An Introduction to Classification of Four-Manifolds

One of the central problem in topology is to classify n-dimensional manifolds. In this talk, before focusing on the nature of four-manifolds, we shall discuss a number of possible techniques that have been developed to classify n-dimensional manifolds in an historical order. Classically, we will start with compact connected manifolds of dimension two without boundary; i.e. compact connected surfaces without boundary, and we define possible topological invariants. After the classification for surfaces, we introduce a number of additional mathematical objects with the aid of algebraic topology for the classification problem of n-dimensional manifolds where $n \geq 3$. As we shall see, because of several technical reasons we restrict our attention to compact simply connected n-manifolds without boundary so that one can hope to answer the classification problem in a general framework. However, the answer for the dimension 3 and 4 is quite strange and unexpectedly complicated than that of analogous problem in the dimension 5 and higher. In 1980s, there were two breakthroughs that remarkably shaded light on the classification problem for simply-connected four-manifolds: one was the work of M. Friedmann that was completely topological, and the other was the work of S. Donaldson that used instantons arising from Yang-Mills *SU(2)* gauge theory. In the final part of the talk, we will briefly discuss these two breakthroughs and beyond.

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