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Spring 2013-2014

**STAT 552**

**HOMEWORK 2**

**Due 26 May 2014, Monday, 17:00**

You should work on these questions on your own. Please feel free to get help from me, but not from anyone else.

**1.** Let X1, ..., Xn be independent and identically distributed each with Exponential(λ).

a) Find an unbiased estimator of λ based only on Y=min(X1, ..., Xn).

b) Find a better estimator than the one in part a). Prove that it is better.

c) Check the consistencies of the two estimators from parts a) and b).

d) The following data are high-stress failure times (in hours) of Kevlar/epoxy spherical vessels used in sustained pressure environment on the space shuttle:

50.1, 70.1, 137, 166.9, 170.5, 152.8, 80.5, 123.5, 112.6, 148.5, 160, 125.4.

Failure times are often modeled with the exponential distribution. Estimate the mean failure time using the estimators from parts a) and b).

**2.** Find a 1-α confidence interval for θ, given X1, ..., Xn iid with pdf

**3.** Suppose that we have a random sample of four observations from the density function



a) Find the rejection region for the most powerful test of Ho: θ = θ0 against Ha: θ = θa , assuming that θa> θ0. [Hint: Make use of χ2 distribution].

b) Is the test given in (a) uniformly most powerful for the alternative θ> θ0?

**4.** Urban storm water can be contaminated by many sources, including discarded batteries. When ruptured, these batteries release metals of environmental significance. A real data reveals the following summary data for urban areas around Cleveland: A sample of 51 Panasonic AAA batteries gave a sample mean zinc mass of 2.06 g. and a sample standard deviation of 0.141 g. We would like to know whether this data provide compelling evidence for concluding that the population mean zinc mass exceeds 2.0 g. for this region.

a) Clearly write down the null and alternative hypothesis.

b) Calculate the appropriate test statistics. Explain why you used that test statistic.

c) Calculate the p-value.

d) State the conclusion of this test.