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Modelling of a continuous optical discharge stabilized by a gas flow in quasi-optical approximation

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Abstract
We consider a continuous optical discharge (COD) sustained by a weakly focused CO₂ laser beam and stabilized by a cold gas flow incident in the direction of the laser radiation propagation. We develop and examine a two-dimensional radiative gas-dynamic model for COD which uses realistic quasi-optics and takes refraction of the laser radiation in the plasma properly into account in describing the laser beam propagation. The model is applied to calculate the parameters of COD in the air flow as functions of the laser power and inlet velocity of the incident flow.

1. Introduction

The investigation of a continuous optical discharge (COD) in an air flow is of considerable scientific and practical interest. (The process of propagation of COD in a gas medium is also known as the ‘laser supported combustion wave’.) It is stimulated by various applications in plasma chemistry, ecology, optics, rocket technology, etc [1, 2]. The efficient adoption of COD as a plasma source for these applications requires control over the COD plasma properties and an accurate determination of the conditions under which a stable COD can exist in a gas flow.

Experimental investigations of COD in a focused CO₂ laser beam [3–7] as well as its theoretical investigations on the basis of the radiative gas-dynamic models [8–17] reveal a picture of burning of the discharge and the features of its interaction with the external gas flow. In several studies (see, e.g. [10, 12]) concentrated on COD in a gas flowing parallel to the laser beam, it has been shown that the gas flow incident to the plasma behaves like a flow over an obstacle. It flows around the hot discharge region and forms the thermal and gas-dynamic fronts of the plasma. Only a small fraction of the cold gas enters the discharge in its axial region. Afterwards, this gas expands as a result of heating and fills the entire high-temperature region. The experimental data on the parameter range in which COD can occur in a focused CO₂ laser beam in an air flow were presented by Generalov et al [4], who showed that the discharge is stable in parallel (co-directional) and perpendicular (with respect to the laser beam) gas flows and is unstable in a gas flowing towards the beam. As the gas-flow velocity increases, the COD contracts and moves towards the beam waist. The conditions for a stable discharge to occur in a gas flow were investigated theoretically in [12–14, 18]. Numerical simulations carried out by Surzhikov and Chentsov [14] on the basis of the radiative gas-dynamic model predicted the domain of existence of the COD in terms of laser power and longitudinal flow velocity; this domain was in good agreement with the experimental data [4]. In [18], Kulumbaev and Lelevkin applied an analogous approach to simulate a three-dimensional COD in a transverse gas flow under the experimental conditions [4]. Furthermore, parameters of a COD excited in crossed CO₂ laser beams in air have been calculated in [19] as functions of the gas-flow velocity and the ratio between the power of the laser beams. The questions concerning the instability and bifurcation of gas flow streaming past the COD have been considered in [20, 21].

The most complete description of COD in a forced convective flow is presented in the literature by means of two-dimensional radiative gas-dynamic models, which include conservation equations for mass, momentum and energy, together with the equation of radiation heat transport. These are supported by realistic transport and thermodynamic properties as well as the radiation emission and absorption coefficients. Also, most consider a focusing geometry, where
radial distribution of the laser radiation intensity, \( I(r, z) \), is Gaussian,

\[
I(r, z) = \frac{P_l}{\pi w^2(z)} \exp\left(-\frac{r^2}{w^2(z)}\right),
\]

where the light channel boundary is fixed and determined by the beam radius \( w(z) \) (\( P_l \) is the laser power, \( w(z) \) is the distance from the beam axis where the intensity drops to \( e^{-1} \) of its axial value and \( r \) and \( z \) are cylindrical coordinates). Effects due to the inhomogeneous refractive index within the plasma are neglected in these calculations. However, we note that refraction in the nonuniform plasma can essentially change the light channel geometry and space distribution of the beam intensity. Inevitably, there can be a change in the discharge processes in gas-dynamic flow under appropriate boundary conditions. This is a complex nonlinear task, which requires extensive numerical calculations.

In [9, 10, 15], refraction of the laser beam in the plasma was approximately taken into account for parabolic radial distributions of the refraction index, absorption coefficient, electron density and Gaussian radial distribution of the intensity in the beam. In [22], a more accurate parabolic equation of beam propagation was used for COD in a stationary gas, but with a rather rough description of the radiative transfer, so that there were both quantitative and qualitative discrepancies in results reported in [9, 22].

A complete two-dimensional theoretical calculation of COD in a gas flow taking the laser beam refraction in the plasma into account on the basis of the beam propagation equation in parabolic approximation of quasi-optics has not been reported yet. This study mainly aims to fill this gap.

The objectives of this investigation are to first develop a two-dimensional radiative gas-dynamic numerical code based on an adequate physical model and, second, predict the properties of COD in a forced gas-dynamic flow. Consequently, the major improvement of the mathematical model in this work compared with the existing models is the application of a more realistic quasi-optical approximation to model the laser radiation propagation through the focusing system. Although the developed computer model has the capability to be applied to more complex flows, for the purpose of examining the model and comparing it with other modelling approaches and existing experimental data, the calculation presented in this paper were performed for a COD in a simple homogeneous flow.

This paper is organized as follows: section 2 introduces the model of COD in a gas flow and the method of solution. As an example of an application for the model, section 3 presents the results of the calculation, namely the characteristics of the COD. Finally, section 4 summarizes the results.

2. Model

We consider a steady-state COD sustained by a weakly focused CO\(_2\) laser beam. We also introduce cold air flowing in the direction of laser radiation propagation, a schematic is presented in figure 1. We assume that the gas flow is subsonic and laminar. Moreover, we assume that the pressure is atmospheric; this allows for a local thermodynamic equilibrium (LTE) to be maintained in the discharge plasma. Therefore, the plasma is described by a single temperature, and all its thermodynamic and transport coefficients are functions of this temperature and pressure. Finally, assuming an axial symmetry, we introduce cylindrical coordinates \((r, z)\), orienting the \(z\)-axis along the optical axis.

### 2.1. Equations

The model takes into account all of the important factors that are of influence in the laser-sustained plasma process. As a further development of the existing radiative gas-dynamic models of COD in forced convective flow, the model includes a parabolic equation for beam propagation in order to take laser radiation refraction in the nonuniform plasma properly into consideration. We determine the thermal and gas-dynamic structure of COD by solving the following set of equations:

**Continuity equation,**

\[
\frac{1}{r} \frac{\partial}{\partial r} (r \rho v) + \frac{\partial}{\partial z} (\rho u) = 0. \tag{1}
\]

The compressible Navier–Stokes equations,

\[
\rho \left( \frac{\partial v}{\partial r} + u \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial r} + 2 \frac{\partial}{\partial r} \left( \eta \frac{\partial u}{\partial r} \right) - \frac{2 \eta v}{r^2} + \frac{2 \eta v}{r} \frac{\partial}{\partial z} \left( \rho u \right), \tag{2}
\]

\[
\frac{\partial v}{\partial z} + \eta \left( \frac{\partial v}{\partial r} + \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial z} + \frac{2}{3} \eta \left( \frac{\partial v}{\partial r} + \frac{\partial u}{\partial z} \right), \tag{2}
\]

\[
\rho \left( \frac{\partial u}{\partial r} + u \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial z} + 2 \frac{\partial}{\partial z} \left( \eta \frac{\partial u}{\partial z} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( \eta \frac{\partial v}{\partial r} + \frac{\partial v}{\partial z} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( \eta \frac{\partial u}{\partial r} + \frac{\partial u}{\partial z} \right). \tag{3}
\]
The equation of energy balance, which is determined by the processes of convection, heat conduction, laser radiation absorption and selective thermal radiation transport,

\[ \rho C_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial r} + \frac{\partial T}{\partial z} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left( r \lambda \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left( \lambda \frac{\partial T}{\partial z} \right) + Q_L - \rho q. \]  

(4)

The amplitude equation of a weakly focused laser beam propagating in the z direction in parabolic approximation of quasi-optics can be written as [22],

\[ 2i k \frac{\partial \mathcal{E}}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \mathcal{E}}{\partial r} \right) + k^2 (\varepsilon_e - 1) \mathcal{E}. \]

(5)

Power density of the heat sources associated with the selective thermal radiation transport is determined in the multigroup diffusion approximation [9, 23]

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial U_m}{\partial r} \right) + \frac{1}{3 \chi_m} \frac{\partial \chi_m}{\partial z} = \chi_m (U_m - U_{eq,m}), \]

\[ m = 1, 2, \ldots, 10. \]

(6)

Here, \( v \) and \( u \) are the projections of the velocity vector onto the \( r \) and \( z \) axes, \( T \) is the temperature, \( p \) is the pressure, \( C_p \) is the heat capacity at a constant pressure, \( \eta \) and \( \lambda \) are the viscosity and the thermal conductivity, respectively. Since the flow is substantially subsonic, the pressure drops are small and the pressure is approximately constant, \( p(\rho, T) \approx p_0 \). Under this condition, the gas density is uniquely related to the temperature of the gas, \( \rho = \rho(T, p_0) \).

We assume a laser field of the form

\[ E(r, z) = \mathcal{E}(r, z) \exp(-ikz + i\omega t), \]

(7)

where \( \mathcal{E} \) is the complex amplitude of the field, \( k = \omega/c, \omega \) is the angular frequency of laser radiation, and \( c \) is the speed of light.

The complex dielectric permittivity of the plasma, \( \varepsilon_e \), is defined by

\[ \varepsilon_e = 1 - \frac{n_e}{n_e^*} - \frac{\varepsilon \mu}{k}, \]

(8)

where the volume coefficient of the CO\(_2\) laser radiation absorption, \( \mu \), is given approximately by the formula [17],

\[ \mu = \frac{2.82 \times 10^{-20} n_e^2}{T^{3/2}} \lg \frac{2.17 \times 10^3 T}{n_e^{3/2}} \]

(9)

\( n_e \) is in \( \text{cm}^{-3} \), \( T \) in K and \( \mu \) in \( \text{cm}^{-1} \), \( n_e \) is the equilibrium electron number density and is determined from the Saha equation. The critical plasma density at which its dielectric permittivity vanishes, \( n_e \), is given as

\[ n_e = \frac{\varepsilon_0 m_e c^2}{e^2}, \]

(10)

where \( e \) and \( m_e \) denote the electron charge and electron mass, respectively.

\( Q_L \) expresses the specific power of the energy release within the plasma caused by the laser radiation absorption,

\[ Q_L = \frac{1}{2} \mu e_0 c |\mathcal{E}|^2, \]

(11)

where \( \epsilon_0 \) is permittivity of vacuum.

\( \chi_m, U_m \) and \( U_{eq,m} \) are the group values of the volume absorption coefficient, the radiation density of plasma, and ideal blackbody, averaged over each of the \( N_m \) spectral intervals, \( m = 1, 2, \ldots, 10 \). The power density of the heat sources associated with thermal radiation transport is

\[ Q_R = \sum_{m=1}^{n_m} c \chi_m (U_{eq,m} - U_m), \]

(12)

where

\[ U_{eq,m} = \frac{4}{c} \sigma_m(T, \nu_m, \nu_{m+1}) T^4 \]

(13)

with

\[ \sigma_m(T, \nu_m, \nu_{m+1}) = \frac{2\pi k_B^4}{c^3 h^5} \left[ \sigma \left( \frac{h \nu_{m+1}}{k_B T} \right) - \sigma \left( \frac{h \nu_m}{k_B T} \right) \right]. \]

(14)

\[ \sigma(x) = \int_0^x \frac{\xi^3}{e^{\xi^2} - 1} d\xi, \]

(15)

where \( h \) is the Planck constant and \( k_B \) is the Boltzmann constant [23].

2.2. Boundary conditions

The boundary conditions at the rectangular contour around the computational region describe the COD in an unbounded free space. The conditions at the upstream boundary (\( z = 0, 0 < r < R \)) describe an unperturbed uniform flow,

\[ u = u_0(r), \quad v = 0, \quad T = T_0, \quad U_m = U_{eq,m}, \]

(18)

\[ \mathcal{E}(r, 0) = \mathcal{E}_0(r), \]

(16)

where \( u_0(r) \) and \( \mathcal{E}_0(r) \) are given functions.

At the outflow boundary (\( z = L, 0 < r < R \)) correspond to a one-dimensional gas flow,

\[ \frac{\partial u}{\partial r} = 0, \quad v = 0, \quad \frac{\partial T}{\partial r} = 0, \quad \frac{\partial U_m}{\partial r} = 0, \]

\[ \frac{\partial \mathcal{E}}{\partial r} = 0, \quad \frac{\partial p}{\partial r} = 0. \]

(19)

\[ \mathcal{E}(r, L) = \mathcal{E}_0(r), \]

(17)

We impose symmetry conditions at the \( z \)-axis (\( r = 0, 0 < z < L \)),

\[ \frac{\partial u}{\partial z} = 0, \quad \frac{\partial T}{\partial z} = 0, \quad \frac{\partial U_m}{\partial z} = 0, \]

\[ \frac{\partial p}{\partial z} = 0. \]

(18)
2.3. Solution method

We solved the above set of equations numerically using physical variables. In calculations, two separated computational grid systems were used. The first one, the ‘staggered’ grid, for the ‘hydrodynamic’ part of the system, which consists of coupled mass, momentum, energy conservation and radiation transport equations (1)–(4) and (6). Another independent grid was used to describe the propagation of the laser beam, described by equation (5). During the iteration process, values of the electric field $E$ (solution of (5)) were projected onto the nodes of the ‘hydrodynamic’ grid.

For the ‘hydrodynamic’ equations, numerical method was based on the control-volume technique and Patankar’s SIMPLE algorithm [24]. The systems of five-point difference equations (finite-difference counterparts of equations (1)–(4) and (6)) were solved by the SIP method [25]. In order to produce better spatial resolution in the high-temperature region, cells of the ‘staggered’ grid decreased exponentially in both the axial and the radial directions towards the focal point of the laser beam. The results plotted in figures 2–5 were computed on the grid with 96 nodes in the $r$ direction by 288 nodes in the $z$ direction.

The solution of equation (5) for the electric field $E$ required a special approach. Numerical tests on the model problem in a vacuum showed that conventional finite-difference methods were not applicable there. Instead, we employed the ‘method of lines’ [26]. Equation (5) was transformed to a system of ODEs by discretization of the right-hand side and the boundary conditions. The ODE solver was applied to the resulting semi-discrete system to integrate it upwards along the variable $z$ starting with initial condition (16) for $E$. A variable step size mechanism of the solver adopted to the behaviour of the
The computed field $E_0$ provided a correct location of the focus and the correct field amplitude. The overall numerical approach will be described in detail elsewhere.

The data on the temperature dependence of the thermodynamic, transport and optical properties of air, which are $\rho(T)$ at $p = p_0$, $C_p(T)$, $\eta(T)$ and $\chi_m(T)$ ($m = 1, 2, \ldots, 10$) were taken from [27–29].

3. Numerical results

To illustrate and examine the model, we applied it to the case of COD sustained by a converging laser beam in an unbounded air space at atmospheric pressure, $p_0 = 10^5$ Pa. We also considered a ‘cold’ ($T_0 = 300$ K) air flow propagating co-directionally with the laser radiation. The gas flow was assumed to be uniform rectilinear and incident on the plasma at velocity $u_0(r) = u_0$, where $u_0$ changed from 0.1 to 1 m s$^{-1}$. The dimensions of the computational domain were taken to be sufficiently large (a cylinder of length 10 cm along the axis and 4 cm in radius) so that the boundary conditions had no influence on the numerical results in the hot region. The discharge was initially located in the middle of the chamber: the distance between the geometrical focal point and the bottom of the chamber was $z_F = 5$ cm. The laser beam was taken as a Gaussian in the initial section, $z = 0$,

$$E_0(r) = E_0 \exp \left( \frac{r^2}{2w^2} + \frac{kz^2}{2z_F^2} \right).$$

The spot size of the laser beam in a vacuum, $w_0$, (where radius of the beam is taken at the $e^{-1}$ laser radiation intensity level) was taken as 0.03 mm, and the corresponding beam radius at the initial section $w = u_0 \sqrt{1 + (z_F/k u_0^2)^2} = 2.81$ mm. The relation between the applied laser power $P_L$ and the electric field amplitude in the initial section is

$$E_0 = \frac{1}{w} \sqrt{\frac{2P_L}{\pi \varepsilon_0 c}}.$$

The plasma was maintained by a CO$_2$ laser with an output power $P_L = 2–6$ kW and burnt in free space at atmospheric pressure.

The computed temperature and gas-dynamic structure of a COD is presented in figures 2–5. The model solutions improve the results of previous modelling approaches [9, 10] and predict well the size, shape and temperature distribution of laboratory plasmas [3] as well as a COD behaviour in the external gas-dynamic flows [4]. Specifically, our model solutions for $P_L = 6$ kW and incident flow velocities about 0.25–0.5 m s$^{-1}$ predict perfectly the discharge location and maximum plasma temperature measured in the experiment [3], where steady-state COD in free air atmosphere was produced by a CO$_2$ laser beam focused along a vertical axis, with $P_L = 6$ kW, a lens focus of 15 cm and laser beam diameter of 2.5 cm. The discharge localizes about 1 cm forward from the focal point and its maximum temperature is about 17.5 K K exactly as in the experiment. Calculated sizes of the discharge in the axial and radial directions are somewhat smaller than in the experiment, which are 14 mm and 3 mm. Although the discharge in [3] is free from the external forced flow, the convective flow (due to buoyancy force) streaming past the discharge plays a somewhat similar role to that of a forced convective flow as in our model.

We also note that appearance of oscillations and eddy structures in the region of cooled jet, reported in [10] (and also mentioned in [15]), has nothing to do with physical reality and seems to be purely computational. Parameter regime in [10], which follows the experimental conditions [4] (the atmospheric pressure air, laser power of $P_L = 6$ kW, a

![Figure 3](image_url)
lens focus of 20 cm, laser beam diameter of 4 cm, incident flow velocity 1 and 3 m s$^{-1}$), has a focusing (ratio of the diameter of the beam to the focal lengths of the lens) close to one used in our calculations (and also close to the focusing in the experiment [3]). However, our calculations under a similar parameter regime with $u_0 = 1$ m s$^{-1}$ did not reveal any instability in the flow. It should be noted that our modelling approach follows the one implemented in [10, 15], but applies a more realistic approximation to describe the laser radiation propagation through the focusing system. Namely, it uses a laser beam propagation equation in parabolic approximation of quasi-optics.

Figure 2 illustrates the typical shapes of the plasma temperature $T$ (a), deviation from the atmospheric pressure $p - p_0$ (d) and electric field amplitude $|E|$ (e) along the $z$-axis in a COD in air for $p = 1$ atm and laser powers $P_L = 2, 3, 4$ and 5 kW. Inlet gas velocity is $u_0 = 0.5$ m s$^{-1}$.

Figures 3 shows the temperature field (isothermal contour lines) and mass flux vector field (gas streamlines, $G_{max} = 2.92$ g s$^{-1}$) in COD for inlet gas velocity $u_0 = 0.5$ m s$^{-1}$ and laser power $P_L = 5$ kW. Temperature contour lines are from 4 to 18 kK in 2 kK increment. Figure 3(b) is a close-up view of the same plasma. The maximum temperature of about 18 028 K is shifted upstream several millimetres from the laser ‘geometrical’ focal point. Laser beam waist in the plasma
of 0.43 mm is about one order as large as that in a vacuum, $w_0 = 0.03$ mm.

Calculations confirm the experimental [3, 4] and theoretical [9, 10, 15] evidence that due to refraction of the laser beam in the discharge plasma and consequently the divergence of the beam in the plasma, the plasma initiated in a ‘geometrical’ focal region is shifted from this region towards the laser radiation source. It shifts together with the focus itself, and stabilizes in the place where the energy source is balanced by the energy losses due to radiation, thermal conductivity and convection.

Formation of steep thermal and gas-dynamics fronts (from the side of the incident laser light) and trailing edges of the high-temperature core of the COD can be seen from figures 2–5. Ahead of the thermal front, high pressure space is formed, which deflects the cold gas, incident to the plasma, in the radial direction, and forces it to flow primarily around the hot plasma (along 10 kK isotherm). Only a small fraction of the axial flow penetrates the highly heated region, where the gas flow is decelerated by increasing the positive pressure gradient as it traverses the front of the discharge core. In the region behind the front, where pressure falls sharply, the gas is heated, expands and fills in the high-temperature region. Inside the discharge core, the pressure is depressed and remains essentially unchanged (figures 4 and 5). Farther along the discharge axis, the change in the discharge parameters is governed by the reverse radial gas motion from the discharge region, where the laser power is dissipated, towards the trailing edge. Due to continuity of the motion, the negative pressure gradient occurs at the trailing edge, which ensures that the same amount of the gas flows from the laser power deposition region towards the trailing edge of the discharge. Specifically,
this is supported by the approximate equality of the maximum and minimum values of the pressure at the front and trailing edge of the discharge core.

3.1. Effect of applied laser power

Figure 4 shows the distributions of temperature $T$ ($a$), laser radiation intensity $I$ ($b$), axial component of velocity $u$ ($c$), deviation from the atmospheric pressure $p - p_0$ ($d$) and electric field amplitude $|\mathbf{E}|$ along the z-axis in the COD sustained by a laser power of $P_L = 2, 3, 4$ and 5 kW. Inlet gas-flow velocity was $u_0 = 0.5 \text{ m s}^{-1}$.

Calculations confirm the existence of minimum and maximum values of applied laser power, between which a steady-state COD is possible [9]. In agreement with [7, 9], the increase in the laser power results in a shift of the plasma along the laser beam towards the laser. Peak plasma temperature, due to expansion of the light channel and, correspondingly, decrease in laser intensity in the focal region, decreases with increase in applied laser power. If COD progresses to a point in the beam where the intensity is no longer sufficient to maintain it, it vanishes. Incidence cold gas flow, directed counter to the direction of COD propagation, can stabilize the COD.

3.2. Effect of inlet velocity

Figure 5 shows the distributions of the same parameters as in figure 4 computed for COD under a laser power of $P_L = 3 \text{ kW}$ and inlet gas velocities of $u_0 = 0.1, 0.25, 0.5$ and 1 m s$^{-1}$. Results show that flow velocity has a strong effect on the plasma parameters and the intensity of laser radiation needed to sustain a stationary plasma. Compared with the effect of the applied laser power, an increase in the convection velocity has the opposite effect. It tends to push the plasma downstream towards the ‘geometrical’ focal point, and thus reduce the path length of the laser beam in the plasma. By these means, forced convective flow can stabilize the COD supported by the weakly converging laser beam.

The position of the plasma front is a function of inlet velocity, applied laser power and focusing geometry. In the framework of the model, focusing is controlled by spot size $w_0$ at $z_F = 5 \text{ cm}$ fixed. Here we presented results specifically for $w_0 = 0.03 \text{ mm}$.

4. Conclusions

We have presented a full two-dimensional radiative gas-dynamic model of a COD, which uses realistic quasi-optics to describe the laser radiation propagation.

The effect of the inlet velocity of ‘cold’ air and applied laser power on the thermal and gas-dynamic structure of discharge plasmas at atmospheric pressure have been studied.

The model solutions improve the results of previous modelling approaches and the model performs well in calculating the size, shape, and temperature distribution of laboratory plasmas. The calculations confirm the experimental and theoretical evidence that the discharge moves towards the radiation source together with the effective focal point and localizes at the position of the beam waist.

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References