```
> # Prof. Dr. Serkan Dağ
  # ME 451 Introduction to Composite Structures
> # File 6.1
  # Example on computation of extensional ([A]), coupling ([B]), and bending ([Dm])
  # stiffness matrices for a laminate
```

> *restart* :
> *with*(*LinearAlgebra*) :

```
> # Enter the number of plies
```

> $n := 3$:

```
> # Define extensional, coupling, and bending stiffness matrices
```

> $A := Matrix(3)$:
> $B := Matrix(3)$:
> $Dm := Matrix(3)$:

```
> # Define ply surface coordinate vector in meters
```

> $$h := \begin{bmatrix} -\frac{7.5}{1000} \\ -\frac{2.5}{1000} \\ \frac{2.5}{1000} \\ \frac{7.5}{1000} \end{bmatrix} :$$

```
> # Define ply angle vector in radians
```

> $$\text{theta} := \begin{bmatrix} 0 \\ \frac{\text{Pi}}{6} \\ -\frac{\text{Pi}}{4} \end{bmatrix} :$$

```
> # Define Qbar array
```

> $Qbar := Array(1..3, 1..3, 1..n)$:
> $ArrayNumElems(Qbar)$;

$$27 \quad (1)$$

```
> # Enter properties of the unidirectional lamina
  # From Table 2.1 for graphite/epoxy (unit = MPa)
```

> $E1 := 181000$:
> $E2 := 10300$:
> $nu12 := 0.28$:
> $G12 := 7170$:

```
> # Calculate elements of the compliance matrix for the unidirectional lamina
```

> $$S11 := \frac{1}{E1} :$$

$$S12 := -\frac{nu12}{E1}:$$

$$S22 := \frac{1}{E2}:$$

$$S66 := \frac{1}{G12}:$$

```
> # Calculate elements of the reduced stiffness matrix for the
unidirectional lamina
```

$$> Q11 := \frac{S22}{S11 \cdot S22 - S12^2}:$$

$$Q22 := \frac{S11}{S11 \cdot S22 - S12^2}:$$

$$Q12 := -\frac{S12}{S11 \cdot S22 - S12^2}:$$

$$Q66 := \frac{1}{S66}:$$

```
> # Calculate elements of transformed reduced stiffness matrix for
each angle lamina
# Unit = MPa
```

> **for** i **from** 1 **by** 1 **to** n
while *true* **do**

$$Qbar[1, 1, i] := Q11 \cdot (\cos(\text{theta}[i, 1]))^4 + Q22 \cdot (\sin(\text{theta}[i, 1]))^4 + 2 \cdot (Q12 + 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2:$$

$$Qbar[1, 2, i] := (Q11 + Q22 - 4 \cdot Q66) \cdot (\sin(\text{theta}[i, 1]))^2 \cdot (\cos(\text{theta}[i, 1]))^2 + Q12 \cdot \left((\cos(\text{theta}[i, 1]))^4 + (\sin(\text{theta}[i, 1]))^4\right):$$

$$Qbar[1, 3, i] := (Q11 - Q12 - 2 \cdot Q66) \cdot (\sin(\text{theta}[i, 1])) \cdot (\cos(\text{theta}[i, 1]))^3 - (Q22 - Q12 - 2 \cdot Q66) \cdot (\sin(\text{theta}[i, 1]))^3 \cdot \cos(\text{theta}[i, 1]):$$

$$Qbar[2, 2, i] := Q11 \cdot (\sin(\text{theta}[i, 1]))^4 + Q22 \cdot (\cos(\text{theta}[i, 1]))^4 + 2 \cdot (Q12 + 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2:$$

$$Qbar[2, 3, i] := (Q11 - Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1])) \cdot (\sin(\text{theta}[i, 1]))^3 - (Q22 - Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^3 \cdot \sin(\text{theta}[i, 1]):$$

$$Qbar[3, 3, i] := (Q11 + Q22 - 2 \cdot Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2 + Q66 \cdot \left((\cos(\text{theta}[i, 1]))^4 + (\sin(\text{theta}[i, 1]))^4\right):$$

$Qbar[2, 1, i] := Qbar[1, 2, i]:$
$Qbar[3, 1, i] := Qbar[1, 3, i]:$
$Qbar[3, 2, i] := Qbar[2, 3, i]:$
end do:

```
> # Calculate elements of extensional stiffness matrix [A],
coupling stiffness matrix [B], and bending stiffness matrix [Dm]
# Units: [A]--> MPa.m; [B]--> MPa.m^2; [Dm]--> MPa.m^3
```

> **for** i **from** 1 **by** 1 **to** 3
while *true* **do**
for j **from** 1 **by** 1 **to** 3
while *true* **do**
$A[i, j] = 0:$
$B[i, j] := 0:$
$Dm[i, j] := 0:$

```
for k from 1 by 1 to n
while true do
A[i,j] := A[i,j] + Qbar[i,j,k]·(h[k+1,1] − h[k,1]) :
B[i,j] := B[i,j] + 1/2·Qbar[i,j,k]·(h[k+1,1]^2 − h[k,1]^2) :
Dm[i,j] := Dm[i,j] + 1/3·Qbar[i,j,k]·(h[k+1,1]^3 − h[k,1]^3) :
end do:
end do:
end do:
> evalf( A );
```

$$\begin{bmatrix} 1739.240863 & 388.3864106 & 56.6337309 \\ 388.3864106 & 453.2535123 & -114.0636096 \\ 56.6337309 & -114.0636096 & 452.4825438 \end{bmatrix} \quad (2)$$

```
> evalf( B );
```

$$\begin{bmatrix} -3.128833804 & 0.9855215539 & -1.071656126 \\ 0.9855215539 & 1.157790698 & -1.071656126 \\ -1.071656126 & -1.071656126 & 0.9855215540 \end{bmatrix} \quad (3)$$

```
> evalf( Dm );
```

$$\begin{bmatrix} 0.03343203414 & 0.006460977237 & -0.005240293688 \\ 0.006460977237 & 0.009319771318 & -0.005595913147 \\ -0.005240293688 & -0.005595913147 & 0.007662779735 \end{bmatrix} \quad (4)$$

```
>
```