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> # Prof. Dr. Serkan Dağ
  # ME 310 Numerical Methods
  # File 8.2
  # Polynomial Regression
  # Finds coefficients of the quadratic approximation a0 + a1 x + a2 x^2
> restart :
  Digits := 16 :
  with(CurveFitting) :
   with(Statistics) :
> # Number of data pairs
> n := 6 :
> # Define the data points
> X := [0, 0.002, 0.006, 0.012, 0.018, 0.024];
  Y := [0, 0.287, 0.899, 1.915, 3.048, 4.299];
                         X := [0, 0.002, 0.006, 0.012, 0.018, 0.024]
                          Y := [0, 0.287, 0.899, 1.915, 3.048, 4.299]                  (1)
> # Generate the polynomial by the MAPLE command
> LeastSquares(X, Y, x, curve = a2·x^2 + a1·x + a0);
  StandardDeviation(Y);

               0.0002363636371076 + 140.0062770562770 x + 1629.329004329004 x^2
                                     1.681430898570223                                 (2)
> # Evaluate required summations
> xt := 0 :
  yt := 0 :
  xiyi := 0 :
  xi2 := 0 :
   xi3 := 0 :
   xi4 := 0 :
   xi2yi := 0 :
> for i from 1 by 1 to n
   while true do

   xt := xt + X[i] :
   yt := yt + Y[i] :
   xiyi := xiyi + X[i]·Y[i] :
   xi2 := xi2 + X[i]^2 :
   xi3 := xi3 + X[i]^3 :
   xi4 := xi4 + X[i]^4 :
   xi2yi := xi2yi + X[i]^2·Y[i] :

   end do:
> # Form the linear system
> with(LinearAlgebra) :
> A := Matrix(3, 3) :
   B := Matrix(3, 1) :
> A[1, 1] := n :
   A[1, 2] := xt :
   A[1, 3] := xi2 :
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  A[2, 1] := xt :
  A[2, 2] := xi2 :
  A[2, 3] := xi3 :
  A[3, 1] := xi2 :
  A[3, 2] := xi3 :
  A[3, 3] := xi4 :
> B[1, 1] := yt :
  B[2, 1] := xiyi :
  B[3, 1] := xi2yi :
> # Determine the coefficients
> C := LinearSolve(A, B);
  a0 := C[1, 1] :
  a1 := C[2, 1] :
  a2 := C[3, 1] :
```

$$C := \begin{bmatrix} 0.000236363636361666\,7 \\ 140.0062770562775 \\ 1629.329004328990 \end{bmatrix} \qquad \textbf{(3)}$$

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> # Arithmetic means
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$$> \ xbar := \frac{\mathrm{xt}}{n} :$$

$$ybar := \frac{yt}{n} :$$

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> # Sums needed in error quantification
> St := 0 :
  Sr := 0 :
> for i from 1 by 1 to n
  while true do
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$$St := St + (Y[i] - ybar)^2 :$$

$$Sr := Sr + \left(Y[i] - a0 - a1 \cdot X[i] - a2 \cdot X[i]^2\right)^2 :$$

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  end do:
> # Sum of squares of residuals (relative to mean)
> St;
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$$14.13604933333333 \qquad \textbf{(4)}$$

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> # Sum of squares of residuals (relative to regression line)
> Sr;
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$$1.974025974027055\ 10^{-7} \qquad \textbf{(5)}$$

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> # Standard deviation
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$$> \ sy := \mathrm{sqrt}\left(\frac{St}{n-1}\right);$$

$$sy := 1.681430898570223 \qquad \textbf{(6)}$$

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> # Standard error of the estimate
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$$> \ syx := \mathrm{sqrt}\left(\frac{Sr}{n-3}\right);$$

$$syx := 0.0002565167943837242 \qquad \textbf{(7)}$$

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> # Correlation coefficient
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$$> r := \mathrm{sqrt}\left(\frac{(St - Sr)}{St}\right);$$

$$r := 0.9999999930177592 \qquad \textbf{(8)}$$

> # Plot the data and the regression approximation

$$> y := a0 + a1 \cdot x + a2 \cdot x^2;$$

$$y := 1629.329004328990\, x^2 + 140.0062770562775\, x + 0.0002363636363616667 \qquad \textbf{(9)}$$

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> with(plots) :
  p1 := plot(X, Y, style = point, color = red, symbol = solidcircle, symbolsize = 12) :
  p2 := plot(y, x = 0 ..0.025, color = green) :
  display({p1, p2});
```

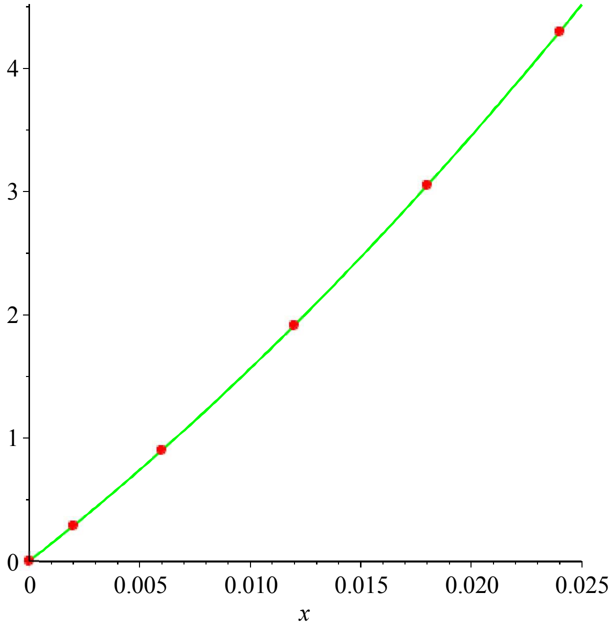

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