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SOLBOX-02

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SOLBOX-03

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1. Introduction

In this issue, we have two new problems, SOLBOX-02 and SOLBOX-03, as well as a solution of SOLBOX-03 by its submitters.

SOLBOX-02, which was submitted by Bruno Carpentieri, involves a satellite geometry that is modeled with perfectly conducting surfaces. The scattering problem is formulated with the electric-field integral equation (EFIE), leading to an ill-conditioned matrix equation that is difficult to iteratively solve, despite its small size. The aim and the challenge for this problem is to reduce the number of iterations. The associated matrix is available for downloading by interested readers.

SOLBOX-03, which is submitted by Manouchehr Takrimi and Vakur B. Ertürk, involves just a small sphere. However, the sphere is densely discretized, and the discretization is extremely nonuniform, so that conventional solvers become inefficient. The solution provided by the submitters was achieved by using a broadband multilevel fast multipole algorithm (MLFMA), incorporating incomplete tree structures. The model of the sphere can be downloaded by interested readers who would like to test their solvers.

With the submission of these new problems, we now have a total of three problems that can be solved with alternative implementations. We are looking forward to receiving submissions of new problems, as well as alternative solutions to these interesting three problems. Please remember that the aim of this column is to seek the most optimum solutions to electromagnetic problems, allowing us to share our knowledge and know-how. All electromagnetic code developers and users are invited to provide submissions.

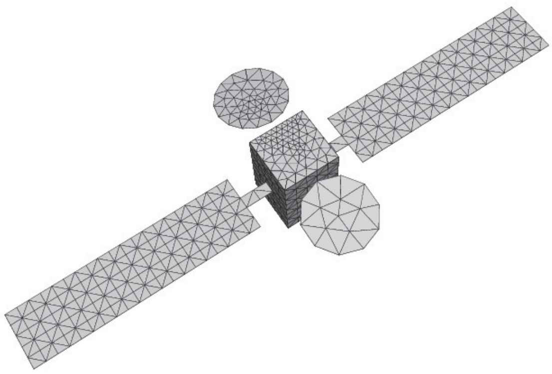

Figure 1a. The satellite problem for SOLBOX-02.

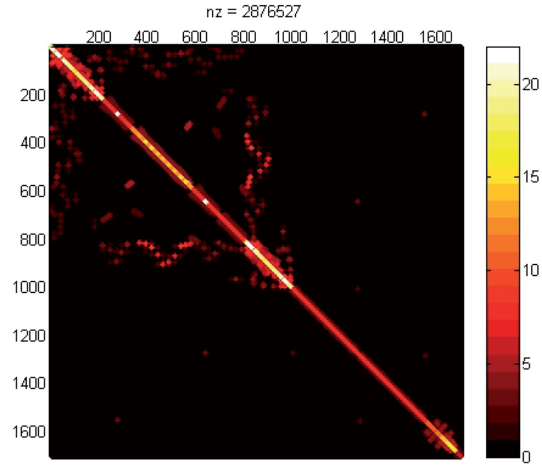

Figure 1b. An illustration of the related matrix for the problem in Figure 1a.

2. Problems

2.1 Problem SOLBOX-02 (by B. Carpentieri)

This problem concerns the design of fast iterative methods for solving dense linear systems arising from the boundary-element discretization of electromagnetic scattering problems expressed in an integral formulation. We concentrate our attention on the electric-field integral-equation formulation for perfectly conducting objects. The geometry is a satellite, depicted in Figure 1a, which is illuminated by an incident plane wave at 222 MHz. We consider the Galerkin discretization of the electric-field integral equation using the Rao-Wilton-Glisson basis functions for the surface-current expansion, which gives rise to a dense, complex, symmetric (but non-Hermitian) linear system. In Figure 1b, the pattern of the large entries of the matrix is plotted. Despite the small size (only 1699 unknowns), the problem is representative of the general trend for this class of problems. As a reference, in *MATLAB* (*version R2013a*), the un-restarted GMRES method required 389 iterations with no preconditioning, and 309 iterations with diagonal preconditioning, to solve the pertinent linear system. The computational cost increased further when GMRES was restarted: Using a restart value equal to 50, convergence was achieved after 40 outer and 36 inner GMRES iterations with no preconditioning, and 32 outer and 25 inner iterations with diagonal preconditioning. The particular aim is to find better preconditioners to accelerate Krylov-subspace methods for this problem class. The matrix data for the satellite problem are available for downloading in *MATLAB* at

https://www.dropbox.com/s/z9zlkz5letirsry/satellite.mat?dl=0.

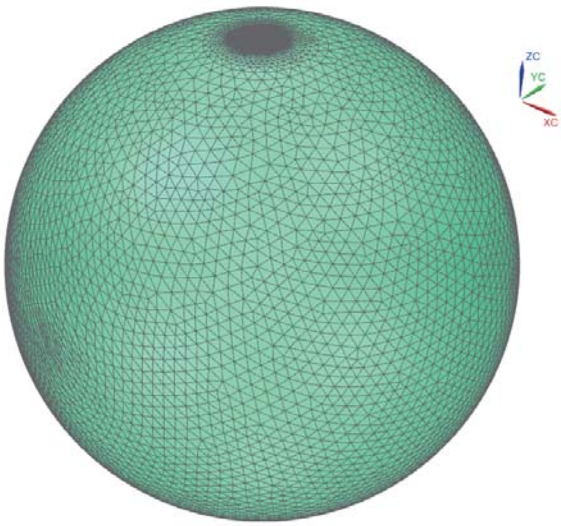

Figure 2. An isometric view of the sphere model possessing a dense discretization around the north pole with a multi-scale factor of 110 (SOLBOX-03).

2.2 Problem SOLBOX-03 (by M. Takrimi and V. B. Ertürk)

Figure 2 illustrates a perfectly electrically conducting sphere of radius 5 cm ($\lambda/20$) at 300 MHz with a highly nonuniform triangulation. The edge size for the triangles varied from 0.034 mm ($\lambda/30000$) at the north pole up to 3.8 mm ($\lambda/263$) at the south pole, resulting a nonuniform mesh that had a multi-scale factor of 110. The multi-scale factor is defined as the ratio of the largest edge length to the smallest edge length over the entire meshed surface. The total number of triangles was 120,992. A `.unv` model file of the sphere generated by the Siemens *NX* (*version 8.5*) program is available at

https://www.dropbox.com/s/m9bntgze75mqqtt/Sphere_R%3Dp05m_multiscale%3D110_Un%3D181K_unv.rar?dl=0

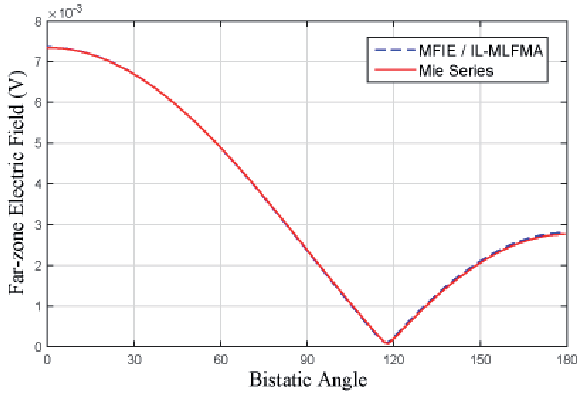

Figure 3. The solution of the problem SOLBOX-03 using a broadband implementation of MLFMA, compared to the Mie-series results.

The sphere was illuminated by a unit plane wave, polarized in the *x* direction, and propagating in the *z* direction. It was desired to find the far-zone electric field (the electric field times the distance from the object, when the distance approached infinity) scattered from the sphere.

3. Solution to Problem SOLBOX-03

3.1 Solution Summary

Solver Type (e.g., noncommercial, commercial): Noncommercial research-based code developed at Bilkent University, Ankara, Turkey, and CEMMETU, Ankara, Turkey.

Solution core algorithm or method:
Frequency-domain incomplete-leaf MLFMA (IL-MLFMA) [1]

Programming language or environment (if applicable): *MATLAB + MEX*

Computer properties and resources used:
Single core of 3 GHz Intel Xeon X5472, *Windows 7* (64-bit), 32 GB ECC RAM

Total time required to produce the results shown (categories: < 1 sec, < 10 sec, < 1 min, < 10 min, < 1 hour, < 10 hours, < 1 day, < 10 days, > 10 days): < 10 hours

3.2 Short Description of the Numerical Solution

Problem SOLBOX-03 was solved using the incomplete-leaf version [1] of the scaled MLFMA [2, 3] in the frequency domain. The solver was based on a nonuniform clustering, where only the overcrowded boxes were divided into smaller boxes for a given basis-function population threshold (100, for this solution). The problem was formulated with the magnetic-field integral equation (MFIE) [4], which is suitable for low-frequency perfectly conducting objects. The magnetic-field integral equation was discretized with the Rao-Wilton-Glisson functions [5] over multi-scale triangles. The model file that was provided with the problem definition was used, leading to a total of 181,488 unknowns. Both near-field and far-field interactions were computed using second-order Gaussian quadrature rules. Two digits of accuracy and 4×4-point interpolations were also considered for the MLFMA computations. The number of iterations to reach a residual error of 0.001 was 66, using GMRES without preconditioning. The field intensities and the Mie-series solution were calculated in the far-zone at 360 points.

3.3 Results

Figure 3 depicts the far-zone electric field (V) as a function of the bistatic angle, along with the Mie-series solution for the problem SOLBOX-03. The data samples were plotted using the plot command of *MATLAB*. There was fairly good agreement between the simulated results and the analytic Mie-series solution. The small discrepancy originated from the approximate method (diagonalization of the Green's function) in the low-frequency regime, due to the use of the scaled MLFMA [2].

4. References

1. M. Takrimi, Ö. Ergül, and V. B. Ertürk, "A Novel Broadband Multilevel Fast Multipole Algorithm with Incomplete-Leaf Tree Structures for Multiscale Electromagnetic Problems," *IEEE Transactions on Antennas and Propagation*, submitted for publication.

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3. W. C. Chew, J.-M. Jin, E. Michielssen, and J. Song, *Fast and Efficient Algorithms in Computational Electromagnetics*, Norwood, MA, Artech House, 2001.

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5. S. M. Rao, D. R. Wilton, and A. W. Glisson, "Electromagnetic Scattering by Surfaces of Arbitrary Shape," *IEEE Transactions on Antennas and Propagation*, **AP-30**, 3, May 1982, pp. 409-418.