ME 522
PRINCIPLES OF ROBOTICS

PROBLEM SET 2

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PROBLEM 1

The figure shows a 6-dof mobile robot. It consists of a carriage and an onboard manipulator. The position and orientation of the carriage with respect to the *foundation frame* $\mathcal{F}_f$ is described by the coordinates s_1, s_2, and the heading angle θ_3. The joint variables of the manipulator are θ_4, θ_5, and θ_6. The variables s_1, s_2, and θ_3 can be treated as if they are associated with hypothetical prismatic and revolute joints.

a) When $\vec{u}_3^{(0)}$ of the *base frame* $\mathcal{F}_0$, which is defined according to the D-H convention, is not vertically upward as in the present case, it may be more convenient to define a *foundation frame* $\mathcal{F}_f$, whose $\vec{u}_3^{(f)}$ is vertically upward in the accustomed way. As seen in the figure, $\mathcal{F}_f$ and $\mathcal{F}_0$ are coincident but their axes are indexed differently so that

$$\vec{u}_1^{(f)} = \vec{u}_3^{(0)}, \quad \vec{u}_2^{(f)} = \vec{u}_1^{(0)}, \quad \vec{u}_3^{(f)} = \vec{u}_2^{(0)}.$$

Show that the transformation matrix between $\mathcal{F}_f$ and $\mathcal{F}_0$ can be expressed as follows:

$$\hat{C}^{(f,0)} = e^{\tilde{u}_1\pi/2} e^{\tilde{u}_2\pi/2}.$$

b) Determine the *link parameters*, *joint variables*, and the *link frame origins* associated with this mobile robot according to the D-H convention. Meanwhile, indicate the relevant common normals and the joint variables on the auxiliary views drawn below the main figure.

c) Let the *orientation of the hand* and the *location of the wrist point* with respect to $\mathcal{F}_0$ and $\mathcal{F}_f$ be expressed by the following 3×3 and 3×1 matrices:

$$\hat{C} = \hat{C}^{(0,6)}, \quad \hat{C}^* = \hat{C}^{(f,6)}; \quad \bar{r} = \bar{r}^{(0)}, \quad \bar{r}^* = \bar{r}^{(f)}.$$

Carrying out the necessary matrix manipulations, show that

$$\hat{C}^* = e^{\tilde{u}_1\pi/2} e^{\tilde{u}_2\theta_3} e^{\tilde{u}_3\theta_4} e^{\tilde{u}_2\theta_5} e^{\tilde{u}_3\theta_6},$$

$$\bar{r}^* = \bar{u}_1(s_1 + a_4 c\theta_3 c\theta_4) + \bar{u}_2(s_2 + a_4 s\theta_3 c\theta_4) + \bar{u}_3(d_3 + a_4 s\theta_4).$$

As a check, note that $\bar{r}^*$ could also be obtained in a purely geometrical way.

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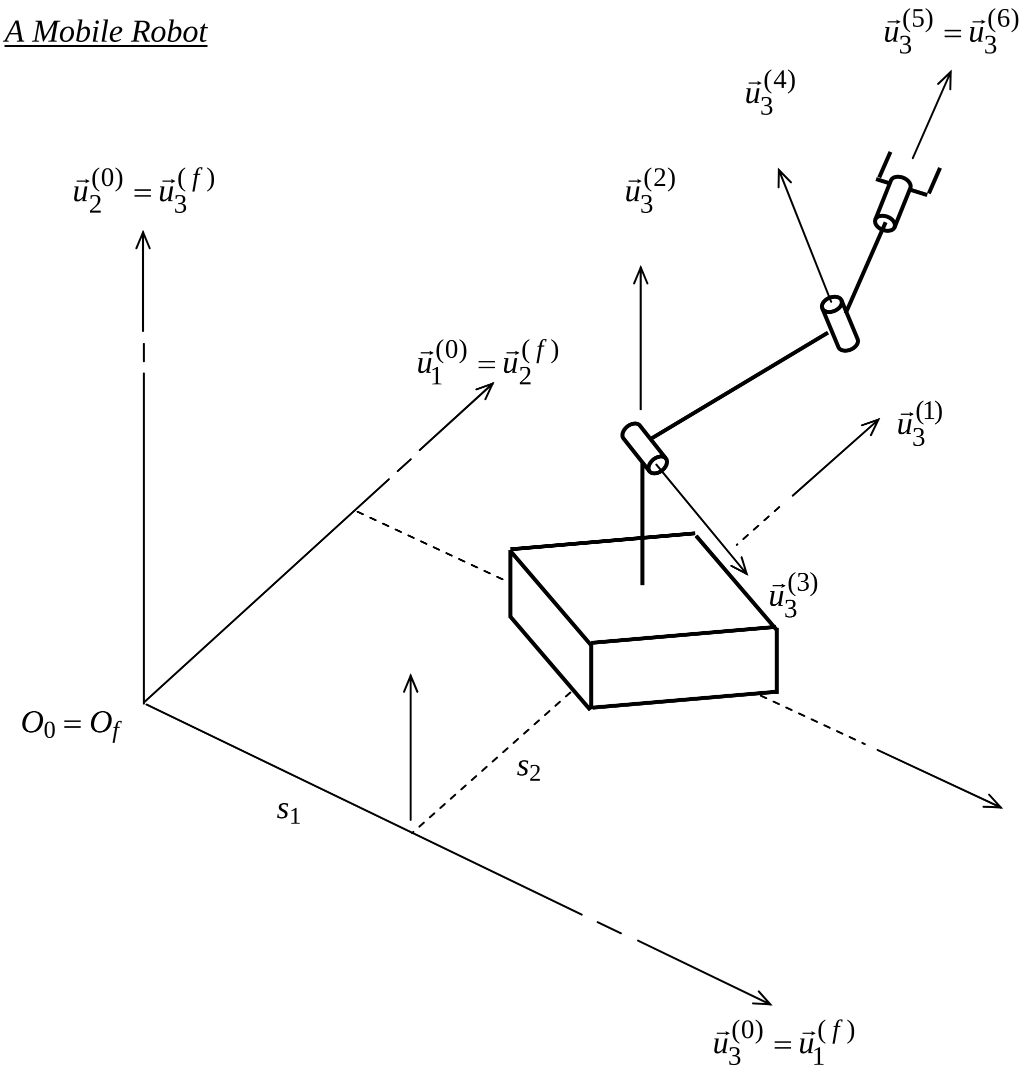
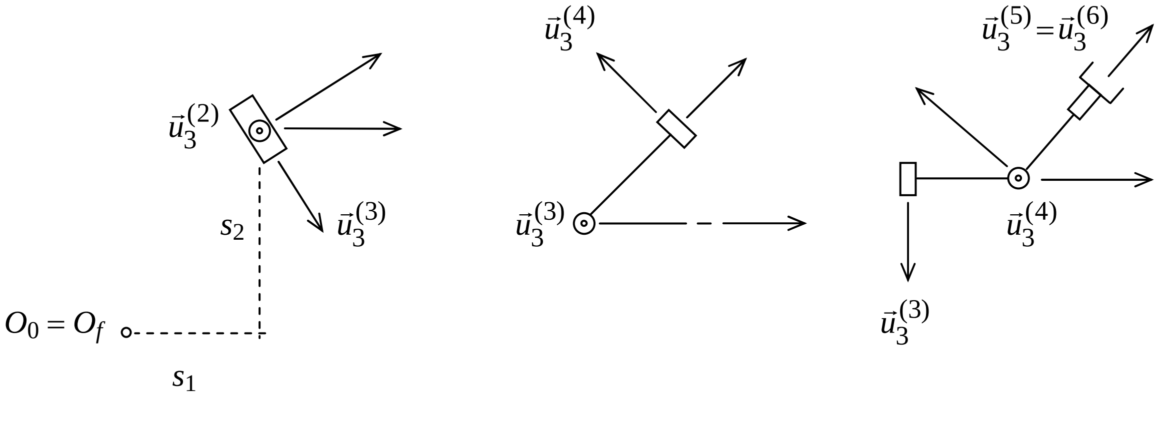
A Mobile Robot



Auxiliary Views



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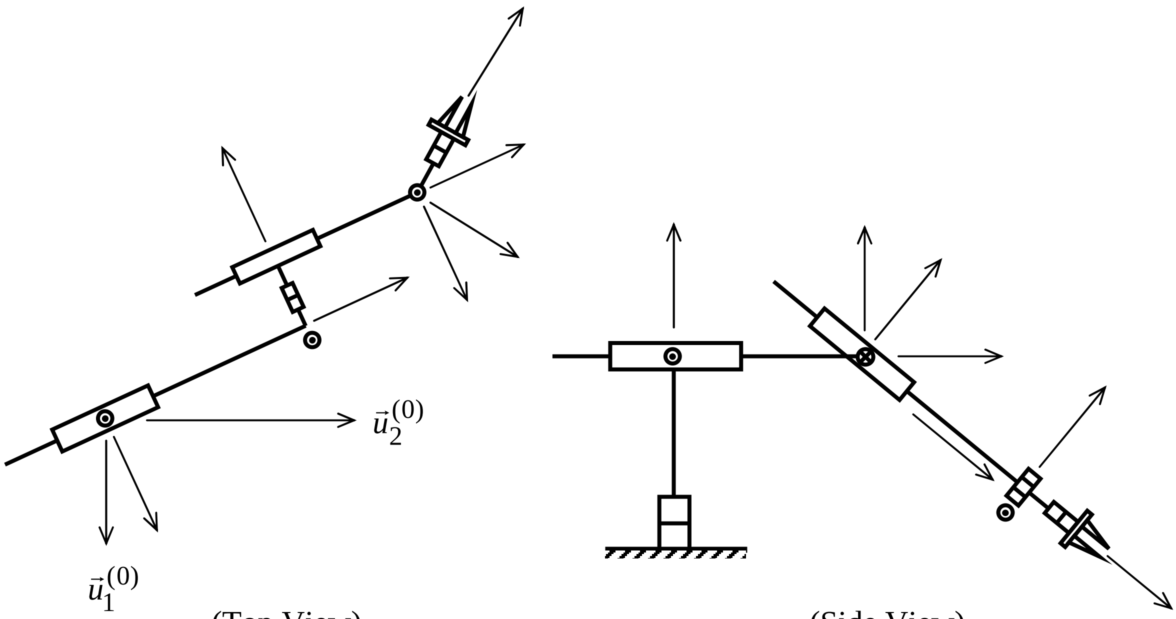
PROBLEM 2



(Top View) (Side View)

The figure shows the top and side views of an RPRPRR manipulator. The unit vectors associated with the joint axes and the common normals are also shown.

a) Label the unit vectors. Identify the link frames and mark their origins. Determine the Denavit-Hartenberg parameters of each link and display them in a tabular form. Identify the joint variables and indicate them clearly on the figure.

b) Show that the hand orientation matrix $\hat{C} = \hat{C}^{(0,6)}$ and the column matrix representation $\bar{r} = \bar{r}^{(0)}$ of the wrist point position vector can be simplified to the following forms:

$$\hat{C} = e^{\tilde{u}_3\theta_1} e^{-\tilde{u}_1\theta_3} e^{\tilde{u}_3\theta_5} e^{\tilde{u}_2\theta_6} e^{-\tilde{u}_1\pi/2} \,;$$

$$\bar{r} = e^{\tilde{u}_3\theta_1}\left[-d_3\bar{u}_1 + (s_2 + s_4\cos\theta_3)\bar{u}_2 - (s_4\sin\theta_3)\bar{u}_3\right] .$$

c) Obtain the inverse kinematic solution for the joint variables corresponding to $\hat{C}$ and $\bar{r}$.

d) Draw simple sketches to illustrate the multiple solutions.

e) Draw simple sketches to illustrate the singularities and discuss their consequences.

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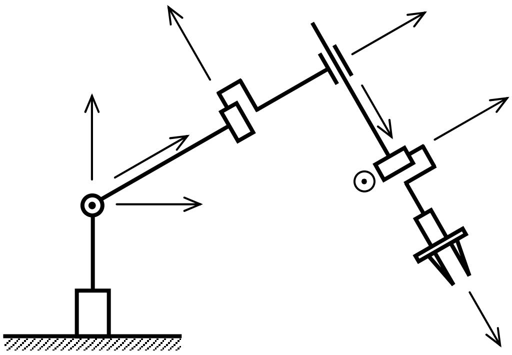
PROBLEM 3



a) The R^3PR^2 manipulator sketched above does not have any shoulder offset. Label the indicated unit vectors. Then, accordingly, identify the joint variables, the link parameters, and indicate the link frame origins. Show the joint variables clearly by drawing auxiliary diagrams, if necessary.

b) Verify the following equations for the hand orientation matrix and the wrist point position vector:

$$\hat{C} = \hat{C}^{(0,6)} = e^{\tilde{u}_3\theta_1} e^{-\tilde{u}_2\theta_2} e^{\tilde{u}_3\theta_3} e^{\tilde{u}_1\theta_5} e^{-\tilde{u}_3\theta_6'} e^{\tilde{u}_1\pi}, \quad \theta_6' = \theta_6 + \pi/2 ;$$

$$\bar{r} = \bar{r}^{(0)} = e^{\tilde{u}_3\theta_1} e^{-\tilde{u}_2\theta_2} [\bar{u}_1(a_2 + a_3\cos\theta_3) + \bar{u}_2(a_3\sin\theta_3) - \bar{u}_3 s_4] .$$

c) Note that the inverse kinematic solution for this manipulator can not be obtained analytically if $a_3 \neq 0$. So, take $a_3 = 0$ and obtain the inverse kinematic solution following the guideline described below:

- First, determine s_4 using the fact that an equation such as $\bar{y} = \hat{R}\bar{x}$ implies that $\bar{y}^t\bar{y} = \bar{x}^t\bar{x}$ if $\hat{R}$ is an orthonormal matrix.
- Next, taking the relevant matrix to the other side of the equation, determine θ_1.
- Then, determine θ_2.
- Finally, determine the remining angles θ_5, θ_3, and θ_6.

d) Noting that s_4 can not be negative, show that there are only two multiplicities, which are associated with θ_1 and θ_5. Illustrate these multiplicities by drawing simple sketches.

e) Identify the singular configurations. Discuss what happens in these configurations. Illustrate these configurations also by drawing simple sketches.

f) If $a_3 \neq 0$, obtain the inverse kinematic solution semi-analytically and write a computer program (e.g. in MATLAB) in order to determine the joint variables for a range of specified positions of the gripper. Of course, you must make sure that this range is within the working space of the manipulator.

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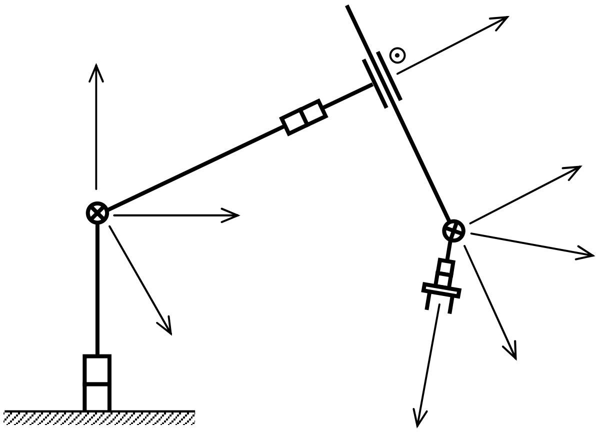
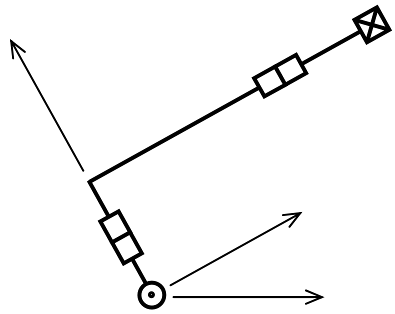
PROBLEM 4



(Side View)



(Partial Top View)

Consider the line diagram of the R^3PR^2 manipulator shown in the figure with the side view and a partial top view. The unit vectors along the joint axes and the common normals are also shown.

(a) Using the Denavit-Hartenberg convention, label the unit vectors, mark the link frame origins, identify the link parameters and the joint variables, and also indicate the joint variables on the figure.

(b) Let the orientation of the gripper and the location of the wrist point be denoted by $\hat{C} = \hat{C}^{(0,6)}$ and $\bar{r} = \bar{r}^{(0)}$. Show that

$$\hat{C} = e^{\tilde{u}_3\theta_1} e^{\tilde{u}_2\theta_2} e^{\tilde{u}_3\theta_3} e^{-\tilde{u}_1\theta_5} e^{\tilde{u}_2\theta_6'} e^{-\tilde{u}_1\pi/2}, \ \theta_6' = \theta_6 - \pi/2;$$

$$\bar{r} = e^{\tilde{u}_3\theta_1} e^{\tilde{u}_2\theta_2} (d_2\bar{u}_2 + d_3\bar{u}_3 + s_4 e^{\tilde{u}_3\theta_3}\bar{u}_2).$$

(c) If $d_2 = 0$, the shoulder configuration of the manipulator will be *spherical*. Then, show that the expression of the wrist point location simplifies to

$$\bar{r} = e^{\tilde{u}_3\theta_1} e^{\tilde{u}_2\theta_2} e^{\tilde{u}_3\theta_3} (d_3\bar{u}_3 + s_4\bar{u}_2).$$

(d) Obtain the *inverse kinematic solution* analytically for *each* of the joint variables in the *spherical shoulder case*. Indicate and discuss the *multiplicities* and *singularities*.

Hint: You may start the solution by finding s_4 from $\bar{r}^t\bar{r}$. Then, you may find θ_5 and θ_6' from the following equation obtainable from $\bar{r}$ and $\hat{C}$ with known $\bar{x}$ and $\bar{y}$:

$$e^{\tilde{u}_1\theta_5}\bar{x} = e^{\tilde{u}_2\theta_6'}\bar{y}.$$

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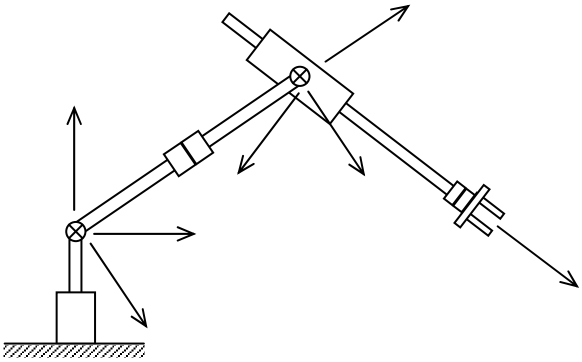
PROBLEM 5



Consider the robotic manipulator shown above with the joint arrangement R^4PR. Since the penultimate joint is prismatic, the wrist and tip points are taken to be coincident.

a) Some of the typical unit vectors are shown on the figure. You may add few more. Identify the unit vectors and indicate the joint variables clearly by drawing auxiliary diagrams, whenever necessary.

b) Show that the orientation of the hand and the location of the tip point can be expressed as follows in the base frame:

$$\hat{C} = \hat{C}^{(0,6)} = e^{\tilde{u}_3\theta_1} e^{\tilde{u}_2\theta_2} e^{\tilde{u}_3\theta_3} e^{\tilde{u}_2\theta_4} e^{\tilde{u}_3\theta_6} .$$

$$\bar{p} = \bar{p}^{(0)} = d_3 e^{\tilde{u}_3\theta_1} e^{\tilde{u}_2\theta_2} \bar{u}_3 + s_5 \hat{C} \bar{u}_3 .$$

c) Determine the joint variables corresponding to given $\bar{p}$ and $\hat{C}$. *Note*: It is advisable to start with s_5 by writing the relevant equation as

$$\bar{p} - s_5 \hat{C} \bar{u}_3 = d_3 e^{\tilde{u}_3\theta_1} e^{\tilde{u}_2\theta_2} \bar{u}_3 .$$

d) During the solution, identify the multiplicities and singularities. Discuss them and illustrate them with simple sketches.

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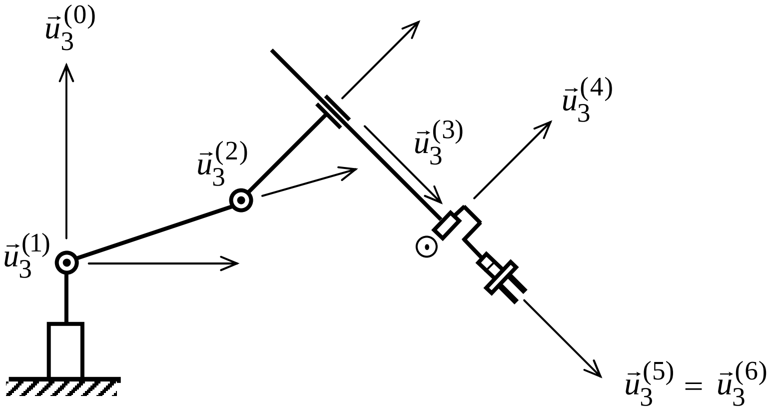
PROBLEM 6



a) Considering the R^{3}PR^2 manipulator shown above in the figure. Show that the *hand orientation matrix* with respect to the *base frame* can be expressed as follows:

$$\hat{C} = \hat{C}^{(0,6)} = e^{\tilde{u}_3\theta_1} e^{-\tilde{u}_2\theta_{23}} e^{\tilde{u}_1\theta_5} e^{-\tilde{u}_3\theta_6'} e^{\tilde{u}_1\pi},$$

where $\theta_{23} = \theta_2 + \theta_3$ and $\theta_6' = \theta_6 + \pi/2$.

b) Show that the *wrist point position vector* can be expressed with respect to the *base frame* as follows:

$$\bar{r} = \bar{r}^{(0)} = e^{\tilde{u}_3\theta_1} e^{-\tilde{u}_2\theta_{23}} \left[\bar{u}_1(a_2\cos\theta_3 + a_3) - \bar{u}_3(a_2\sin\theta_3 + s_4)\right].$$

c) For given $\hat{C}$ and $\bar{r}$, determine the corresponding joint variables. *Hint*: To facilitate the solution, you may write the wrist point position equation as

$$e^{\tilde{u}_2\theta_{23}} e^{-\tilde{u}_3\theta_1}\bar{r} = \bar{u}_1(a_2\cos\theta_3 + a_3) - \bar{u}_3(a_2\sin\theta_3 + s_4).$$

Then, start the solution by determining θ_1 from this equation.

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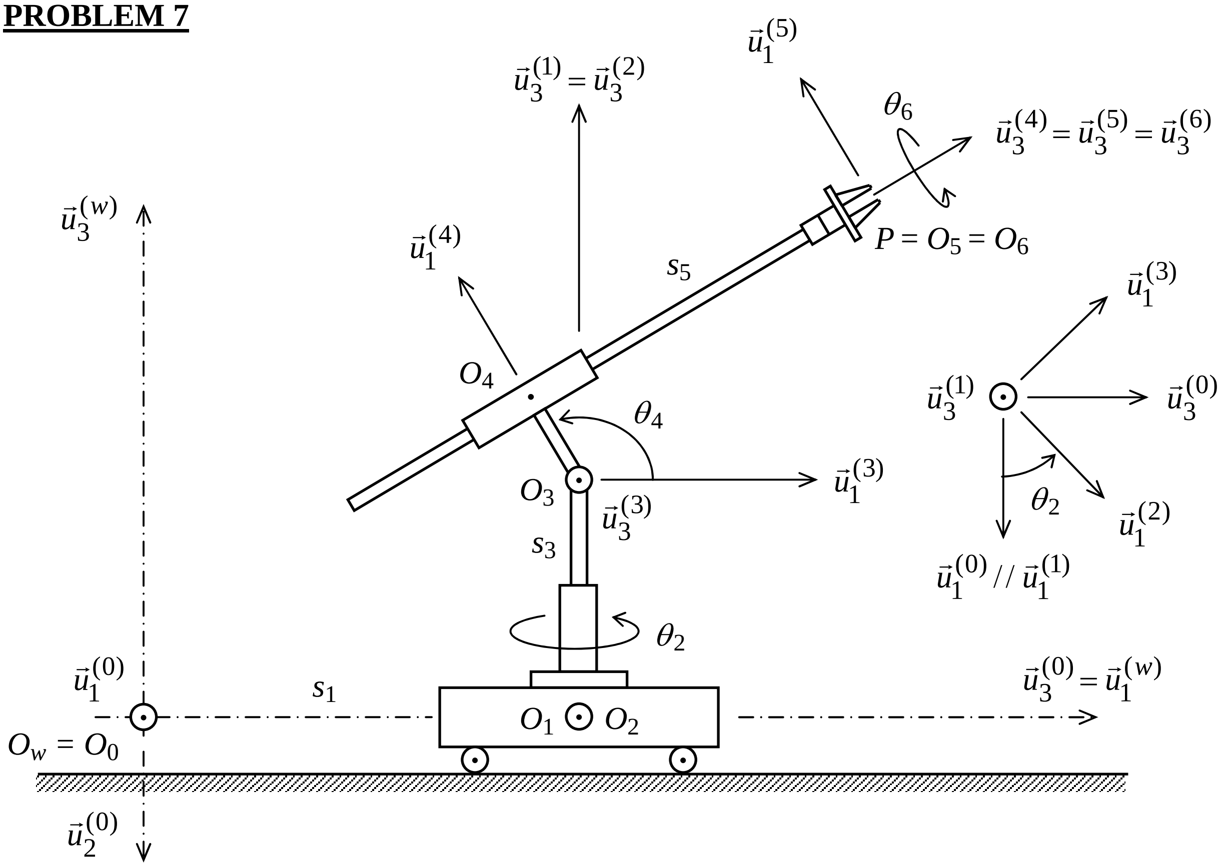
PROBLEM 7



Although it can be used for any arbitrary task as well, the PRPRPR robotic manipulator shown in the figure is designed particularly to carry out *screwing operations* at various locations and orientations along a rather long *assembly line*. That is why it has a mobile base moving on rails parallel to the assembly line. Also, the *cylindrical arrangement* with coincident axes of the last two joints allows the robot to make screwing in the simplest way by using only the last two joints while the others are kept fixed.

For this manipulator, the *base frame* $\mathcal{F}_0$, which is defined according to the *Denavit-Hartenberg* convention, is not oriented in the accustomed way of having its third axis pointing vertically upward. Therefore, $\mathcal{F}_0$ is not taken here to be coincident with the *world frame* $\mathcal{F}_w$. The origins of these frames are the same, i.e. $O_w = O_0$, but their basis vectors are different and they related so that $\vec{u}_1^{(w)} = \vec{u}_3^{(0)}$, $\vec{u}_2^{(w)} = -\vec{u}_1^{(0)}$, and $\vec{u}_3^{(w)} = -\vec{u}_2^{(0)}$.

a) Verify that $\hat{C}^{(w,0)} = e^{\tilde{u}_2 \pi/2} e^{-\tilde{u}_3 \pi/2}$.

b) Let the orientation and the tip point location of the end-effector are described by $\hat{C}^{\circ} = \hat{C}^{(0,6)}$ and $\overline{p}^{\circ} = \overline{p}^{(0)}$ in the base frame $\mathcal{F}_0$. Note that the wrist and tip points of this manipulator happen to be the same. Show that

$$\hat{C}^{\circ} = e^{-\tilde{u}_2 \theta_2'} e^{-\tilde{u}_3 \theta_4} e^{\tilde{u}_2 \theta_6} e^{-\tilde{u}_1 \pi/2}; \quad \theta_2' = \theta_2 + \pi/2.$$

$$\overline{p}^{\circ} = s_1 \overline{u}_3 - s_3 \overline{u}_2 + e^{-\tilde{u}_2 \theta_2'} e^{-\tilde{u}_3 \theta_4} (a_4 \overline{u}_1 + s_5 \overline{u}_2).$$

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c) As mentioned above, it is more customary and convenient to use $\mathcal{F}_w$ rather than $\mathcal{F}_0$. Therefore, $\hat{C} = \hat{C}^{(w,6)}$ and $\overline{p} = \overline{p}^{(w)}$ are preferred to $\hat{C}^\circ = \hat{C}^{(0,6)}$ and $\overline{p}^\circ = \overline{p}^{(0)}$ in describing the orientation and location of the end-effector. Show that

$$\hat{C} = e^{\tilde{u}_3\theta'_2} e^{-\tilde{u}_1\theta'_4} e^{-\tilde{u}_2\theta'_6} e^{\tilde{u}_1\pi/2}\,; \quad \theta'_4 = \theta_4 - \pi/2\,, \quad \theta'_6 = \theta_6 + \pi/2\,.$$

$$\overline{p} = s_1\overline{u}_1 + s_3\overline{u}_3 + e^{\tilde{u}_3\theta'_2} e^{-\tilde{u}_1\theta'_4}(a_4\overline{u}_3 - s_5\overline{u}_2)\,.$$

d) For specified $\hat{C}$ and $\overline{p}$, determine the corresponding joint variables.

e) Show that there are *only two* possible solutions; one with O_4 above O_3 and the other with O_4 below O_3. Illustrate these solutions and indicate your opinion about choosing one of these solutions.

f) Show that there are *only two* distinct types of singularities. As it can also be noticed by inspection, in one of them (*which one?*), s_1 and s_5 become undistinguishable; and in the other (*which one?*), s_3 and s_5 become undistinguishable. Show also that *all three* of the angular joint variables can be determined without any problem in both of the singularities. Additionally, determine the task space limitations corresponding to these singularities.