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Radiative gas-dynamic model of a continuous optical discharge in a gravitational field: quasi-optical approximation

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Abstract
We consider a continuous optical discharge (COD) sustained by a vertically directed weakly focused CO₂ laser beam, in a gravitational field. We used a full two-dimensional radiative gas-dynamic numerical model for the COD, which uses realistic quasi optics and takes refraction of the laser radiation in the plasma properly into account in the description of the laser beam propagation. The model is applied to calculate the parameters of the COD in a converging CO₂ laser beam in free air atmosphere as a function of the laser power. We also demonstrate the effect of the selection of spectral groups, used in the multi-group diffusion approximation of the thermal radiation transport, on the model solutions.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

In a continuous optical discharge (COD) in gaseous media, the plasma is maintained by continuous laser radiation, usually from a CO₂ laser. In order to facilitate plasma formation, some initial ionization is produced either by irradiating a solid target and creating some ionization near the target surface or by using an electric discharge as a source of initial radiation. When formed, depending on the degree to which the beam is focused as well as the output power of the laser and external gas flow conditions, the plasma either propagates towards the laser in the direction of the beam ("laser supported combustion wave") and vanishes at a point where the intensity is no longer sufficient to maintain it or stops at some distance from the focal point and continues to exist as long as the laser power remains.

Aside from a purely scientific interest, the COD phenomenon is important for several reasons involving the practical applications of laser and plasma technology and material processing [1, 2]. The efficient adoption of COD as a plasma source for these applications requires control over the COD plasma properties and determination of the conditions under which a stable COD may exist in a quiescent and moving gaseous medium. One of the ways of solving these problems is to numerically model the processes occurring in the optical discharge plasma.

The most complete description of COD in gaseous media is presented in the literature by means of two- and three-dimensional radiative gas-dynamic models, which include equations of conservation of mass, momentum, energy and equations for radiation heat transfer on the basis of multi-group diffusion approximation. These equations are usually provided with the realistic transport, thermodynamic and optical properties of the plasma. However, most of these models use geometric optics to trace the laser beam through the optical system. More precisely, these models apply either geometric ray tracing to calculate the laser beam path [3–7] or assume a Gaussian laser beam in a prescribed and fixed light channel, with spatial distribution of the radiation intensity given by

\[ J(r, z) = \frac{P_L}{\pi w^2(z)} \exp \left( -\frac{r^2}{w^2(z)} \right), \]

where \( P_L \) is the laser power, \( w(z) \) determines the light channel border (it is the distance from the beam axis where the intensity...
drops to $e^{-1}$ of its axial value), $r$ and $z$ are cylindrical coordinates [8–16]. Therefore, under this approximation, model solutions determine the thermal and gas-dynamic structure of the discharge at a prescribed laser radiation electric field. Consequently, electrodynamic and thermal-gas-dynamic components of these models are not really self-consistent. The main drawback of the geometric optics models is that they are unable to take refraction of the beam in the plasma properly into account. However, laser radiation refraction in a nonuniform plasma (and consequently divergence of the beam in the plasma) can essentially change the light channel geometry and spatial distribution of the beam intensity. As a consequence, it causes a change in the discharge geometry, the temperature distribution and finally the discharge’s dynamics [17–19, 21]. The degree of the laser beam divergence itself, in turn, is determined by the discharge plasma. As a consequence, a strong coupling exists between the laser radiation field and the plasma discharge characteristics.

A self-consistent description of this problem requires the simultaneous solution of the laser radiation propagation equation together with those describing the discharge processes under appropriate boundary conditions. In [17–19], refraction of the laser beam in a nonuniform plasma was approximately taken into account for parabolic radial distributions of the refraction index, absorption coefficient, electron density and Gaussian radial distribution of the intensity in the beam. In [20], a parabolic equation of beam propagation was used for COD in a stationary gas, but with a rather rough description of the radiative transfer.

Results of a complete two-dimensional modelling of an optical discharge in a forced gas flow, employing a beam propagation equation in parabolic approximation of quasi-optics, have been reported for the first time in [21]. The objective of this work is to apply the numerical model [21] for COD, free of the external gas flow, and maintained by a vertical CO$_2$ laser beam, in the Earth’s gravitational field. We also demonstrate the effect of the selection of spectral groups, used in the calculation of thermal radiation transfer, on the model solutions. (We modelled thermal radiation on the basis of the most common and preferred method, namely the multi-group diffusion approximation [17, 21, 22].) For this purpose, we performed calculations of COD properties with different numbers of groups (bands or spectral intervals), namely with 6, 10 and 37 spectral groups [23–25].

The paper is organized as follows: section 2 describes the model of COD. Section 3 presents the results of the calculation of COD characteristics. Finally, section 4 summarizes the results.

2. Model

We consider a steady-state COD sustained by a weakly focused 10.6 μm CO$_2$ laser beam in free space at atmospheric pressure. The beam is focused vertically from below, the discharge is free of the external gas flow and the free-fall acceleration is acted vertically downwards (see figure 1). Moreover, we assume that the gas flow is subsonic and laminar. Furthermore, we assume that the discharge plasma is in local thermodynamic equilibrium (LTE). Therefore, the plasma can be described by a single temperature, and its physical properties are only a function of this temperature and pressure. Finally, assuming that all the processes are axially symmetric, we use cylindrical coordinates $(r, z)$.

2.1. Equations

We determine the thermal and gas-dynamic structure of COD by solving the set of equations consisting of the continuity equation, the Navier–Stokes equations, the equation of energy balance, the equation for the selective thermal radiation transport in the multigroup diffusion approximation [17, 21, 22] and the amplitude equation of a weakly focused laser beam propagating in the positive $z$ direction in parabolic approximation of quasi-optics [21, 22].

\[ \nabla \cdot (\rho \mathbf{v}) = 0, \]
\[ \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla \left( p + \frac{2}{3} \eta \mathbf{v} \cdot \mathbf{v} \right) + 2 \nabla \cdot (\eta \mathbf{S}) - \rho \mathbf{g}, \]
\[ \rho \mathbf{v} \cdot C_p \nabla T = \nabla \cdot (\lambda \nabla T) + Q_L - Q_R, \]
\[ \nabla \cdot \left( \frac{1}{3 \chi_m} \nabla U_m \right) = \chi_m \left( U_m - U_{eq,m} \right) \quad (m = 1, 2, \ldots, N_m) \]
\[ 2i k \frac{\partial \epsilon}{\partial z} = \nabla_E^2 \epsilon + k^2 (\epsilon_0 - 1) \epsilon. \]

Here, $v(r, z) = (v_r, v_z) = (u, v)$ is the mean mass velocity, $T$ is the temperature, $\rho$ is the pressure, $g(0, 0, g_z = g)$ is the gravity acceleration, $\mathbf{S} = \frac{1}{2} (\nabla \mathbf{v} + \nabla \mathbf{v}^T)$ is the deformation rate tensor, $\nabla_E^2$ denotes the $r$-term of the Laplacian operator, $(1/r)(\partial / \partial r)(r \partial / \partial r)$ is the deformation rate tensor. Parameters $\rho$, $C_p$, $\eta$ and $\lambda$ are the density, heat capacity, viscosity and thermal conductivity, respectively. Since the flow is substantially subsonic, the pressure drops are small and the pressure is approximately constant, $p(\rho, T) \approx p_0$, where $p_0 = 10^5$ Pa. Under this
condition, the gas density is uniquely related to the temperature of the gas, \( \rho = \rho(T, p_0) \). \( \mathcal{E} \) is the complex amplitude of the laser radiation field,

\[
E(r, z) = \mathcal{E}(r, z) \exp(-i k z + i \omega t),
\]

where \( k = \omega/c \), where \( \omega \) is the angular frequency of laser radiation, \( c \) is the speed of light. The complex dielectric permittivity of the plasma, \( \epsilon_c \), is defined as

\[
\epsilon_c = 1 - \frac{n_e}{n_e^*} - i \frac{\mu}{k},
\]

where \( \mu \) is the volume coefficient of the laser radiation absorption [11],

\[
\mu = \frac{2.82 \times 10^{-29} \mu^2}{T^{3/2}} \log \frac{2.17 \times 10^3 T}{n_e^{1/3}},
\]

\( n_e \) is in cm\(^{-3}\), \( T \) in K and \( \mu \) in cm\(^{-1}\). \( n_e^* \) is the equilibrium electron number density and is determined from the Saha equation and \( n_e \) is the critical plasma density at which its dielectric permittivity vanishes,

\[
\eta(T) = \frac{\epsilon_0 m_e \omega^2}{e^2}.
\]

Quantity \( Q_l \) expresses the specific power of the energy release within the plasma caused by the laser radiation absorption,

\[
Q_l = \mu \frac{\epsilon_0 |\mathcal{E}|^2}{2c} = \mu J,
\]

where \( J \) is laser radiation intensity, \( \epsilon_0 \) is permittivity of vacuum. The power density of the hot sources associated with thermal radiation transport is

\[
Q_R = \sum_{m=1}^{N_m} c \chi_m(U_{eq,m} - U_m),
\]

where \( \chi_m, U_m \) and \( U_{eq,m} \) are the group values of the volume absorption coefficient and of the radiation density of the plasma and ideal blackbody, averaged over each of the \( N_m \) spectral intervals [17,22].

The data on the temperature dependence of the thermodynamic, transport and optical properties of air, which are \( \rho(T) \) at \( p = p_0 \), \( C_p(T) \), \( \eta(T) \) and \( \chi_m(T) \) (\( m = 1, 2, \ldots, N_m \)) were taken from [23–27]. We tested models with \( N_m = 6, 10 \) and 37 spectral intervals [23–25].

2.2. Boundary conditions

The boundary conditions at the rectangular contour around the computational region describe the COD in an unbounded free space. Along the lower boundary (\( z = 0, 0 < r < R \)) we set

\[
p = p_0, \quad \frac{\partial v}{\partial z} = 0, \quad T = T_0, \quad U_m = U_{eq,m}, \quad \mathcal{E}(r, 0) = \bar{\mathcal{E}}(r),
\]

where \( m = 1, 2, \ldots, N_m \) and \( \bar{\mathcal{E}}(r) \) is the given function. Along the outer cylindrical surface (\( 0 < z < L, r = R \)) we set

\[
p = p_0, \quad \frac{\partial U_m}{\partial r} = 0, \quad \frac{\partial T}{\partial r} = 0, \quad \frac{\partial \mathcal{E}}{\partial r} = 0.
\]

We impose symmetry conditions at the \( z \)-axis (\( r = 0, 0 < z < L \)),

\[
\frac{\partial p}{\partial r} = 0, \quad \frac{\partial v}{\partial r} = 0, \quad v = 0, \quad \frac{\partial T}{\partial r} = 0, \quad \frac{\partial U_m}{\partial r} = 0, \quad \frac{\partial \mathcal{E}}{\partial r} = 0.
\]

The conditions at the outflow boundary (\( z = L, 0 < r < R \)) correspond to a one-dimensional gas flow,

\[
\frac{\partial p}{\partial z} = 0, \quad \frac{\partial v}{\partial z} = 0, \quad v = 0, \quad \frac{\partial T}{\partial z} = 0, \quad \frac{\partial U_m}{\partial z} = 0.
\]

3. Numerical results

We calculated the parameters of a COD in a vertical CO\(_2\) laser beam, free of external forced gas flow, as functions of the laser power \( P_0 = 2–6 \) kW at \( p_0 = 10^5 \) Pa and \( T_0 = 300 \) K. We used the numerical model and the corresponding numerical code (realized in Fortran 90) of the previous work [21]. The ‘hydrodynamic’ part of the system, which consists of coupled mass, momentum, energy conservation and radiation transport equations (2)–(4), (5), was solved by means of Patankar’s pressure-correction SIMPLE algorithm [28] with application of SIP iterations [29]. Control-volume cells of the ‘staggered’ grid decayed exponentially in both the \( r \) and the \( z \) directions towards the focal point to ensure sufficient spatial resolution in the hot plasma region. For the laser beam propagation equation (6) we employed the ‘method of lines’ [30]. This equation was transformed into the ODE system by a spatial discretization of the right-hand side,

\[
2ik \frac{d\mathcal{E}_j(z)}{dz} = F_j(\mathcal{E}_{j-1}, \mathcal{E}_j, \mathcal{E}_{j+1}),
\]

\( j = 2, \ldots, N_t - 1 \)

and boundary conditions. The grid employed for it was finer in the radial direction (compared with the grid for the ‘hydrodynamic’ part of the system) and uniform. The ODE-solver was applied to the resulting semi-discrete system (17) to integrate it upwards along the variable \( z \) starting with the initial condition (13). A variable step size mechanism of the solver, adapted to the behaviour of the computed field \( \mathcal{E} \), ensured a correct location of the focus and the correct field amplitude.

The dimensions of the computational domain were taken to be sufficiently large (a cylinder of length 10 cm along the axis and 2 cm in radius) to ensure a weak effect of the boundary conditions on the COD parameters in the hot region. The results plotted in figures 2–6 were computed on the (‘hydrodynamic’) grid with 96 nodes in the \( r \) direction by 288 nodes in the \( z \) direction.
The discharge was initially located in the middle of the chamber: the distance between the geometrical focal point and the bottom of the chamber was \( z_\text{f} = 5 \) cm. The laser beam was of Gaussian shape in the initial section, \( z = 0 \),

\[
E_\text{0}(r) = E_\text{0} \exp \left( -\frac{r^2}{2w^2} + \frac{kr^2}{2z_\text{f}} \right),
\]

(18)

The focusing geometry was chosen to be close to the conditions of the experiment [31], where the COD was produced in a free air atmosphere by a carbon-dioxide laser, operated at 10.6 \( \mu \)m, focused by a 15 cm focal-length lens along the vertical axis. The focusing used in the calculations corresponds to the case of the beam of diameter 2.5 cm (at \( e^{-2} \) intensity level). The spot size of the laser beam in a vacuum, \( w_0 \), (where the radius of the beam is taken at the \( e^{-1} \) laser radiation intensity level) was taken as 0.0284 mm, the beam radius at the initial section

\[
w = w_0 \sqrt{1 + (z_\text{f}/ku_0^2)^2} = 2.97 \text{ mm}.
\]

Figure 2(a) shows the temperature field and the mass flux vector field (\( G_{\text{max}} = 0.58 \text{ g s}^{-1} \)) in COD for a laser power \( P_L = 3 \) kW. Figure 2(b) is a close-up view of the plasma and gas-dynamic flow from panel (a). Temperature contour lines are from 2 to 18 kK in 2 kK increments. The maximum temperature of about 19 kK is shifted several millimetres upstream from the laser’s ‘geometrical’ focal point. The laser beam waist in the plasma of length 0.45 mm is about one order as large as that in a vacuum, \( w_0 = 0.0284 \text{ mm} \). Plasma behaviour and flow pattern around the hot plasma resemble those from previous modelling [21], where optical discharge was considered in the external gas flow: the convective flow (due to buoyancy force) streaming past the plasma plays a somewhat similar role to that of a forced convective flow under conditions [21]. Note that convective air flow around the discharge behaves like a flow over an obstacle: it flows primarily around the hot plasma, while a small fraction of the gas flow penetrates the highly heated region.

### 3.1. Effect of applied laser power

Figure 3 demonstrates the effect of increase in the laser power from \( P_L = 2–5 \) kW on the axial distributions of (a) plasma temperature \( T \), (b) laser radiation intensity \( I \), (c) axial component of velocity \( u \), (d) pressure \( p - p_0 \) and (e) electric field amplitude \( |E| \). In agreement with the experimental observations [4,31, 32] and theoretical [17–19,21] evidence, the increase in the laser power results in a shift of the plasma, initiated in the ‘geometrical’ focal region, along the laser beam towards the laser. It should be noted that models employing focusing geometry with a Gaussian beam (1) cannot capture this discharge behaviour (see, e.g. [14], where the calculation was performed under similar conditions in the framework of geometric optics). Peak plasma temperature, due to divergence of the laser beam in the plasma and consequently the expansion of the light channel and decrease in the laser intensity in the focal region, decreases with increase in applied laser power.

Calculations confirm the existence of minimum and maximum values of applied laser power, between which a steady-state COD is possible. At larger applied powers, when COD progresses to a point in the beam where the intensity is no longer sufficient to maintain it, it vanishes. However the power limits are not as strong as for the less realistic models with a Gaussian beam and prescribed light channel approach, for which under similar conditions the power limits reported were between \( P_L = 2 \) and 10 kW [17]. For example, at a laser power \( P_L = 1 \) kW our model still has a converging solution describing a tiny plasma formation of dimensions of about 4 mm in the axial and 2 mm in the radial direction.
Figure 3. Plasma temperature $T$ (a), laser radiation intensity $I$ (b), axial component of velocity $u$ (c), deviation from the atmospheric pressure $p - p_0$ (d) and electric field amplitude $|E|$ (e) along the $z$-axis in a COD in air for $p_0 = 1$ atm and laser powers $P_L = 2, 3, 4$ and 5 kW.

Note that figures 2(a), (b), together with the light channel borders at a laser power $P_L = 3$ kW, contain those also for the powers $P_L = 2$ and 4 kW (panel (b)), for which the locations of the minimum radii along the beam propagation direction (‘beam waists’) are just above and below the beam waist at the power level $P_L = 3$ kW. This illustrates the fact that with an increase in the laser power, plasma shifts from the initial focal point with the beam focus itself, and stabilizes in the place where the energy source is balanced by the energy losses due to radiation, thermal conductivity and convection. However, since the plasma cools down and shrinks as it moves upstream towards the laser, its effect on the divergence of the laser beam weakens with the increase in the applied laser power. Calculations showed that for higher laser powers, the discharge may not necessarily stabilize in the waist of the beam. With the degree of the beam focusing which we considered, already for $P_L = 5$ kW and higher powers, the discharge localizes in the place below the beam waist, where the beam intensity is still sufficient to maintain it (see figure 4). Here we should clarify that we determined the beam radius as a radius of a circular cross section in the transverse plane, so that the fraction of the laser power, passing across it, was equal to $1 - 1/e = 0.63$ of the total passing power. This is the level corresponding to $e^{-1}$ of the laser radiation intensity for the case of a Gaussian beam.

In the framework of our model, the location of the steady-state plasma front is a function of the applied laser power and focusing geometry, where the degree of beam focusing
is controlled by spot size $w_0$ at $z_F = 5$ cm in equation (18). Here we presented results specifically for $w_0 = 0.0284$ mm.

3.2. Effect of the method for the thermal radiation transport

Figures 5(a) and (b) illustrate the comparison of calculated radial and axial plasma temperature profiles with the experimental ones from [31]. Note that numerical temperature profiles were calculated under different approximations used for the thermal radiation (12), with the number of groups (bands or spectral intervals) $N_m = 6, 10$ and $37$, where data for group absorption coefficients $\chi_m(T)$ and limits of these spectral intervals were taken from [23–25], respectively.

All the group models tested perfectly predict the plasma temperature in the axial region, but discrepancy from the experimental data increases with increasing distance in the radial direction, most noticeably for the model with $N_m = 10$ (figures 5(a), (b)). Figure 5(b) demonstrates the location of the plasma temperature peak for models with $N_m = 6, 10$ and $37$. It should be noted that no attempts of parameter fitting have been aimed. Computed size, shape and location of steady-state plasma are very sensitive to the degree to which the laser beam is focused. However, the focusing degree is not clear from [31].

Although the approach [23] uses a smaller number of groups, $N_m = 6$, the data for $\chi_m(T)$ in [23] are also tabulated as a function of the plasma thickness $R_p$, which, as is suggested in [23], is roughly about the radial distance where the maximum axial temperature has dropped to 80% of its axial value. Figures 5 also illustrate the influence of the radial distance where the maximum axial temperature has dropped to 80% of its axial value. Larger $R_p$ leads to overestimation of the reabsorption of radiation in plasma, a more flat intensity distribution and a greater shift of the discharge towards the laser.

Figure 6 presents a comparison of the same parameters as in figure 3, calculated at laser powers $P_L = 3$ kW and with radiation transport approximated by means of $N_m = 6$ (with $R_p = 0.5$ mm), 10 and 37 spectral group models. Calculations showed that the approach used in the modelling of the thermal radiation transfer and reabsorption of the radiation in the plasma (negative term in equation (12)) has a noticeable effect on the energy balance in the plasma. Note that the peak radiation intensity, calculated with $N_m = 10$, is twice as high as that for $N_m = 37$, which, in turn, is nearly twice as high as the peak value of the intensity calculated with $N_m = 6$ (figure 6(b)).

4. Conclusions

We have presented the results for a full two-dimensional radiative gas-dynamic modelling of a COD, maintained by a vertical CO$_2$ laser beam in free air atmosphere, free of
the external gas flow, in the Earth’s gravitational field. The model takes into account all of the important factors that are of influence in laser-sustained plasma processes. As a further development of the existing radiative gas-dynamic models of COD, in order to take laser radiation refraction in the nonuniform plasma properly into consideration, it includes the amplitude equation for the laser radiation field in parabolic approximation of quasi-optics.

The effect of an increase in the applied laser power on the thermal and gas-dynamic structure of the discharge plasma has been shown. We also demonstrated the effect of the selection of spectral groups, used in the approximation of the thermal radiation, on the model solutions, performing calculations with 6, 10 and 37 spectral group models for the radiation transfer.

The model solutions improve the results of previous modelling approaches and it performs well in calculating the size, shape and temperature distribution of laboratory plasmas. The calculation confirms the experimental and theoretical evidence that the discharge plasma moves towards the radiation as the laser power increases and stabilizes in the place where the energy source is balanced by the energy losses due to radiation, thermal conductivity and convection.

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