The Minimal Number of Lefschetz Fibrations

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Donaldson and Gompf results give the relation between symplectic 4-manifolds and Lefschetz fibrations, which are a fibering of a 4-manifold by surfaces, with a finite number of singularities of a prescribed type. Their results say that symplectic 4-manifolds (after perhaps blowing up) admit the structure of a Lefschetz fibration and a genus-g Lefschetz fibration with a fiber genus $g \geq 2$ over the Riemann surface admits a symplectic structure. Hence, Lefschetz fibrations provide a combinatorial way to study symplectic 4-manifolds. The results on the minimal number of vanishing cycles of a Lefschetz fibration provide some results on symplectic 4- manifolds, which also gives a connection between symplectic topology and geometric group theory.

In this talk, after giving some background on the mapping class group, we will discuss the minimal number of Lefschetz fibrations.