ME 522
PRINCIPLES OF ROBOTICS

PROBLEM SET 1

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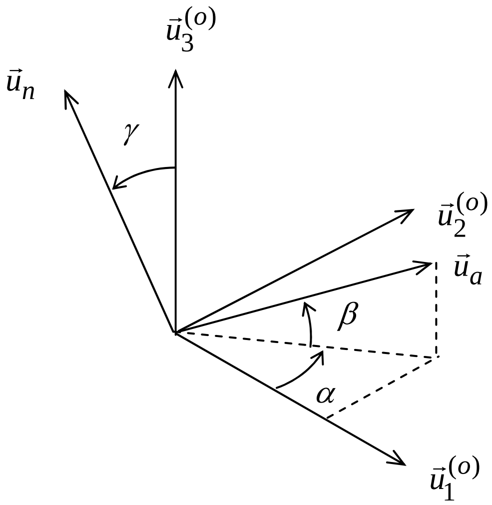
PROBLEM 1



During the operation of a robotic manipulator, the orientation of the gripper is described with respect to the base frame by means of the *approach* and *normal* vectors as shown in the figure. The angles α and β denote the *azimuth* and *elevation* angles of the approach vector and the angle γ denotes the *declination* angle of the normal vector. At an instant of the operation, the gripper is oriented in such a way that $\alpha = 40^o$, $\beta = 30^o$, $\gamma = 50^o$ and, as an additional information, the angle between the normal vector and the first axis of the base frame is an acute angle (i.e. positive but less than 90^o).

a) Determine the matrix $\hat{C} = \hat{C}^{(o,g)}$ that describes the orientation of the gripper with respect to the base frame.

b) As an alternative way to describe the orientation of the gripper with respect to the base frame, determine the Euler Angles (ϕ_1, ϕ_2, ϕ_3) according to the 3-2-3 sequence. Note that you can determine ϕ_1 and ϕ_2 (but not ϕ_3) even by *inspection* (explain how). Hence, check your analytically found results with those found by inspection.

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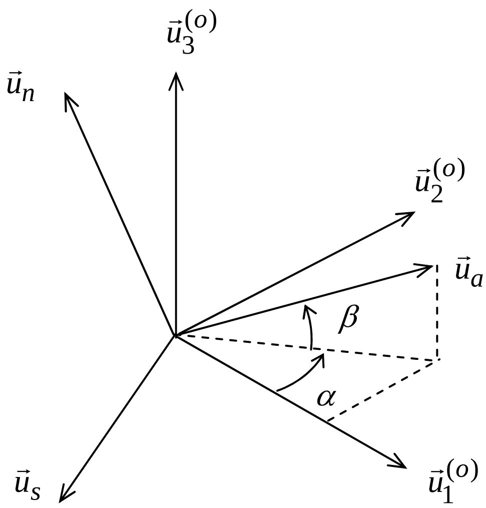
PROBLEM 2



At an instant of a task performed by a robotic manipulator, the orientation of the gripper frame $\mathcal{F}_g$ with respect to the base frame $\mathcal{F}_o$ is specified so that the azimuth (α) and elevation (β) angles of the *approach vector* ($\vec{u}_a$) are $\alpha = 40^\circ$ and $\beta = 30^\circ$, the *side vector* ($\vec{u}_s$) lies in the 1-2 plane of $\mathcal{F}_o$, and the *normal vector* ($\vec{u}_n$) deviates from $\vec{u}_3^{(o)}$ by an *acute* angle as illustrated in the figure.

(a) Determine the orientation matrix $\hat{C} = \hat{C}^{(o,g)}$ of $\mathcal{F}_g$ with respect to $\mathcal{F}_o$.

(b) Determine the corresponding Euler Angles (ϕ_1, ϕ_2, ϕ_3) of 1-2-3 sequence.

PROBLEM 3

During a certain task, the approach vector of the gripper of the manipulator used is required to be oriented with respect to the base frame by the yaw (ϕ_1) and pitch (ϕ_2) angles as follows:

$$\vec{u}_a = \vec{u}_1^{(0)} \cos\phi_1 \sin\phi_2 + \vec{u}_2^{(0)} \sin\phi_1 \sin\phi_2 + \vec{u}_3^{(0)} \cos\phi_2 .$$

Meanwhile, the normal vector ($\vec{u}_n$) of the gripper is required to remain always in the vertical plane pointing downward.

Determine the required orientation matrix $\hat{C} = \hat{C}^{(0,g)}$ of the gripper.

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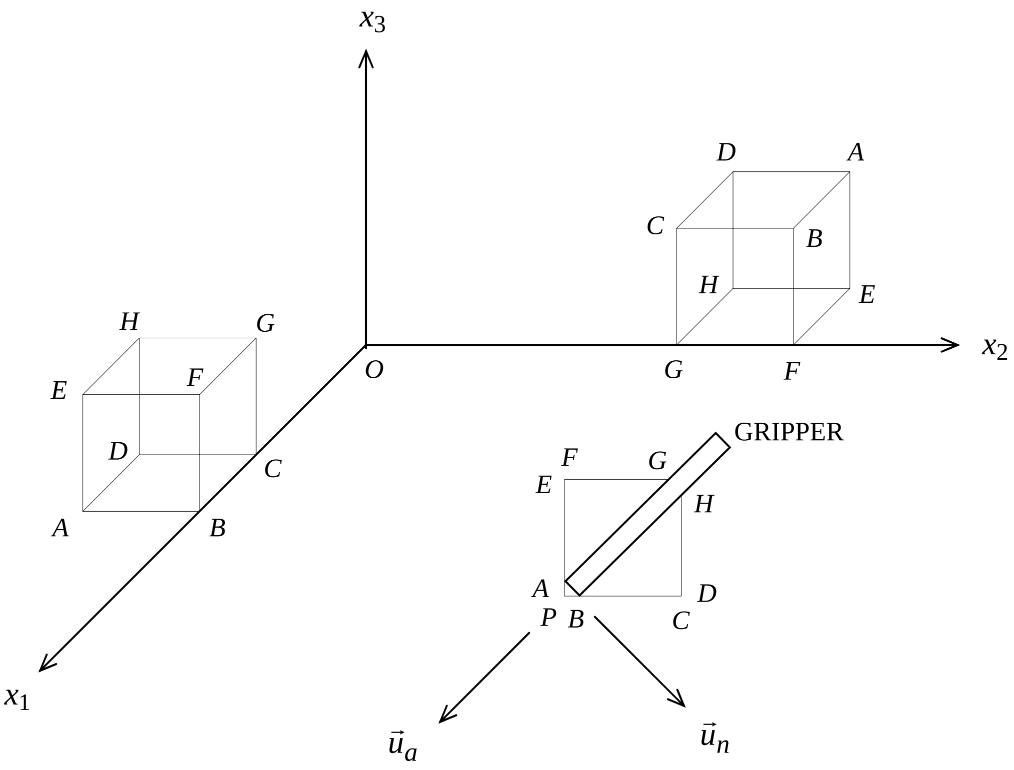
PROBLEM 4



The figure shows the initial and final positions of a cube. The length of each edge of the cube is 10 cm. In the initial position, the edge BC is coincident with the first axis of the base frame $\mathcal{F}_b(O)$ with OC = 20 cm. In the second position, the edge GF is coincident with the second axis of $\mathcal{F}_b(O)$ with OG = 25 cm. The reference frame $\mathcal{F}_c(A)$, which is fixed to the cube, is oriented so that $\vec{u}_1^{(c)} \,//\, AB$, $\vec{u}_2^{(c)} \,//\, AD$, and $\vec{u}_3^{(c)} \,//\, AE$. The cube is taken from the initial to final position by the gripper of a robotic manipulator. Throughout the task, the gripper holds the cube tightly on the faces $BCGF$ and $ADHE$ having the tip point P on the edge AB as shown in the figure.

a) Write out the homogeneous transformation matrix $\hat{H}_{A_1A_2}^{(c_1,c_2)}$ that describes the second position of the cube with respect to its first position.

b) In order to program the robot's computer for this task, determine the 123 (yaw-pitch-roll) Euler Angles (ϕ_1, ϕ_2, ϕ_3) of the gripper in its both initial and final positions with respect to the base frame $\mathcal{F}_b(O)$.

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