P. 1) LOAD UNDER A RECTANGULAR AREA (1)

Question:
The footing shown in the figure below exerts a uniform pressure of 300 kN/m\(^2\) to the soil. Determine **vertical stress increase due to uniform pressure**, at a point of 4 m directly under; (a) point A, (b) point B.

Solution:
a) Point A:

\[ \Delta \sigma_z = q.I_r \]

By the use of Figure 1.6 in Lecture Notes, page 10;
For area 1: $A(abcfg)$

$z = 4 \text{ m}$  \hspace{1cm} mz = 4  \hspace{1cm} m = 4/4 = 1

$nz = 2$  \hspace{1cm} n = 2/4 = 0.5  \hspace{1cm} I_r = 0.12

For area 2: $A(cdef)$

$z = 4 \text{ m}$  \hspace{1cm} mz = 2  \hspace{1cm} m = 2/4 = 0.5

$nz = 2$  \hspace{1cm} n = 2/4 = 0.5  \hspace{1cm} I_r = 0.085

$\Delta \sigma_z = 300 \times (0.12 + 0.085) = 61.5 \text{ kPa}$

the stress at 4 m depth under point A due to 300 kN/m$^2$ uniform pressure

b) Point B:

Area 1 = Area 2 = Area 3

$mz = nz = 2$  \hspace{1cm} m = n = 2/4 = 0.5  \hspace{1cm} I_r = 0.085

$\Delta \sigma_z = 300 \times 3 \times 0.085 = 76.5 \text{ kPa}$

the additional stress at 4 m depth under point B due to 300 kPa uniform pressure
P. 2) LOAD UNDER A RECTANGULAR AREA (2)

**Question:**
A rectangular footing as shown in figure below exerts a uniform pressure of 420 kN/m². Determine the vertical stress due to uniform pressure at point A for a depth of 3 m.

**Solution:**

For area (abkh):

\[
\begin{align*}
    z &= 3 \text{ m} \\
    mz &= 4 \\
    nz &= 3.5
\end{align*}
\]

\[
\begin{align*}
    m &= 4 / 3 = 1.33 \\
    n &= 3.5 / 3 = 1.17
\end{align*}
\]

\[I_r = 0.195\]

For area (bcdk):

\[
\begin{align*}
    mz &= 3.5 \\
    nz &= 2
\end{align*}
\]

\[
\begin{align*}
    m &= 3.5 / 3 = 1.17 \\
    n &= 2 / 3 = 0.67
\end{align*}
\]

\[I_r = 0.151\]

For area (defk):

\[
\begin{align*}
    mz &= 2 \\
    nz &= 1.5
\end{align*}
\]

\[
\begin{align*}
    m &= 2 / 3 = 0.67 \\
    n &= 1.5 / 3 = 0.5
\end{align*}
\]

\[I_r = 0.105\]
For area (fghk):

\[
\begin{align*}
mz &= 4 \\
nz &= 1.5 \\
m &= 4 / 3 = 1.33 \\
n &= 1.5 / 3 = 0.5 \\
I_r &= 0.133
\end{align*}
\]

For area (ijkm):

\[
\begin{align*}
mz &= nz = 2 \\
m &= n = 2 / 3 = 0.67 \\
I_r &= 0.117
\end{align*}
\]

\[
\Delta \sigma_z = \sigma \cdot I_r
\]

\[
= 420 \left[ I_{r1} + I_{r2} + I_{r3} + I_{r4} - I_{r5} \right]
\]

\[
= 420 \left[ 0.195 + 0.151 + 0.105 + 0.133 - 0.117 \right]
\]

\[
\Delta \sigma_z = 196.14 \text{ kPa}
\]

**Note:** Where do we use the vertical stress increase, \( \Delta \sigma_z \), values?

For example, in a consolidation settlement problem, stress increase, \( \Delta \sigma_z \), values are needed to calculate settlement under a foundation loading. We make the following calculations for a point located under the foundation at a certain depth (for example, at the mid-depth of the compressible layer):

1. First, calculate the initial effective vertical stress, \( \sigma'_{v,o} \), before the building was constructed,
2. Then, find the vertical stress increase \( \Delta \sigma_z \) at that depth, by using Boussinesq stress distribution or by approximate methods (for example, 2V: 1H approximation)
3. Find the final effective vertical stress, \( \sigma'_{v,f} = \sigma'_{v,o} + \Delta \sigma_z \), after the building is constructed.
4. Use these values in calculating the settlement under the foundation.
P. 3) IMMEDIATE SETTLEMENT

**Question:**
A foundation $4 \text{ m} \times 2 \text{ m}$, carrying a net uniform pressure of 200 kN/m$^2$, is located at a depth of 1.5 m in a layer of clay 5 m thick for which the value of $E_u$ is 45 MN/m$^2$. The layer is underlain by a second layer, 10 m thick, for which the value of $E_u$ is 80 MN/m$^2$. A hard stratum lies below the second layer. Ground water table is at the depth of foundation. Determine the average immediate settlement under the foundation.

**Hint:** Since soil is SATURATED CLAY, $\nu_s=0.5$. So the following equation can be used:

$$S_i = \mu_0 \cdot \mu_i \cdot \frac{q \cdot B}{E_u}$$

**Solution:**

![Diagram](image-url)

$$S_i = \mu_0 \cdot \mu_i \cdot \frac{q \cdot B}{E_u}$$
We obtain,

\( \mu_0 \) from \( D / B \)

\( \mu_1 \) from \( H / B \) and \( L / B \)

\[ \frac{D}{B} = \frac{1.5}{2} = 0.75 \rightarrow \mu_0 = 0.95 \text{ (Figure 3.3, p.62 Lecture Notes)} \]

(1) Consider the upper layer with \( E_u = 45 \) MPa.

\[ \frac{H}{B} = \frac{3.5}{2} = 1.75 \quad \text{and} \quad \frac{L}{B} = \frac{4}{2} = 2 \]

\[ S_{i_1} = \mu_0 \cdot \mu_1 \cdot \frac{q \cdot B}{E_u} = (0.95) \cdot (0.65) \cdot \frac{(200) \cdot 2}{45} = 5.49 \text{mm} \]

(2) Consider the two layers combined with \( E_u = 80 \) MPa.

\[ \frac{H}{B} = \frac{(3.5 + 10)}{2} = 6.75 \quad \text{and} \quad \frac{L}{B} = \frac{4}{2} = 2 \]

\[ S_{i_2} = \mu_0 \cdot \mu_1 \cdot \frac{q \cdot B}{E_u} = (0.95) \cdot (0.9) \cdot \frac{(200) \cdot 2}{80} = 4.28 \text{mm} \]

(3) Consider the upper layer with \( E_u = 80 \) MPa.

\[ \frac{H}{B} = \frac{3.5}{2} = 1.75 \quad \text{and} \quad \frac{L}{B} = \frac{4}{2} = 2 \]

\[ S_{i_3} = \mu_0 \cdot \mu_1 \cdot \frac{q \cdot B}{E_u} = (0.95) \cdot (0.65) \cdot \frac{(200) \cdot 2}{80} = 3.08 \text{mm} \]
Using the principle of superposition, the settlement of the foundation is given by:

$$S_i = S_{i1} + S_{i2} - S_{i3}$$

$$S_i = 5.49 + 4.28 - 3.08$$

$$S_i = 6.69 \text{ mm}$$

**P. 4) SCHMERTMAN**

**Question:**

A soil profile consists of deep, loose to medium dense sand ($\gamma_{\text{dry}} = 16 \text{ kN/m}^3$, $\gamma_{\text{sat}} = 18 \text{ kN/m}^3$). The ground water level is at 4 m depth. A 3.5 m x 3.5 m square footing rests at 3 m depth. The **total (gross) load** acting at the foundation level (footing weight + column load + weight of soil or footing) is 2000 kN. Estimate the elastic settlement of the footing 6 years after the construction using influence factor method (Schmertman, 1978).

End resistance values obtained from static cone penetration tests are:

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>$q_c$ (kN/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00 – 2.00</td>
<td>8000</td>
</tr>
<tr>
<td>2.00 - 4.75</td>
<td>10000</td>
</tr>
<tr>
<td>4.75 - 6.50</td>
<td>8000</td>
</tr>
<tr>
<td>6.50 – 12.00</td>
<td>12000</td>
</tr>
<tr>
<td>12.00 – 15.00</td>
<td>10000</td>
</tr>
</tbody>
</table>

*Note that:*

- for square footing; $z$ (depth) (from foundation level) $\quad \ell_i$ (strain factors)

<table>
<thead>
<tr>
<th>$z$</th>
<th>$\ell_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>B/2</td>
<td>0.5</td>
</tr>
<tr>
<td>2B</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Where; $B$ : width of footing

- $E_s = 2.0 \ q_c$
Solution:

\[ S_i = C_1 C_2 q_{\text{net}} \sum \frac{L}{E} \Delta z \]

\( q_{\text{net}} = \) net foundation pressure

\[ C_1 = 1 - 0.5 \frac{\sigma'_o}{q_{\text{net}}} \] correction factor for footing depth

\( \sigma'_o = \) effective overburden pressure at foundation level

\[ C_2 = 1 + 0.2 \log \frac{t}{0.1} \] correction factor for creep

\( t = \) time at which the settlement is required (in years)

\[ q_{\text{gross}} = 2000 \text{ kN} \]

4m 3m

\( \gamma_{\text{dry}} = 16 \text{ kN/m}^3 \)

\( \gamma_{\text{sat}} = 18 \text{ kN/m}^3 \)

\[ q_{\text{net}} = \frac{2000}{3.5 \times 3.5} - 3 \times 16 = 115.26 \text{ kPa} \] gross pressure, initial effective overburden pressure

\( \sigma'_o = 3 \times 16 = 48 \text{ kPa} \)

\[ C_1 = 1 - 0.5 \frac{48}{115.26} = 0.792 \]

\[ C_2 = 1 + 0.2 \log \frac{6}{0.1} = 1.356 \]
Width of foundation, 
B = 3.5 m

\[ E_s = 2.0 \times q_c \]

<table>
<thead>
<tr>
<th>Layer No</th>
<th>Depth(m)</th>
<th>( \Delta z )(m)</th>
<th>( q_c )(kPa)</th>
<th>( E_s )(kPa)</th>
<th>( I_z )</th>
<th>( (I_z/E_s) \times \Delta z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.00-4.75</td>
<td>1.75</td>
<td>10.000</td>
<td>20.000</td>
<td>0.3</td>
<td>2.65x10^{-5}</td>
</tr>
<tr>
<td>2</td>
<td>4.75-6.50</td>
<td>1.75</td>
<td>8.000</td>
<td>16.000</td>
<td>0.416</td>
<td>4.55x10^{-5}</td>
</tr>
<tr>
<td>3</td>
<td>6.50-8.25</td>
<td>1.75</td>
<td>12.000</td>
<td>24.000</td>
<td>0.249</td>
<td>1.82x10^{-5}</td>
</tr>
<tr>
<td>4</td>
<td>8.25-10.00</td>
<td>1.75</td>
<td>12.000</td>
<td>24.000</td>
<td>0.083</td>
<td>0.605x10^{-3}</td>
</tr>
</tbody>
</table>

\[ \Sigma = 9.625x10^{-5} \]

\[ S_i = (0.792) (1.356) (115.26) (9.625x10^{-5}) \]

\[ = 0.01191 \text{ m} \phantom{.} \rightarrow S_i = 11.91 \text{ mm} \]
P5. CONSOLIDATION SETTLEMENT

Question:
Ignore the immediate settlement, and calculate total consolidation settlement of soil profile composed of two different types of clay, i.e. Clay 1 and Clay 2 due to 150 kPa net foundation loading. Take unit weight of water as 10 kN/m\(^3\) and assume that Skempton-Bjerrum Correction Factor is \(\mu = 0.7\) for both clay layers. Note that \(\sigma'_c\) (or sometimes shown as \(\sigma'_p\)) is the preconsolidation pressure.

Solution:
Settlement will take place due to loading (q\(_{\text{net}}\) = 150 kPa) applied at a depth of 2 m. Thus, all (consolidation) settlement calculations will be performed for clayey soil beneath the foundation (z > 2 m).

Reminder: General equation of 1D consolidation settlement (one dimensional vertical consolidation) for an overconsolidated clay is;

\[
S_{c,1D} = \frac{C_r}{1 + e_o} H \log \left( \frac{\sigma'_c}{\sigma'_{\text{v,0}}} \right) + \frac{C_c}{1 + e_o} H \log \left( \frac{\sigma'_{\text{w,f}}}{\sigma'_c} \right)
\]
Note that, all calculations are done for the mid-depth of the compressible layers under the loading.

Consolidation settlement in Clay 1:

Initial effective overburden stress, \( \sigma'_{v,o} = (2*19) + (3*(20-10)) = 68 \) kPa

Stress increment due to foundation loading, \( \Delta \sigma = \frac{150*(10*10)}{[(10+3)*(10+3)]} = 88.8 \) kPa

Final stress, \( \sigma'_{v,f} = 68 + 88.8 = 156.8 \) kPa

This is an overconsolidated clay (overconsolidation ratio OCR = \( \sigma'_c / \sigma'_{v,o} = 80 / 68 > 1.0 \)) ; and the final stress, \( \sigma'_{v,f} \) is greater than \( \sigma'_c \) ( \( \sigma'_{v,f} > \sigma'_c \) ) therefore we should use both \( C_r \) and \( C_c \) in consolidation settlement calculation (see figure below).

\[
\begin{align*}
S_{c,1D} &= \left\{ \frac{0.05}{1 + 0.80} (6) \log \left( \frac{80}{68} \right) \right\}_1 + \left\{ \frac{0.15}{1 + 0.80} (6) \log \left( \frac{156.8}{80} \right) \right\}_2 = 0.158m = 15.8 \text{ cm}
\end{align*}
\]

Consolidation settlement in Clay 2:

Initial effective overburden stress, \( \sigma'_{v,o} = (2*19) + (6*(20-10)) + (3*(20-10)) = 128 \) kPa

Stress increment due to foundation loading, \( \Delta \sigma = \frac{150*(10*10)}{[(10+9)*(10+9)]} = 41.6 \) kPa
Final stress, $\sigma_{v,f} = 128 + 41.6 = 169.6$ kPa

This is an overconsolidated clay (overconsolidation ratio $OCR = \sigma'_c / \sigma'_{v,o} = 200 / 128 > 1.0$); and the final stress, $\sigma_{v,f}$ is less than $\sigma_c$ ($\sigma_{v,f} < \sigma_c$) therefore we should use only $C_r$ in consolidation settlement calculation (see figure below).

[Note: If a soil would be a normally consolidated clay ($OCR = \sigma'_c / \sigma'_{v,o} = 1.0$), we would use only $C_c$ in consolidation settlement calculation.]

$$S_{c,1D} = \left\{ \frac{0.03}{1 + 0.60} \right\log \left( \frac{169.6}{128} \right) = 0.014 \, m = 1.4 \, cm$$

Total Consolidation Settlement (1D):

$$S_{c,1D} = 15.9 + 1.4 = 17.3 \, cm$$

Corrected Settlement for 3D Consolidation (Skempton-Bjerrum Factor):

$$S_{c,3D} = S_{c,1D} \times \mu = 17.3 \times 0.7 = 12.1 \, cm$$