elements of transformed reduced stiffness matrix for each angle lamina
> # Unit = MPa

> **for** i **from** 1 **by** 1 **to** n
while *true* **do**

$Qbar[1, 1, i] := Q11 \cdot (\cos(\text{theta}[i, 1]))^4 + Q22 \cdot (\sin(\text{theta}[i, 1]))^4 + 2 \cdot (Q12 + 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2$:

$Qbar[1, 2, i] := (Q11 + Q22 - 4 \cdot Q66) \cdot (\sin(\text{theta}[i, 1]))^2 \cdot (\cos(\text{theta}[i, 1]))^2 + Q12 \cdot \left((\cos(\text{theta}[i, 1]))^4 + (\sin(\text{theta}[i, 1]))^4 \right)$:

$Qbar[1, 3, i] := (Q11 - Q12 - 2 \cdot Q66) \cdot (\sin(\text{theta}[i, 1])) \cdot (\cos(\text{theta}[i, 1]))^3 - (Q22 - Q12 - 2 \cdot Q66) \cdot (\sin(\text{theta}[i, 1]))^3 \cdot \cos(\text{theta}[i, 1])$:

$Qbar[2, 2, i] := Q11 \cdot (\sin(\text{theta}[i, 1]))^4 + Q22 \cdot (\cos(\text{theta}[i, 1]))^4 + 2 \cdot (Q12 + 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2$:

$Qbar[2, 3, i] := (Q11 - Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1])) \cdot (\sin(\text{theta}[i, 1]))^3 - (Q22 - Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^3 \cdot \sin(\text{theta}[i, 1])$:

$Qbar[3, 3, i] := (Q11 + Q22 - 2 \cdot Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2 + Q66 \cdot \left((\cos(\text{theta}[i, 1]))^4 + (\sin(\text{theta}[i, 1]))^4 \right)$:

$Qbar[2, 1, i] := Qbar[1, 2, i]$:
$Qbar[3, 1, i] := Qbar[1, 3, i]$:
$Qbar[3, 2, i] := Qbar[2, 3, i]$:
end do:

> # Calculate elements of extensional stiffness matrix [A], coupling stiffness matrix [B], and bending stiffness matrix [Dm]
> # Units: [A]--> MPa.m; [B]--> MPa.m^2; [Dm]--> MPa.m^3

> **for** i **from** 1 **by** 1 **to** 3
while *true* **do**
for j **from** 1 **by** 1 **to** 3
while *true* **do**
$A[i, j] = 0$:
$B[i, j] := 0$:
$Dm[i, j] := 0$:
for k **from** 1 **by** 1 **to** n
while *true* **do**

$A[i,j] := A[i,j] + Qbar[i,j,k]\cdot(h[k+1,1] - h[k,1])$:

$B[i,j] := B[i,j] + \frac{1}{2}\cdot Qbar[i,j,k]\cdot(h[k+1,1]^2 - h[k,1]^2)$:

$Dm[i,j] := Dm[i,j] + \frac{1}{3}\cdot Qbar[i,j,k]\cdot(h[k+1,1]^3 - h[k,1]^3)$:

end do:
end do:
end do:

> *evalf*(*A*);

$$\begin{bmatrix} 1869.842182 & 43.45386666 & 0. \\ 43.45386666 & 1012.517281 & 0. \\ 0. & 0. & 107.5500000 \end{bmatrix} \quad (2)$$

> evalf(B);

$$\begin{bmatrix} 0. & 0. & 0. \\ 0. & 0. & 0. \\ 0. & 0. & 0. \end{bmatrix} \quad (3)$$

> evalf(Dm);

$$\begin{bmatrix} 0.04934828925 & 0.0008147599997 & 0. \\ 0.0008147599997 & 0.004695950685 & 0. \\ 0. & 0. & 0.002016562500 \end{bmatrix} \quad (4)$$

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> # Total thickness
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> $h := 0.015$:

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> # Astar (in 1/(MPa.m)) and Dstar (in 1/(MPa.m^3)) matrices
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> $Astar := MatrixInverse(A)$:
$Dstar := MatrixInverse(Dm)$:

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> # In-plane engineering constants in GPa
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> $Ex := \frac{1}{Astar[1,1]\cdot h}\cdot\frac{1}{1000}$;
$Ey := \frac{1}{Astar[2,2]\cdot h}\cdot\frac{1}{1000}$;
$Gxy := \frac{1}{Astar[3,3]\cdot h}\cdot\frac{1}{1000}$;
$nuxy := -\frac{Astar[1,2]}{Astar[1,1]}$;
$nuyx := -\frac{Astar[1,2]}{Astar[2,2]}$;

$$Ex := 124.531819125896$$
$$Ey := 67.4338295034442$$
$$Gxy := 7.17000000000000$$
$$nuxy := 0.0429166666835388$$
$$nuyx := 0.0232393231248647 \quad (5)$$

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> # Flexural engineering constants in GPa
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> $Exf := \frac{12}{Dstar[1, 1] \cdot h^3} \cdot \frac{1}{1000};$
$Eyf := \frac{12}{Dstar[2, 2] \cdot h^3} \cdot \frac{1}{1000};$
$Gxyf := \frac{12}{Dstar[3, 3] \cdot h^3} \cdot \frac{1}{1000};$
$nuxyf := -\frac{Dstar[1, 2]}{Dstar[1, 1]};$
$nuyx := -\frac{Dstar[1, 2]}{Dstar[2, 2]};$

$$Exf := 174.957959863389$$
$$Eyf := 16.6488841650510$$
$$Gxyf := 7.17000000000000$$
$$nuxyf := 0.173502673761575$$
$$nuyx := 0.0165104000986215 \quad \mathbf{(6)}$$

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