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> # Prof. Dr. Serkan Dağ
  # ME 310 Numerical Methods
  # File 11.1
  # Derivation of two-point Gauss-Legendre points and weights
> restart :
> # Define the polynomial
> f := p0 + p1·x + p2·x^2 + p3·x^3;
```

$$f := p3\,x^3 + p2\,x^2 + p1\,x + p0 \tag{1}$$

```
> # Define the left- and right-sides
> eq1 := int(f, x = -1 ..1);
```

$$eq1 := \frac{2}{3}\,p2 + 2\,p0 \tag{2}$$

```
> eq2 := expand( c0·subs(x = x0, f) + c1·subs(x = x1, f) );
```

$$eq2 := c0\,p3\,x0^3 + c1\,p3\,x1^3 + c0\,p2\,x0^2 + c1\,p2\,x1^2 + c0\,p1\,x0 + c1\,p1\,x1 + c0\,p0 + c1\,p0 \tag{3}$$

```
> # Find coefficients of p_i
> A1 := coeff(eq1, p3);
  A2 := coeff(eq1, p2);
  A3 := coeff(eq1, p1);
  A4 := coeff(eq1, p0);
```

$$A1 := 0$$
$$A2 := \frac{2}{3}$$
$$A3 := 0$$
$$A4 := 2 \tag{4}$$

```
> B1 := coeff(eq2, p3);
  B2 := coeff(eq2, p2);
  B3 := coeff(eq2, p1);
  B4 := coeff(eq2, p0);
```

$$B1 := c0\,x0^3 + c1\,x1^3$$
$$B2 := c0\,x0^2 + c1\,x1^2$$
$$B3 := c0\,x0 + c1\,x1$$
$$B4 := c0 + c1 \tag{5}$$

```
> # Derive the equations
> res1 := B1 - A1;
  res2 := B2 - A2;
  res3 := B3 - A3;
  res4 := B4 - A4;
```

$$res1 := c0\,x0^3 + c1\,x1^3$$
$$res2 := c0\,x0^2 + c1\,x1^2 - \frac{2}{3}$$
$$res3 := c0\,x0 + c1\,x1$$
$$res4 := c0 + c1 - 2 \tag{6}$$

```
> # Symbolic solution
> final := solve( {res1 = 0, res2 = 0, res3 = 0, res4 = 0}, {c0, x0, c1, x1} );
```

$$final := \{c0 = 1, c1 = 1, x0 = RootOf(3\,\_Z^2 - 1), x1 = -RootOf(3\,\_Z^2 - 1)\} \tag{7}$$

```
> convert( final, radical );
```

$$\left\{c0 = 1, c1 = 1, x0 = \frac{1}{3}\sqrt{3}, x1 = -\frac{1}{3}\sqrt{3}\right\} \tag{8}$$

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