Multi-objective Decision Making Using Fuzzy Discrete Event Systems: A Mobile Robot Example

Yiannis Boutalis and Klaus Schmidt

***Abstract*— In this paper, we propose an approach for the *multi-objective control* of sampled data systems that can be modeled as *fuzzy discrete event systems* (FDES). In our work, the choice of a "fuzzy" system representation is justified by the assumption of a controller realization that depends on various potentially imprecise sensor measurements. Our approach consists of three basic steps that are performed in each sampling instant. First, the current fuzzy state of the system is determined by a *sensor evaluation*. Second, the future fuzzy state is *predicted* for the possible control actions, and finally, a particular *multi-objective weighting* strategy allows to determine the control action to be applied. We demonstrate the features of our method by a mobile robot example.**

I. INTRODUCTION

Fuzzy discrete event systems (FDES), generalize the well-known modeling framework of discrete event systems (DES) [2], which are discrete both in time and state space and change their state due to asynchronous events. In conventional DES, both states and state transitions are crisp, that is, no uncertainties arise. In contrast, in FDES, states and state changes via transitions are determined by a *possibility degree*. The formal framework of FDES was initially set in [5] based on existing work on *fuzzy finite automata* [6], [11]. Considering the control of FDES, approaches that achieve the fulfillment of a given *fuzzy language* [1], [8], [9] and *optimal control* of FDES [5] are studied.

In this paper, we develop an optimal control strategy that extend the FDES approach in our previous work [10] to a multi-objective framework. It is based on a sensor evaluation that determines the current (fuzzy) state of the system with respect to multiple control objectives. This state is then used by a set of FDES models to predict the future system states in respect to the different objectives, when pre-specified control actions are applied. In each sampling instant, the most appropriate control action is selected by using a multi-objective optimization step. Different from our previous work, a particular multi-objective weighting strategy is introduced to achieve this task. The proposed approach is illustrated by simulations of a mobile robot, which has to follow a specified path in an environment with unspecified obstacles. Different formulations of the control objectives show how the framework can easily be adapted to the specific requirements of the respective control problem.

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It has to be noted that *distributed FDES* have been recently used in [3], [4] for robotic sensor information processing and robotic behavior modulation. However, that work does not allow a clear separation of the sensor processing and the control action computation but rather uses a heuristic to retrieve the control output from the sensor measurements.

The paper is organized as follows. Section II provides basic notations used in FDES, while Section III presents our multi-objective control strategy. The application of our method to a mobile robot example is performed in IV including a simulation experiment. Conclusions are given in Section V.

II. NOTATION AND FORMAL FRAMEWORK

A. Fuzzy Sets

Each *fuzzy set* $\mathcal{A}$ is defined in terms of a *universal set* X by a *membership function* assigning to each element $x \in X$ a value $\mathcal{A}(x)$ in the interval [0, 1]. We denote by $\mathcal{F}(X)$ the set of all fuzzy subsets of X. For any $\mathcal{A}, \mathcal{B} \in \mathcal{F}(X)$, we say that $\mathcal{A}$ is contained in $\mathcal{B}$, denoted by $\mathcal{A} \subseteq \mathcal{B}$, if $\mathcal{A}(x) \leq \mathcal{B}(x)$ for all $x \in X$. We say that $\mathcal{A} = \mathcal{B}$ if and only if $\mathcal{A} \subseteq \mathcal{B}$ and $\mathcal{B} \subseteq \mathcal{A}$. A fuzzy set is empty, denoted by $\mathcal{O}$, if its membership function is identically zero on X.

B. Formalism for FDES

Let the crisp state set $\mathcal{P}$ of a DES consist of the states $\mathcal{P} = \{p_1, p_2, \ldots, p_n\}$. Then each *fuzzy state* in the setting of FDES can be written as a vector $q = [q_1, q_2, \ldots, q_n]$, where $q_i \in [0, 1]$. This way, each fuzzy state can be considered as a possibility distribution or alternatively as a fuzzy set $q \subseteq \mathcal{F}(\mathcal{P})$ determining the degree q_i by which the system participates in each crisp state p_i, provided it is in the current fuzzy state. Similarly, a *fuzzy event* σ is characterized by a matrix $[a_{ij}]_{nxn}$, in which every element $a_{ij} \in [0, 1]$ indicates the possibility of a transition in the FDES from the current state p_i to the new state p_j when the event σ occurs. Using the above notation, the successor fuzzy state can be evaluated using the max-min operation $\odot$ [8], [9]

$$q \odot \sigma = \left[\max_{l=1}^{n} \min\{q_l, a_{l1}\}, \cdots, \max_{l=1}^{n} \min\{q_l, a_{ln}\} \right]. \quad (1)$$

We employ the formalism in [10] to describe the behavior of a FDES model as employed in our work. Each FDES models the occurrence of an event σ starting from an *initial fuzzy state* q_σ by a tuple $F_\sigma = (Q_\sigma, \Phi_\sigma, q_\sigma)$ with the *set of crisp states* Q_σ, where the dimension is $|Q_\sigma| = n$. Accounting for the distributed nature of the FDES, the event σ is associated with the *fuzzy event set* Φ_σ in the sense that all events in Φ_σ contribute to the occurrence of σ: given the inital fuzzy state $q_\sigma \subseteq \mathcal{F}(Q_\sigma)$, the successor fuzzy state $\hat{q}_\sigma$ after the occurrence of σ evaluates to

$$\hat{q}_\sigma = q_\sigma \otimes \sigma := \max_{f \in \Phi_\sigma}[q_\sigma \odot f], \tag{2}$$

where $\max$ takes the component-wise maximum of the respective successor state vectors.

Example 1: We study the FDES $F_\sigma = (Q_\sigma, \Phi_\sigma, q_\sigma)$ with three crisp states ($Q_\sigma = \{q_1, q_2, q_3\}$), $\Phi_\sigma = \{\mathtt{f}^1_\sigma, \mathtt{f}^2_\sigma\}$, $q_\sigma = \begin{bmatrix}0.3 & 0.8 & 0.4\end{bmatrix}$ and the fuzzy event matrices

$$\mathtt{f}^1_\sigma = \begin{bmatrix} 0 & 0.2 & 0.7 \\ 0 & 0.8 & 0 \\ 0.4 & 0 & 0.8 \end{bmatrix}, \qquad \mathtt{f}^2_\sigma = \begin{bmatrix} 0 & 0.4 & 0 \\ 0.6 & 0 & 0.9 \\ 0 & 0.8 & 0 \end{bmatrix}$$

Then, the evaluation of $q_\sigma \otimes \sigma$ yields

$$\hat{q}_\sigma = \max([0.4\ 0.8\ 0.4], [0.6\ 0.4\ 0.8]) = [0.5\ 0.6\ 0.6].$$

Finally, we introduce the *fuzzy state product* for two fuzzy states q, p with the dimension n as

$$q \oslash p := \begin{bmatrix}\min\{q_1, p_1\} & \cdots & \min\{q_1, p_n\} & \min\{q_2, p_1\} \\ \min\{q_n, p_1\} & \cdots & \min\{q_n, p_n\}\end{bmatrix}.$$

Hence, $q \oslash p$ describes the fuzzy state corresponding to the product state space of the crisp states of q and p.

Example 2: For example, let $q = [0.5\ \ 0.6\ \ 0.6]$ and $p = [0.6\ \ 0.4\ \ 0.4]$. Then, $q \oslash p = \begin{bmatrix}\min\{0.5, 0.6\} & \cdots & \min\{0.5, 0.4\} & \cdots & \min\{0.6, 0.4\}\end{bmatrix} = \begin{bmatrix}0.5\ 0.4\ 0.4\ 0.6\ 0.4\ 0.4\ 0.6\ 0.4\ 0.4\end{bmatrix}$.

III. MULTI-OBJECTIVE DECISION MAKING USING FDES

A. Problem Statement

We consider a *multi-objective* sampled data control problem with a *set of objectives* $\mathcal{G} = \{1, \ldots, G\}$. It is assumed that the control system can take several measurements per sampling instant in order to determine its adherence to the different objectives. To this end, for each objective $g \in \mathcal{G}$, a *set of measurement events* Σ_g is introduced such that the event $\sigma \in \Sigma_g$ occurs whenever the corresponding measurement supplies a new *sensor value* in the *measurement range* $\mathcal{R}_\sigma$. In addition, the control system offers a *set of control actions*. Defining the *set of control action combinations* $\mathcal{A} \subseteq 2^\Gamma$ in the power set of Γ, the control system can apply a set of control actions in $\mathcal{A}$ in each sampling instant.

Regarding the control system design, it is desired in each sampling instant to evaluate the measurements with respect to the different objectives and find an appropriate control action combination that fulfills the objectives best. In this context, it has to be noted that our model allows the description of the measurement actions Σ_g for $g \in \mathcal{G}$ and the control actions Γ in the form of discrete sets. In addition, we assume that the measurements and the application of the control actions can be uncertain and it has to be considered that data from different sources have to be fused to evaluate each objective. Together, we propose a fuzzy DES (FDES) approach in order to address the stated control problem.

B. Solution Method

Our solution method consists of three consecutive steps that have to be executed in each sampling instant. First, FDES models that characterize the *sensor evaluation* are used to determine the (fuzzy) current system state with respect to the given objectives. In the second step, we perform a *prediction* of the future fuzzy system state with respect to each objective after applying each control action combination in $\mathcal{A}$. Then, the third step allows to find the most appropriate control action combination by *weighting* the state predictions from step two with respect to the different objectives.

C. Sensor Evaluation

We introduce a separate FDES model $S_{g,\sigma} = (Q_{g,\sigma}, \Phi_{g,\sigma}, q_{g,\sigma})$ per objective $g \in \mathcal{G}$ and measurement $\sigma \in \Sigma_g$. Semantically, such model captures how the control system fulfills the objective g if the current measurement σ is taken. As defined earlier, $q_{g,\sigma}$ denotes the initial state of the FDES and $\Phi_{g,\sigma}$ represents a set of fuzzy events that occur when measuring σ. Here, each $f \in \Phi_{g,\sigma}$ describes a characteristic of the measured value (e.g., "large", "medium", "small"). The entries of the respective fuzzy event matrices are determined by an automaton graph on the state space of the sensor evaluation and a membership function. For each fuzzy event matrix $f \in \Phi_{g,\sigma}$, the entry in row i and column j is nonzero if there is a transition with f in the automaton graph from state j to state i. Furthermore, the nonzero entries are denoted as n and quantified by the evaluation of the membership function $\mu_{g,\sigma} : \mathcal{R}_\sigma \times \Phi_{g,\sigma} \to [0,1]$. In words, $\mu_{g,\sigma}(r, f)$ for a measurement value $r \in \mathcal{R}_\sigma$ and a fuzzy event $f \in \Phi_{g,\sigma}$ characterizes the contribution of f to the evaluation of the measurement σ for the objective g. Then, the fuzzy state product as introduced in Section II-B allows the current fuzzy state evaluation q_g for each objective $g \in \mathcal{G}$:

$$q_g = \oslash_{\sigma \in \Sigma_g}(q_{g,\sigma} \otimes \sigma). \tag{3}$$

D. Fuzzy State Prediction

Considering the fuzzy state prediction, we employ an FDES model $C_{g,\gamma} = (Q_{g,\gamma}, \Phi_{g,\gamma}, q_{g,\gamma})$ for each $g \in \mathcal{G}$ and $\gamma \in \Gamma$. In each sampling instant, it characterizes the effect of applying γ with respect to g. Here, it has to be noted that each $C_{g,\gamma}$ resides on the state space of the corresponding sensor evaluation. Hence, the initial fuzzy state $q_{g,\gamma}$ can be set to the current fuzzy state in (3), i.e., $q_{g,\gamma} = q_g$. Furthermore, we associate a measurement that yields a sensor value in $\mathcal{R}_\gamma$ to the control action γ. Then, the entries of the fuzzy event matrices $f \in \Phi_{g,\gamma}$ are computed using the membership functions $\mu_{g,\gamma} : \mathcal{R}_\gamma \times \Phi_{g,\gamma} \to [0,1]$. $\mu_{g,\gamma}(r, f)$ quantifies how f contributes to the state prediction for g and the γ considering the current measurement $r \in \mathcal{R}_\gamma$. Together, we define the predicted fuzzy state for $\gamma \in \Gamma$ and $g \in \mathcal{G}$ and the predicted fuzzy state for each combination $a \in \mathcal{A}$ as

$$\hat{q}_{g,\gamma} = q_{g,\gamma} \otimes \gamma = q_q \otimes \gamma, \tag{4}$$

$$\hat{q}_{g,a} = \max_{\gamma \in a} \hat{q}_{g,\gamma} = \max_{\gamma \in a}(q_g \otimes \gamma), \tag{5}$$

taking the component-wise maximum of the fuzzy state predictions for each control action in a.

In summary, the fuzzy state prediction allows to evaluate all future (predicted) fuzzy states for all objectives in $\mathcal{G}$ when applying the different control action combinations in $\mathcal{A}$. The remaining task is hence to choose the most appropriate control action combination in the sense that all objectives are met in a satisfactory way. The subsequent section introduces a particular weighting scheme in order to perform this task.

E. Multi-objective Weighting

For each objective $g \in \mathcal{G}$, we introduce a *weight vector* w_g. It gives more weight to the components of the predicted fuzzy state that are desirable for the objective g. Hence,

$$A_{g,a} := \hat{q}_{g,a} \cdot w_g \tag{6}$$

represents the the evaluation of the *acceptance* for each control action combination $a \in \mathcal{A}$. It determines the quality of the control action combination in achieving the objective g. In addition, it has to be considered that our goal is the compliance with multiple potentially conflicting objectives such that the same control action will probably lead to different levels of acceptance for different objectives. In order to resolve this issue, we perform a further sensor evaluation that decides about the relevance of the different objectives. To this end, we introduce a set of measurements Λ, where each $\lambda \in \Lambda$ is related to a sensor value in $\mathcal{R}_\lambda$ and an FDES $W_\lambda = (Q_\lambda, \Phi_\lambda, q_\lambda)$. Again, the entries of the fuzzy events $f \in \Phi_\lambda$ are defined by a membership function $\mu_\lambda : \mathcal{R}_\lambda \times \Phi_\lambda \to [0,1]$ such that $\mu_\lambda(r,f)$ quantifies the contribution of f based on the sensor value $r \in \mathcal{R}_\lambda$. Then, we assess the overall situation of the control system with respect to the objectives by the fuzzy state

$$q = \oslash_{\lambda \in \Lambda}(q_\lambda \otimes \lambda), \tag{7}$$

and introduce one weight vector v_g for each objective g that captures the relevance of g for the next control action. That is, $q \cdot v_g$ has a large value whenever the objective g is well achieved. Combining the evaluation in (6) and (7), we propose to solve the following optimization problem so as to find the most appropriate control action $a^* \in \mathcal{A}$.

$$a^* = \arg\max_{a \in \mathcal{A}} \sum_{g \in \mathcal{G}} [(q \cdot v_g)\, A_{g,a}]. \tag{8}$$

IV. Mobile Robot Example

We now illustrate the control concept proposed in the previous section by a mobile robot example. Since a similar example was already used in our previous work [10], we only give a brief description of the basic setup and focus more on the novel ideas presented in this paper.

A. Basic Setup

We consider a mobile robot that has to follow a given path and that can encounter unknown obstacles on its way. Hence, the robot has two objectives ($\mathcal{G} = \{1,2\}$). It has to reach the next subgoal on its path (objective 1) while avoiding the obstacles (objective 2). The situation is depicted in Fig. 1.

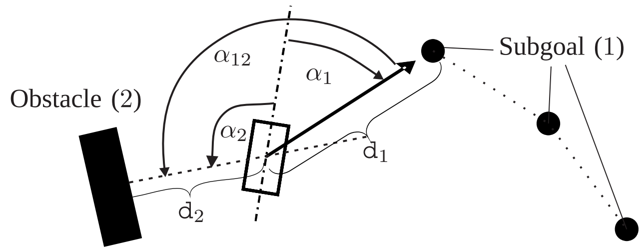

Fig. 1. Mobile robot: subgoals and obstacle.

Objective 1 is related to the events $\Sigma_1 = \{\mathtt{d}_1, \alpha_1\}$ with the corresponding measurement of the distance $r_{\mathtt{d}_1} \in \mathcal{R}_{\mathtt{d}_1} = \mathbb{R}$ in pixels (one pixel corresponds to an area of 20 mm × 20 mm) and the angle $r_{\alpha_1} \in \mathcal{R}_{\alpha_1} = [0, 360°]$ to the subgoal. Likewise, objective 2 is captured by $\Sigma_2 = \{\mathtt{d}_2, \alpha_2\}$. In addition, the mobile robot offers 6 control actions. It can turn to the right ($\mathtt{r}$), to the left ($\mathtt{l}$) or go straight ($\mathtt{s}$) and accelerate ($\mathtt{a}$), decelerate ($\mathtt{d}$) or stay at a constant speed ($\mathtt{c}$). Hence, $\Gamma = \{\mathtt{r}, \mathtt{l}, \mathtt{s}, \mathtt{a}, \mathtt{d}, \mathtt{c}\}$. In this example, angle changes happen in discrete steps of 5°, while speed changes occur in discrete steps of 3% of a predefined maximum speed. Considering the meaning of the control actions, it is only reasonable to combine one angle change with one speed action. Hence, the set of possible control action combinations evaluates to $\mathcal{A} = \{\{\mathtt{r},\mathtt{a}\}, \{\mathtt{r},\mathtt{d}\}, \{\mathtt{r},\mathtt{c}\}, \{\mathtt{l},\mathtt{a}\}, \{\mathtt{l},\mathtt{d}\}, \{\mathtt{l},\mathtt{c}\}, \{\mathtt{s},\mathtt{a}\}, \{\mathtt{s},\mathtt{d}\}, \{\mathtt{s},\mathtt{c}\}\}$. We next apply our three-step procedure to the mobile robot example.

B. Sensor Evaluation

We introduce the FDESs $S_{1,\mathtt{d}_1}$ and S_{1,α_1} for the sensor evaluation with respect to objective 1. For each measurement $\sigma \in \Sigma_1$, we introduce three fuzzy events in $\Phi_{1,\sigma}$ that correspond to a "large", "medium" and "small" value of the respective measurement. For example, $\mathtt{d}_1$ is associated with the fuzzy events $f^{\mathrm{l}}_{\mathtt{d}_1}$ (large distance), $f^{\mathrm{m}}_{\mathtt{d}_1}$ (medium) and $f^{\mathrm{s}}_{\mathtt{d}_1}$ (small). The nonzero entries of the respective fuzzy event matrices are determined by the automata graph in Fig. 2 (a) and the membership function $\mu_{f_{\mathtt{d}_1}, \Phi_{1,\mathtt{d}_1}}$ depicted in Fig. 3 (a) depending on the sensor value $r_{\mathtt{d}_1}$. A similar construction holds for the measurement α_1 of objective 1 and the evaluation of objective 2.

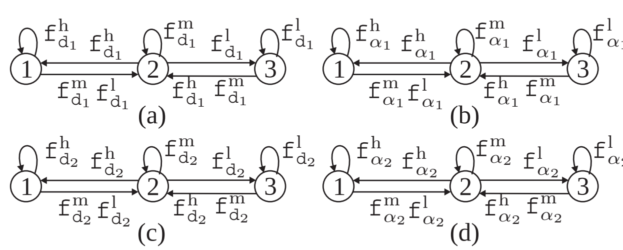

Fig. 2. Automata graphs for both objectives.

The current fuzzy state for objective 1 and 2 is then computed in each time instant as

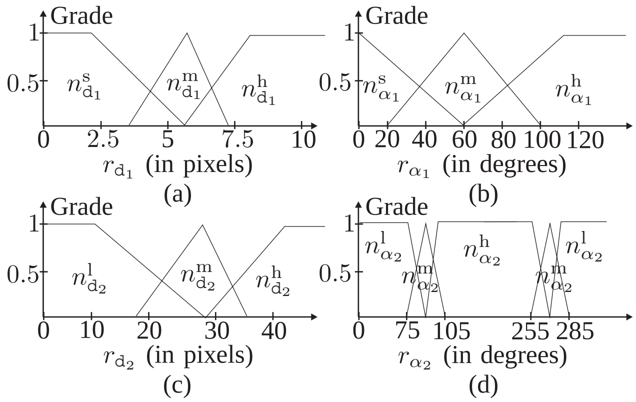

Fig. 3. Membership functions for both objectives.

$$q_1 = (q_{1,\mathrm{d}_1} \otimes \mathrm{d}_1) \oslash (q_{1,\alpha_1} \otimes \alpha_1),$$
$$q_2 = (q_{2,\mathrm{d}_2} \otimes \mathrm{d}_2) \oslash (q_{2,\alpha_2} \otimes \alpha_2).$$

In this example, an appropriate choice of the initial states is $q_{1,\mathrm{d}_1} = q_{1,\alpha_1} = q_{2,\mathrm{d}_2} = q_{2,\alpha_2} = \begin{bmatrix} 0.1 & 0.8 & 0.1 \end{bmatrix}$.

C. Fuzzy State Prediction

Our fuzzy state prediction introduces 6 FDES models $C_{g,\gamma}$ for each of the two objectives $g \in \{1, 2\}$. As specified in Section III-D, we associate a measurement with each control action in Γ. Here, the turning actions are related to the current angle measurement (α_1 for objective 1 and α_2 for objective 2), while the speed actions depend on the current speed s as a new measurement variable. Then, for example, the model $C_{1,\mathrm{r}}$ predicts the future fuzzy state with respect to objective 1 if the control action r is applied. It resides on the product state space of the sensor evaluations for objective 1 that captures the distance (d_1) and the angle (α_1) measurement. The control action r is related to three fuzzy events in $\Phi_{1,\mathrm{r}} = \{\mathrm{r}_1^{\mathrm{g}}, \mathrm{r}_1^{\mathrm{n}}, \mathrm{r}_1^{\mathrm{b}}\}$, where $\mathrm{r}_1^{\mathrm{g}}$, $\mathrm{r}_1^{\mathrm{n}}$, $\mathrm{r}_1^{\mathrm{b}}$ describe the fuzzy state prediction if the angle measurement decides that going to the right is “good”, “neutral” or “bad”, respectively. As usual, the entries of the fuzzy event matrices are determined by the corresponding automaton graph in Fig. 9 and the membership function $\mu_{1,\mathrm{r}}$ as shown in Fig. 10. Analogous considerations lead to the remaining FDES models (see also Fig. 7 and Fig. 8).[1]

D. Multi-objective Weighting

Having obtained a model for the fuzzy state prediction in the previous section, we now apply the idea of weighting to our mobile robot example. We first introduce the weight vectors for the different objectives. Here, we choose $w_1 = \begin{bmatrix} 0.01 & 0.2 & 1.6 & 0.05 & 0.4 & 3.2 & 0.1 & 0.8 & 6.4 \end{bmatrix}$ and $w_2 = \begin{bmatrix} 1.6 & 6.4 & 0.8 & 0.4 & 3.2 & 0.2 & 0.1 & 2.0 & 0.01 \end{bmatrix}$ to capture that the most desired states for objective 1 are the states 3, 6, 8, 9 in Fig. 9, while the preferable states for objective 2 are 1, 2, 5, 8. Hence, the acceptance $A_{g,a}$ can be evaluated for each objective $g \in \{1, 2\}$ and each control action combination $a \in \mathcal{A}$.

[1]For a more detailed description, please consult [10].

In order to address the relevance of the conflicting objectives 1 and 2, we use the measurement d_2 and introduce a new measurement α_{12} that senses the angle between the direction to the next subgoal and the obstacle as shown in Fig. 1. Hence, $\Lambda = \{\mathrm{d}_2, \alpha_{12}\}$. According to the approach in Section III-E, we define two FDES models W_{d_2} and $W_{\alpha_{12}}$ with the respective fuzzy events in $\Phi_{\mathrm{d}_2} = \{f_{\mathrm{d}_2}^{\mathrm{l}}, f_{\mathrm{d}_2}^{\mathrm{m}}, f_{\mathrm{d}_2}^{\mathrm{s}}\}$ and $\Phi_{\alpha_{12}} = \{f_{\alpha_{12}}^{\mathrm{l}}, f_{\alpha_{12}}^{\mathrm{m}}, f_{\alpha_{12}}^{\mathrm{s}}\}$. The corresponding automata graphs and membership functions are shown in Fig. 4 (a), (b) and Fig. 5 (a), (b).

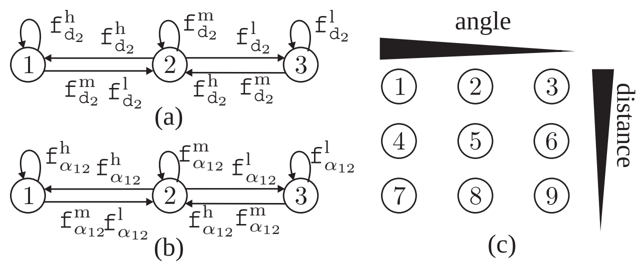

Fig. 4. Multi-objective weighting: graphs

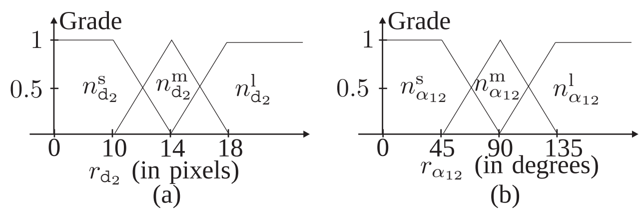

Fig. 5. Multi-objective weighting: membership functions

Moreover, the state space of the fuzzy state product

$$(q_{\mathrm{d}_2} \otimes \mathrm{d}_2) \oslash (q_{\alpha_{12}} \otimes \alpha_{12})$$

is depicted in Fig. 4 (c). It represents the overall state space that is employed to characterize the relevance of the two objectives in each sampling instant. In particular, objective 2 is most relevant in the states 9, 8 and 6, where a small distance to the obstacle and a small angle indicate that the robot should better turn away from the obstacle. In contrast, objective 1 is preferred in the states 1, 2, 4, 7 since the route can be followed without any danger of hitting an obstacle. Accordingly, we choose the weight vectors $v_1 = \begin{bmatrix} 3.2 & 1.6 & 1.6 & 1.6 & 1.2 & 0.8 & 1.6 & 0.4 & 0.2 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 0.2 & 0.4 & 0.4 & 0.4 & 0.8 & 0.4 & 3.2 & 6.4 \end{bmatrix}$. Together, we determine the most appropriate control action combination in each time instant by computing

$$a^* = \arg\max_{a \in \mathcal{A}} [(q_{\mathrm{d}_2} \otimes \mathrm{d}_2) \oslash (q_{\alpha_{12}} \otimes \alpha_{12})](v_1 A_{1,a} + v_2 A_{2,a}). \tag{9}$$

E. Simulation Experiment

We now apply the optimization described in the previous section to the mobile robot scenario in Fig. 6 (a). The robot has to move in a ’corridor’ starting from an initial position and orientation, displayed by the rectangular shape of the robot, in order to reach its goal (small circle at $(325, 0)$). On its route, it has to go around some unknown obstacles, being displayed by dark squares scattered in the corridor. The path that has to be followed is marked by small circles. They are determined by using a well-established procedure ([7]) to determine the safe path between the initial position of the robot and the target (obstacles are not taken into account). With a sampling period of 10 ms, the maximum speed of the robot is 100 pixels/second (about 7.5 km/h).

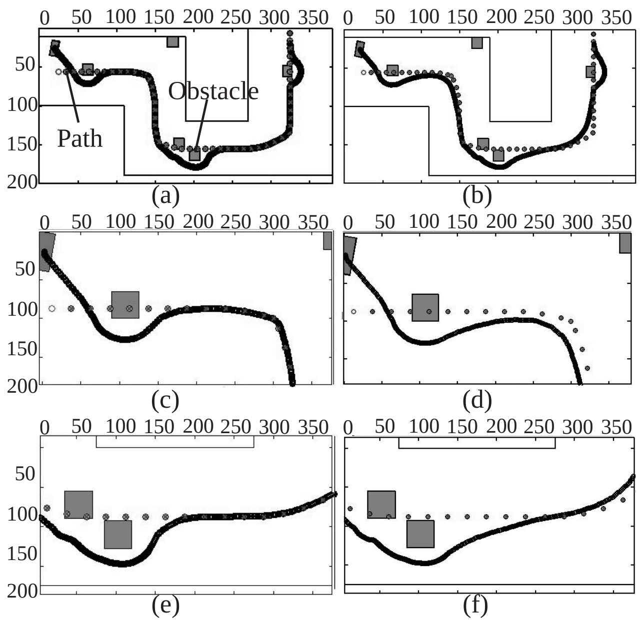

Fig. 6. Experiment and comparison: (a), (c), (e) single subgoal; (b) (d), (f) 3 subgoals.

Fig. 6 (a) shows the result of or multi-objective FDES control approach, where the relevance of the respective objectives is decided by the weighting as proposed in Section IV-D. It can be observed that the robot follows the given path and surrounds obstacles that are encountered on its way. Here, it has to be noted that the robot precisely sticks to the given path according to objective 1 (see for example the segment between position (100,50) and (150,150)), while it tries to move perpendicular to obstacles in a safe distance (e.g., between (50,50) and (80,50)).

F. Route follow with multiple subgoals

Considering the simulation experiment, it can be observed that the robot has to reach the closest subgoal according to the definition of objective 1 as long as no obstacle is encountered. Fig. 6 (a) shows that this can lead to unnecessarily long paths if it is known that there is no obstacle on the way to the desired path. We now modify objective 1 by including multiple (M) subgoals such that the robot can choose a shorter path as long as it does not encounter obstacles. To this end, instead of performing the sensor evaluation and the state prediction for the closest subgoal as suggested by (3) and (5), we evaluate these equations for each of the M closest subgoals. Hence, we arrive at state predictions $\hat{q}^m_{1,a}$ for objective 1, control action combination $a \in \mathcal{A}$ and the subgoal m with $1 \leq m \leq M$. Then, it is possible to choose an appropriate weight w^m_g for each subgoal m and again evaluate its acceptance using (6): $A^m_{1,a} = \hat{q}^m_{1,a} \cdot w^m_g$. The overall acceptance for objective 1 is now defined as $A_{1,a} := \sum_{m=1}^{M} A^m_{1,a}$. In case the robot is in the vicinity of an obstacle the multiple subgoals also lead to different α_{12} angle measurements and therefore different states $(q_{\mathrm{d}_2} \otimes \mathrm{d}_2) \oslash (q_{\alpha_{12}} \otimes \alpha_{12})$. Each such state might produce a different decision when taken into account in (8). Then, it is possible to choose an appropriate weight v^m_g for each subgoal m and take $v_g := \sum_{m=1}^{M} v^m_g$, $g = 1, 2$. After these modifications, the presented approach is applicable as before, i.e., the optimal control action can be computed using (8). In our example, we use the 3 closest subgoals and introduce the corresponding weights $w^1_1 = 0.25 w_1$, $w^2_1 = 0.25 w_1$ and $w^3_1 = 0.5 w_1$, where w_1 was given in Section IV-D. Similarly, using v_1, v_2 in Section IV-D we choose $v^m_1 = \frac{1}{3} v_1$, and $v^m_2 = \frac{1}{3} v_2$, where $m = 1, 2, 3$. The robot simulation for this case is shown in Fig. 6 (b) and a comparison to the case with only one subgoal is performed in Fig. 6 (c), (d) and (e), (f) for different path segments. Here, it is evident that choosing multiple subgoals leads to a more direct path as long as no obstacle has to be surrounded.

V. CONCLUSIONS

A *multi-objective control* scheme for systems that can be modeled as *fuzzy discrete event systems* (FDES) was presented in this paper. This control scheme allows to determine the current system state from possibly imprecise measurements, and choose appropriate control actions based on a prediction of the future system state. As a characteristic feature, the selection of the control action is based on a novel weighting strategy that tries to accommodate multiple control objectives. The simulation study of a mobile robot that has to follow a given path while avoiding unknown obstacles demonstrates the applicability of our approach.

REFERENCES

[1] Y. Cao and M. Ying, "Supervisory control of fuzzy discrete event systems," *IEEE Transactions on Systems, Man and Cybernetics, Part B*, vol. 35, no. 2, pp. 366–371, 2005.

[2] C. G. Cassandras and S. Lafortune, *Introduction to Discrete Event Systems*. Secaucus, NJ, USA: Springer-Verlag New York, Inc., 2006.

[3] R. Huq, G. K. I. Mann, and R. G. Gosine, "Behavior modulation technique in mobile robotics using fuzzy discrete event system," *IEEE Transanctions on Robotics*, vol. 22, no. 5, pp. 903–916, October 2006.

[4] ——, "Distributed fuzzy discrete event system for robotic sensory information processing," *Expert Systems*, vol. 23, no. 5, pp. 273–289, November 2006.

[5] F. Lin and H. Ying, "Modeling and control of fuzzy discrete event systems," *IEEE Transactions on Systems, Man and Cybernetics, Part B*, vol. 32, no. 4, pp. 408–415, August 2002.

[6] J. N. Mordeson and D. S. Malik, *Fuzzy Automata and Languages: Theory and Applications*. London, Boca Raton, FL: Chapman & Hall, CRC, 2002.

[7] R. Murphy, *Introduction to AI Robotics*. Cambridge: MA: MIT Press, 2000.

[8] D. Qiu, "Supervisory control of fuzzy discrete event systems: A formal approach," *IEEE Transactions on Systems, Man and Cybernetics, Part B*, vol. 35, no. 1, pp. 72–88, February 2005.

[9] K. Schmidt and Y. Boutalis, “Computation of supremal controllable sublanguages and infimal controllable superlanguages for fuzzy discrete event systems,” *Control and Automation, Mediterranean Conference on*, June 2008.

[10] G. Tampakis, K. Schmidt, and Y. Boutalis, “Mobile robot navigation based on fuzzy discrete event systems,” in *Conference on Intelligent Control Systems and Signal Processing*, Istanbul, Turkey, 2009.

[11] W. Wechler, *The Concept of Fuzziness in Automata and Language Theory*. Berlin: Akademie-Verlag, 1978.

APPENDIX

STATE PREDICTION FOR OBJECTIVE 2

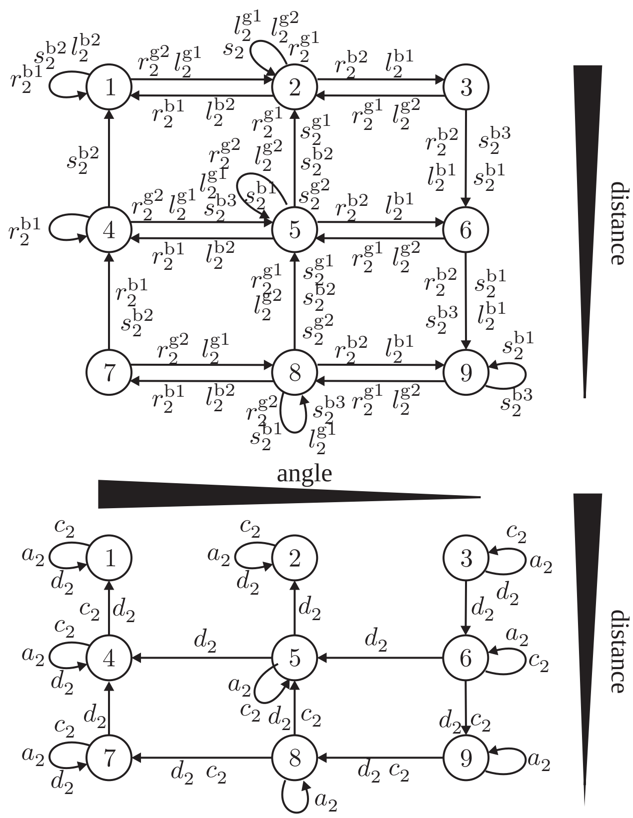

Fig. 7. State prediction for objective 2.

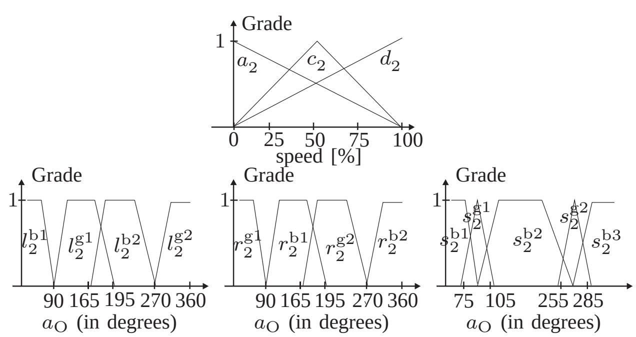

Fig. 8. Membership functions for objective 2.

STATE PREDICTION FOR OBJECTIVE 1

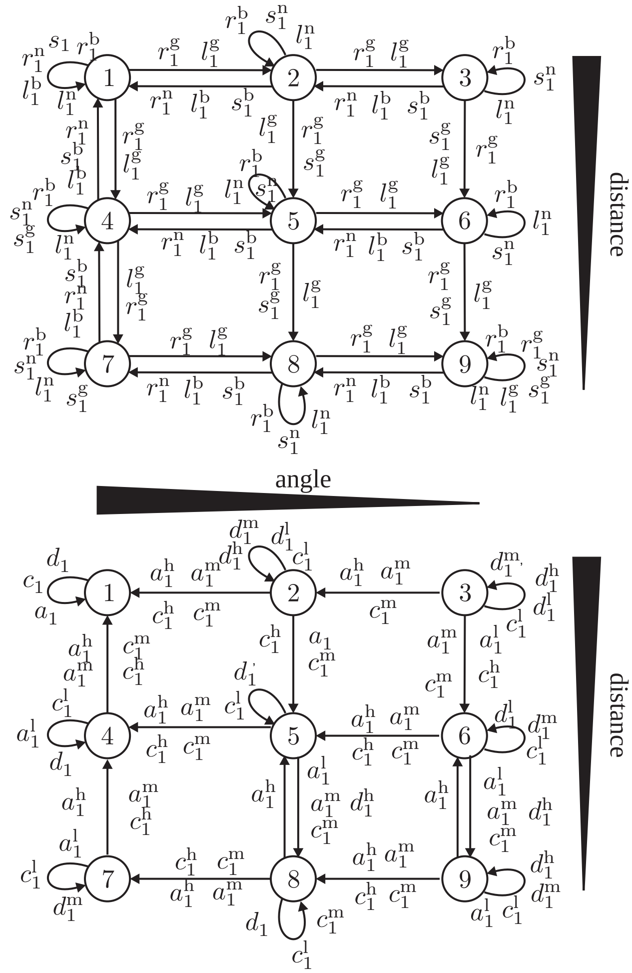

Fig. 9. State prediction for objective 1

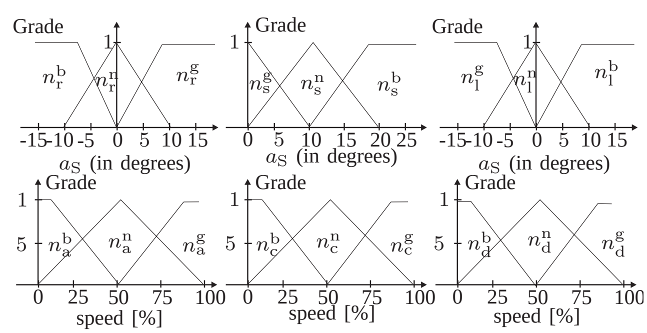

Fig. 10: Membership functions for objective 1.