```
# Prof. Dr. Serkan Dağ
# ME 310 Numerical Methods
# File 9.1
# Newton's Interpolating Polynomials
# Generates nth-order Newton's interpolating polynomial for (n+1) data points
restart :
Digits := 16 :
with(CurveFitting) :
# Order of the polynomial
n := 5 :
# Enter the data points
# (n+1) points needed for an nth-order polynomial
X := [24., 32., 16., 40., 8, 0.];
Y := [8.418, 7.305, 9.870, 6.413, 11.843, 14.621];
```

$$X := [24., 32., 16., 40., 8, 0.]$$

$$Y := [8.418, 7.305, 9.870, 6.413, 11.843, 14.621] \quad (1)$$

```
# Determine the polynomial first by MAPLE command for comparison
fmaple := expand( PolynomialInterpolation(X, Y, x, form = Newton) );
```

$$fmaple := -9.663899739583335\ 10^{-9}\ x^5 + 0.00000181070963541666 7\ x^4 - 0.000163899739583333\ x^3 + 0.009485677083333333\ x^2 - 0.4135333333333333\ x + 14.62100000000000 \quad (2)$$

```
subs(x = 27, fmaple);
```

$$7.968238960266114 \quad (3)$$

```
# Define divided-differences matrix
A := Matrix(n + 1, n + 1) :
# Coefficients matrix
b := Matrix(n + 1, 1) :
# Evaluate divided-differences
for i from 1 by 1 to n + 1
while true do

A[i, 1] := Y[n + 2 − i] :

end do:
for i from 2 by 1 to n + 1
while true do
  for j from 1 by 1 to n + 2 − i
  while true do

A[j, i] := (A[j, i − 1] − A[j + 1, i − 1]) / (X[n + 2 − j] − X[n + 1 − j − (i − 2)]) :

  end do:

end do:
# Print divided-differences matrix
A;
```

$$[[14.621, -0.3472500000000000, 0.004439062500000000, -0.00007708333333333362, 8.829752604166756\ 10^{-7}, -9.663899739584450\ 10^{-9}], \quad (4)$$

```
[11.843, -0.1696875000000000, 0.003205729166666662, -0.00004882812500000000,
6.510416666666488 10^-7, 0],
[6.413, -0.1440416666666667, 0.002033854166666662, -0.00003841145833333362, 0, 0],
[9.870, -0.1603125000000000, 0.002648437500000000, 0, 0, 0],
[7.305, -0.1391250000000000, 0, 0, 0, 0],
[8.418, 0, 0, 0, 0, 0]]
```

```
> # Evaluate the coefficients b0,b1,...,bn
> for i from 1 by 1 to n + 1
  while true do

  b[i, 1] := A[n + 2 - i, i] :

  end do:
> # Print coefficients matrix
> b;
```

$$\begin{bmatrix} 8.418 \\ -0.1391250000000000 \\ 0.002648437500000000 \\ -0.00003841145833333362 \\ 6.510416666666488\ 10^{-7} \\ -9.663899739584450\ 10^{-9} \end{bmatrix} \quad (5)$$

```
> # Evaluate the interpolating polynomial
> pro := 1 :
  f := 0 :
  for i from 1 by 1 to n + 1
  while true do

  f := f + b[i, 1]·pro :

  pro := pro·(x - X[i]) :

  end do:
> # Calculate f for desired x
> subs(x = 27, f);
```

$$7.968238960266113 \quad (6)$$

```
>
```