ME 522

PRINCIPLES OF ROBOTICS

PROBLEM SET 3

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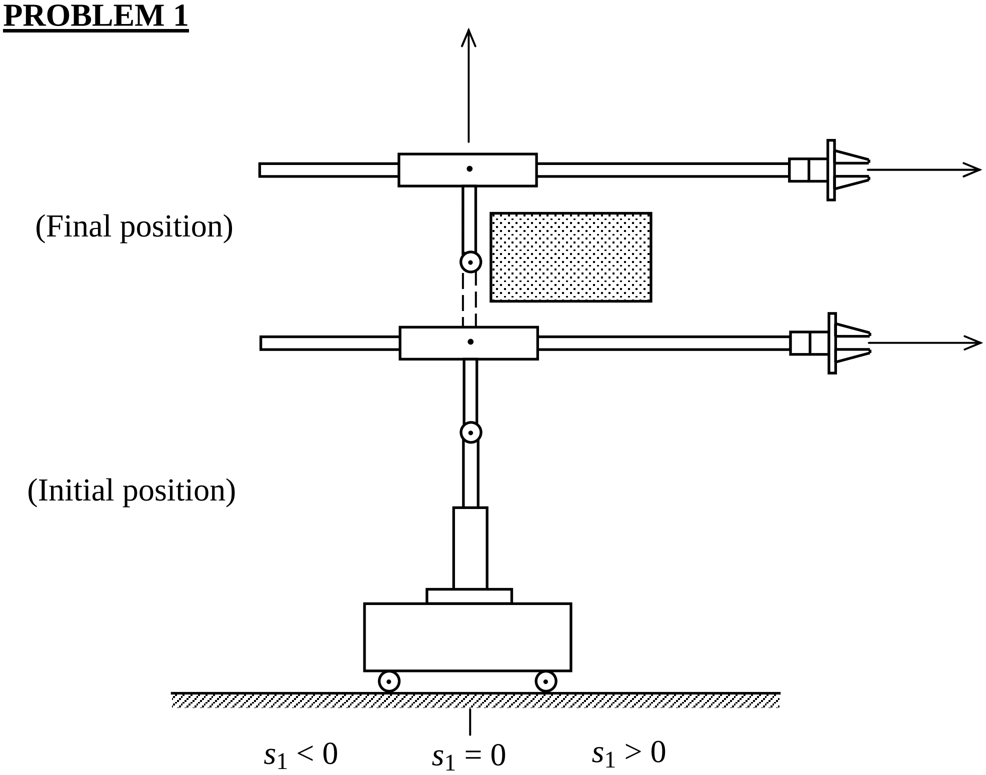
PROBLEM 1



As one of the tasks to be conducted, two successive positions of the robot in Problem 7 of Set 2 are shown in the figure. The shaded rectangle represents an obstacle that must be avoided by the robot as it conducts this task.

a) As noted, it is not necessary to use all the joints of the robot in order to conduct the indicated task. Choose a suitable set of joints to use for this purpose.

b) Plan the task with minimum number of motion steps. Use cubic splines to define the motion steps of the task. Sketch rough plots to show the time histories of the chosen joint variables during the whole task.

c) Describe how you can determine the timing of each motion step. Do this by first developing the necessary equations and/or inequalities considering the cubic splines and the limitations of the actuators on their maximum speeds and accelerations. Then, discuss how to solve these equations and/or inequalities.

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PROBLEM 2

The constant length parameters of the manipulator in Problem 6 of Set 2 are

$$a_2 = 2 \text{ m}, \quad a_3 = 1.5 \text{ m}, \quad d_p = 0.25 \text{ m}.$$

In a particular task, while the *tip point* is positioned as

$$\bar{p} = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix},$$

the hand pulls a rope gripped by its finger tips in such a way that the *normal vector* $\vec{u}_n$ is *vertical*, i.e. $\vec{u}_n = \vec{u}_3^{(0)}$, and the *approach vector* $\vec{u}_a$ is *horizontal*, i.e. parallel to the plane of $\vec{u}_1^{(0)}$ and $\vec{u}_2^{(0)}$, with a *positive component* along $\vec{u}_1^{(0)}$. The tension force applied by the rope to the tip point is $\vec{F} = F\,\vec{u}_2^{(0)}$ with $F = 50$ N.

(a) Show that the hand orientation required as above does *not* specify the matrix $\hat{C}$ completely, but reduces it to a function of only θ_1 in the following form:

$$\hat{C} = e^{\tilde{u}_3\theta_1}\, e^{-\tilde{u}_2\pi/2}\, e^{\tilde{u}_1\pi/2}.$$

This, in turn, leaves the wrist point equation with four unkowns: θ_1, θ_{23}, θ_3, and s_4. However, by expressing $\bar{r}$ in terms of $\bar{p}$ and θ_1, you can show that θ_1 can still be determined from the wrist point equation. Hence, determine θ_1. Then, within the remaining redundancy among θ_{23}, θ_3, and s_4, determine θ_3 and s_4 in terms of θ_{23}. Explain why this redundancy occurs and illustrate its occurence by drawing a local sketch of the manipulator. Also, determine the range of θ_{23} so as to guarantee the existence of a solution.

(b) Ignoring the gravity effect and choosing a suitable value for θ_{23} within the range you determined above in part (a), calculate the actuator force and torques that are necessary to keep the manipulator in static equilibrium.

PROBLEM 3

During a process of motion planning, the initial, the intermadiate, and the final values of a joint variable q are specified as follows:

$$q_0 = 0 \text{ rad}, \quad q_1 = 2 \text{ rad}, \quad q_2 = 1 \text{ rad}.$$

The initial and final values of the joint velocity are required to be zero, i.e.

$$\dot{q}_0 = \dot{q}_2 = 0 \text{ rad/s}.$$

The intermediate value $\dot{q}_1$ of the joint velocity is yet to be determined.

The trajectory between the specified points is to be determined by means of cubic splines with equal time steps so that $t_0 = 0$, $t_1 = T$, and $t_2 = 2T$.

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The acceleration discontinuities at the junction points are defined as follows:

$$\Delta\ddot{q}_0 = \ddot{q}(t_0^+) - \ddot{q}(t_0^-) = \ddot{q}_0 - 0,$$
$$\Delta\ddot{q}_1 = \ddot{q}(t_1^+) - \ddot{q}(t_1^-) = \ddot{q}_1^+ - \ddot{q}_1^-,$$
$$\Delta\ddot{q}_2 = \ddot{q}(t_2^+) - \ddot{q}(t_2^-) = 0 - \ddot{q}_2.$$

a) Express $\Delta\ddot{q}_0$, $\Delta\ddot{q}_1$, and $\Delta\ddot{q}_2$ in terms of $\dot{q}_1$ and *T*.

b) Find $\dot{q}_1$ in terms of *T* so that the function *U* defined below is minimized.

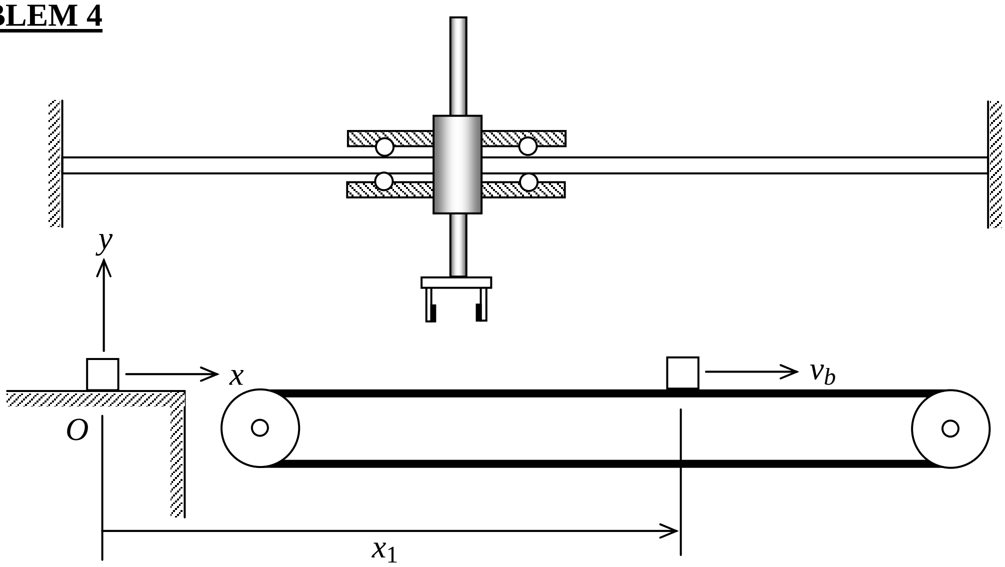
$$U = (\Delta\ddot{q}_1)^2 + (\Delta\ddot{q}_2)^2 + (\Delta\ddot{q}_3)^2.$$

c) The actuator driving this joint has the following speed and acceleration limitations:

$$\dot{q}_{max} = 10\,\text{rad/s} \quad \text{and} \quad \ddot{q}_{max} = 40\,\text{rad/s}^2.$$

Determine the minimum possible *T*.

PROBLEM 4



The figure shows a *gantry robot* having prismatic joints in the *x* and *y* directions. It is to be used for a task of picking up an object from a conveyor belt, carrying and dropping it to the origin *O* of the *xy*-frame, and doing the same for the next object.

Let 2*T* be the *task cycle time*, i.e. the time elapsed between the departure from and arrival back at the origin. The velocity and acceleration at the origin are to be zero at both the departure and arrival instances.

The motion in *x* direction is planned to be composed of two symmetrical quartic splines defined for the two halves of one cycle. That is,

$$x(t) = x^* f(t) \qquad \text{for} \quad 0 \le t \le T,$$

$$x(t) = x^* f(2T - t) \qquad \text{for} \quad T \le t \le 2T;$$

$$f(t) = b_3 t^3 + b_4 t^4, \qquad x^* = x(T) \qquad \text{with} \qquad \dot{x}^* = \dot{x}(T) = 0.$$

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a) Show that $b_3 = 4/T^3$ and $b_4 = -3/T^4$.

b) Show that the maximum acceleration in the x direction occurs at $t = T$ and its value is given for both splines as

$$a_x^* = |a_x(T)| = 12x^* / T^2 .$$

c) Let T_1 ($0 < T_1 < T$) be the pick up instant. The conveyor belt moves at a constant velocity v_b. Therefore, the robot is supposed to have zero acceleration at $t = T_1$. Show that this can be achieved if

$$T_1 / T = 2/3 .$$

d) Let $x_1 = x(T_1)$ be the pick up distance. Express T, T_1, a_x^*, and the overshoot $\Delta x^* = x^* - x_1$ in terms of v_b and x_1. Show that T and Δx^* decrease but a_x^* increases as x_1 decreases.

e) Let $v_b = 2\,\text{m/s}$ and $|a_x(t)|$ be limited by $a_{\max} = 10\,\text{m/s}^2$. Determine the minimum task cycle time and the corresponding pick up distance.

f) As for the motion in the y direction, it is to be planned such that $0 \le y(t) \le y^*$ with the additional conditions that $y(t) = 0$, $v_y(t) = 0$, and $a_y(t) = 0$ whenever $t = 0$, $t = T_1$, and $t = 2T$.

g) Adapting the formula obtained in part (b), determine the maximum acceleration a_y^* in the y direction and the instant at which it occurs.

h) Let $|a_y(t)|$ be limited by $a_{\max} = 10\,\text{m/s}^2$ as in the x direction. Determine the maximum possible elevation y^*.

i) In order to visualize the motion, draw $x(t)$ and $y(t)$ versus t as two roughly sketched plots one above the other.

<u>**PROBLEM 5**</u>

In a motion planning task, a joint variable $q(t)$ is required to obey the following trajectory requirements for $0 \le t \le 2$:

$$q(0) = 0, \qquad \dot{q}(0) = 0, \qquad \ddot{q}(0) = 0;$$

$$q(2) = 0, \qquad \dot{q}(2) = 0, \qquad \ddot{q}(2) = 0;$$

$$q(1-) = q(1+) = 2, \qquad \dot{q}(1-) = \dot{q}(1+), \qquad \ddot{q}(1-) = \ddot{q}(1+).$$

For this purpose, the following quintic splines are used:

$$q(t) = b_0 t^3 + c_0 t^4 + d_0 t^5 \qquad \text{for } 0 \le t \le 1;$$

$$q(t) = 1 + b_2 (t-2)^3 + c_2 (t-2)^4 + d_2 (t-2)^5 \qquad \text{for } 1 \le t \le 2.$$

Note that the initial and terminal conditions are already satisfied with these splines.

a) In order to satisfy the conditions at $t=1$, show that

$$(c_0+c_2)+2(b_0-b_2)=15\,,$$

$$4(c_0-c_2)+7(b_0+b_2)=10\,.$$

b) Define the jerk differences as follows for $k=0,1,2$:

$$\Delta_k=\dddot{q}(k+)-\dddot{q}(k-) \quad \text{with} \quad \dddot{q}(0-)=\dddot{q}(2+)=0\,.$$

Then, show that

$$\Delta_0=6b_0\,,\quad \Delta_2=-6b_2\,,\quad \Delta_1=36(c_0+c_2)+54(b_0-b_2)-180\,.$$

c) In order to even up the jerk differences (why to do this?), take Δ_0 to be positive but as small as possible (why positive?), take $\Delta_2=+\Delta_0$ (why with + sign?), and take $\Delta_1=\sigma\Delta_0$ with the sign variable σ (+1 or –1). Then, determine σ and the coefficients of the splines.