Earnings, Book Values, and Dividends in Equity Valuation*

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Abstract. The paper develops and analyzes a model of a firm's market value as it relates to contemporaneous and future earnings, book values, and dividends. Two owners' equity accounting constructs provide the underpinnings of the model: the clean surplus relation applies, and dividends reduce current book value but do not affect current earnings. The model satisfies many appealing properties, and it provides a useful benchmark when one conceptualizes how market value relates to accounting data and other information.

Résumé. L'auteur élabore et analyse un modèle dans lequel il conceptualise la relation entre la valeur marchande d'une entreprise et ses bénéfices, ses valeurs comptables et ses dividendes actuels et futurs. Deux postulats de la comptabilisation des capitaux propres servent de charpente au modèle : a) la relation du résultat global s'applique et b) les dividendes réduisent la valeur comptable actuelle sans influer, cependant, sur les bénéfices actuels. Le modèle présente de nombreuses propriétés intéressantes et il peut, fort utilement, servir de repère dans la conceptualisation de la relation entre la valeur marchande et les données comptables et autres renseignements.

Accounting assigns an important integrative function to the statement of changes in owners' equity. The statement includes the bottom-line items in the balance sheet and income statement—book value and earnings—and its format requires the change in book value to equal earnings minus dividends (net of capital contributions). We refer to this relation as the clean surplus relation because, as articulated, all changes in assets/liabilities unrelated to dividends must pass through the income statement.[1] Accounting theory generally embraces this scheme without connecting it to a user's perspective on accounting data. Yet the underlying idea that (net) stocks of value reconcile with the creation and distribution of value raises a basic question in an equity valuation context: can one devise a cohesive theory of a firm's value that relies on the clean surplus relation

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to identify a distinct role for each of the three variables, earnings, book value, and dividends?

This paper resolves the question in a neoclassical framework. Thus the analysis starts from the assumption that value equals the present value of expected dividends.[2] One can next assume the clean surplus relation to replace dividends with earnings/book values in the present value formula. Assumptions on the stochastic behavior of the accounting data then lead to a multiple-date, uncertainty model such that earnings and book value act as complementary value indicators. Specifically, the core of the valuation function expresses value as a weighted average of (i) capitalized current earnings (adjusted for dividends) and (ii) current book value. Extreme parameterizations of the model yield either (i) or (ii) as the sole value indicators. Both of these settings have been examined by Ohlson (1991). At its most elementary level, this paper accordingly generalizes prior analysis to derive a convex combination of a "pure" flow model of value and a "pure" stock model of value. The combination is of conceptual interest because it brings both the bottom-line items into valuation through the clean surplus relation.

The development of the model shows the relevance of abnormal (or residual) earnings as a variable that influences a firm's value. This accounting- based performance measure is defined by earnings minus a charge for the use of capital as measured by beginning-of-period book value multiplied by the cost of capital.[3] Abnormal earnings bear on the difference between market and book values, that is, they bear on a firm's goodwill. In fact, a straightforward two-step procedure derives a particularly parsimonious expression for goodwill as it relates to abnormal earnings. First, following Peasnell (1981) and others, the clean surplus relation implies that goodwill equals the present value of future expected abnormal earnings. Second, if one further assumes that abnormal earnings obey an autoregressive process, then it follows that goodwill equals current abnormal earnings scaled by a (positive) constant. The result highlights that one can derive value by assuming abnormal earnings processes that make no reference to past or future expected dividends.

Owners' equity accounting not only subsumes the clean surplus relation, it also implies that dividends reduce book value but leave current earnings unaffected. One exploits this additional feature to examine the (marginal) effects of dividends on value and on the evolution of accounting data. Two closely related Modigliani and Miller (MM)(1958,1961) properties are satisfied. Dividends displace market value on a dollar-for-dollar basis, so that dividend payment irrelevancy applies. Furthermore, dividends paid today influence future expected earnings negatively. The model accordingly separates the creation of wealth from the distribution of wealth. Given the importance one generally attaches to MM properties in valuation analysis, the requirement that dividends reduce book value but not current earnings enhances the economic significance of owners' equity accounting.

The model admits information beyond earnings, book value, and dividends. One motivates the additional information by the idea that some value-relevant events may affect future expected earnings as opposed to current earnings, that is, accounting measurements incorporate some value-relevant events only after a time delay. The feature is of interest because the analysis shows that while the accounting data will be incomplete indicators of value, the weighted average of capitalized earnings and book value still provides the core of the valuation function.

Overall, the paper contributes to the accounting literature a benchmark model that one can use to conceptualize how value relates to the three accounting variables, earnings, book value, and dividends. The model satisfies a number of appealing properties and allows for a certain realism in the accounting: the theory rests directly on the clean surplus relation and the feature that dividends reduce book value but leave current earnings unaffected.

Overview of assumptions, concepts, and results

The primary issue at hand concerns the function that relates a firm's market value to the contemporaneous accounting/information variables. To derive this valuation function, the model relies on a parametric setup. The approach used has the advantage of not only yielding a closed-form valuation function, but also providing a concrete and complete framework to deal with value and accounting data. While some assumptions may seem relatively restrictive, it turns out that many of the model's key features apply under more general circumstances. Subsequent analyses delineate how various accounting constructs link up with properties of the valuation function. In this context one can also examine the broad issue of the different ways value reflects anticipated, rather than current, realizations of accounting data.

Three analytically straightforward assumptions formulate the valuation model.

First, as is standard in neoclassical models of security valuation, the present value of expected dividends (PVED) determines the market value. The underlying probabilistic framework implies an "objective beliefs" setting. To keep matters simple, risk neutrality applies so that the discount factor equals the risk-free rate.

Second, regular owners' equity accounting applies: accounting data and dividends satisfy the clean surplus relation, and dividends reduce book value without affecting current earnings.

Third, a linear model frames the stochastic time-series behavior of abnormal earnings. As already noted, this variable is defined as current earnings minus the risk-free rate times the beginning of period book value, that is, earnings minus a charge for the use of capital. Since PVED and the clean surplus relation imply that the market value equals the book value plus the present value of future expected abnormal earnings (see Peasnell 1981), the valuation analysis can focus on the prediction of abnormal earnings rather than dividends. To extract these predictions, the dynamics specify that date $t+1$ expected abnormal earnings are linear in the date t abnormal earnings, plus a correction for a scalar variable that represents information other than the accounting data and dividends. The variable for "other information" satisfies a (regular) autoregressive process. The two dynamic equations combine with the clean surplus relation to ensure that all value-relevant events will be absorbed by current or subsequent periods' earnings and book values.

The three assumptions lead to a linear, closed-form, valuation solution explaining goodwill, that is, value equals book value plus a linear function of current abnormal earnings and the scalar variable representing other information. A simple restriction eliminates the scalar variable in both the valuation function and the abnormal earnings dynamics; the case shows that current abnormal earnings determine goodwill if, and only if, abnormal earnings satisfy an autoregressive process. One can also derive an alternative expression for value. Disregarding other information, value equates to a weighted average of (i) current earnings capitalized minus current dividends and (ii) current book value. The clean surplus relation reconciles the two expressions for value, and both of them appeal to economic intuition. As a further point, one infers from the model that unexpected (market) returns depend linearly on unexpected earnings and innovations in the scalar variable process.

The unambiguous nature of the expressions relating value and returns to accounting/information raises the issue of their potential in empirical research. While such an inquiry may have its own merits, it is not pursued here. From the perspective of theory, a more critical issue concerns the implications of the second aspect of owners' equity accounting, namely, dividends reduce book value but leave current earnings unaffected. This feature is relevant when one identifies the economics inherent in the abnormal earnings dynamics assumption combined with the clean surplus relation. Initial observations are:

(i) An increase in dividends at any given date reduces the subsequent period's expected earnings. Because risk neutrality obtains, the marginal effect of a dollar of dividends on next period's foregone expected earnings equals the risk-free rate.

(ii) More generally, an increase in dividends reduces the subsequent two periods' aggregate earnings. The two-period compounded interest rate determines this effect.

These two consequences of distributing wealth to the owners extend the more basic requirement that dividends reduce book value but leave current earnings unchanged. Dividends have effects on future accounting data as well as on current accounting data. All of these constructs effectively relate to the idea that earnings in the future partially depend on today's book value. This dependence becomes explicit if one transforms the abnormal earnings dynamics to express next-period expected earnings as a function of current book value, as well as current earnings and dividends.

The quantification (ii) concerning current dividends and future earnings turns out to be central. Rather than deriving the property (ii) from an assumption on the abnormal earnings dynamics, a reverse analysis uses (ii) as an assumption to derive the abnormal earnings dynamics. Thus one can essentially recover the entire model by assuming (ii) in addition to the first two assumptions. The result underscores that to derive a restrictive class of valuation functions the analysis can formalize and exploit accounting constructs in lieu of a stochastic specification relating future dividends to current accounting data.

The model satisfies a number of additional, intuitively appealing properties.

Dividend policy irrelevancy in the spirit of Modigliani and Miller (1961) applies: a dollar of dividends displaces a dollar of market value. This implication obviously follows from "dividends reduce current book value ...," and it provides the other side of the coin of properties (i) and (ii).

Book values are unbiased estimators of market values in that the (unconditional) expected goodwill equals zero. In other words, though goodwill generally has positive serial correlation, over very long periods the average goodwill approximates zero. A similar no-bias time-series property applies when one evaluates the difference between capitalized earnings and value (inclusive of current dividends).

Market value reconciles with future expected aggregate earnings and future expected book values in a sensible fashion. One can also identify conditions such that next-period expected earnings, scaled by the inverse of the risk-free rate, determine value. In this case, the expected earnings for the next period alone provide sufficient information for the present value of all future expected dividends.

Finally, the model embeds an interesting notion of long-run permanent earnings. If one imposes a dividend policy such that dividends equate earnings, then expected earnings in the distant future simply equal current book value times the risk-free rate. This observation, which is due to Ramakrishnan (1990), illustrates that the earnings process integrates with underlying book values.

Development of the valuation model: Assumptions

Consider an economy with risk neutrality and homogenous beliefs. The market value of the firm then equals the present value of future expected dividends. (A subsequent section generalizes for risk aversion.) Given further that the interest rates satisfy a nonstochastic and flat term structure, the first assumption reduces to

$$P_t = \sum_{\tau=1}^{\infty} R_f^{-\tau} E_t[\tilde{d}_{t+\tau}] \qquad \text{(PVED)} \qquad \text{(A1)}$$

where

P_t = the market value, or price, of the firm's equity at date t.
d_t = net dividends paid at date t.
R_f = the risk-free rate plus one.
$E_t[.]$ = the expected value operator conditioned on the date t information.

The model permits negative d_t, that is, capital contributions exceeding dividends may occur. To avoid the cumbersome but more precise expression "dividends net of capital contributions," we simply refer to d_t as "dividends."

The model forces value to depend on accounting data because the data influence the evaluation of the present value of expected dividends. We develop a relatively general framework in which value depends on earnings and book value in addition to current dividends. Each of the three variables will be relevant in its own way, but in no sense does the model rely on "ideal" accounting constructs such as "economic earnings" or "economic earnings plus a random error."

As a matter of notation, let

x_t = earnings for the period $(t\text{-}1,t)$
y_t = (net) book value at date t.

Labeling of x_t and y_t is obviously arbitrary and gratuitous unless the model exploits structural attributes inherent in accounting. Of interest are at least two closely related attributes. First, the change in book value between two dates equals earnings minus dividends, that is, the model imposes the clean surplus relation. Second, dividends reduce current book value, but not current earnings. To formalize these two aspects of owners' equity accounting, we introduce the following mathematical restrictions:

$$y_{t-1} = y_t + d_t - x_t \qquad \text{(A2a)}$$

and

$$\partial y_t/\partial d_t = -1$$
$$\partial x_t/\partial d_t = 0 \qquad \text{(A2b)}$$

Though (A2b) does not follow from (A2a), (A2b) is consistent with (A2a) in the sense that

$$\partial y_{t-1}/\partial d_t = \partial y_t/\partial d_t + \partial d_t/\partial d_t - \partial x_t/\partial d_t$$

$$0 = -1 + 1 - 0$$

We distinguish between (A2a) and (A2b) because many conclusions depend only on (A2a).

One can apply the clean surplus relation (A2a) to express P_t in terms of future (expected) earnings and book values in lieu of the sequence of (expected) dividends in the PVED formula. Define

$$x_t^a \equiv x_t - (R_f - 1)\, y_{t-1}$$

Combined with the clean surplus restriction (A2a), the definition implies

$$d_t = x_t^a - y_t + R_f y_{t-1}$$

Using this expression to replace $d_{t+1}, d_{t+2}, \ldots$ in the PVED formula yields the equation[4]

$$P_t = y_t + \sum_{\tau=1}^{\infty} R_f^{-\tau} E_t[\tilde{x}_{t+\tau}^a] \qquad (1)$$

provided that $E_t[\tilde{y}_{t+\tau}] / R_f^{\tau} \to 0$ as $\tau \to \infty$. We assume that the last regularity condition is satisfied.

That the clean surplus equation implies equivalence of equation (1) and PVED has long been known in the accounting literature. See, for example, Edwards and Bell (1961), and Peasnell (1981), (1982).

We will refer to x_t^a as abnormal earnings. The terminology is motivated by the concept that "normal" earnings should relate to the "normal" return on the capital invested at the beginning of the period, that is, net book value at date t-1 multiplied by the interest rate. Thus one interprets x_t^a as earnings minus a charge for the use of capital.[5] A positive x_{t+1}^a indicates a "profitable" period since the book rate of return, x_{t+1}/y_t, exceeds the firm's cost of capital, R_f-1.

Relation (1) has a straightforward and intuitively appealing interpretation: a firm's value equals its book value adjusted for the present value of anticipated abnormal earnings. In other words, the future profitability as measured by the present value of the anticipated abnormal earnings sequence reconciles the difference between market and book values.

While relation (1) may appeal to one's intuition, its equivalence to PVED depends only on relatively trite algebra. As Peasnell (1982) notes, this formula is peculiar because one interprets it by referring to accounting concepts, yet the formula works regardless of the accounting principles that measure book values and earnings. Accounting constructs beyond the clean surplus restriction are irrelevant, and (1) does not even rely on assumption (A2b).

The third and final assumption concerns the time-series behavior of abnormal earnings. Since any analysis of the valuation function generally depends critically on various aspects of this assumption, it demands careful elaboration. An analytically simple linear model formulates the information dynamics. Two variables enter the specification: abnormal earnings, x_t^a, and information other than abnormal earnings, v_t.

Assume $\{\tilde{x}_\tau^a\}_{\tau\geq 1}$ satisfies the stochastic process (A3)

$$\tilde{x}_{t+1}^a = \omega x_t^a + v_t + \tilde{\varepsilon}_{1t+1} \quad (2a)$$

$$\tilde{v}_{t+1} = \quad + \gamma v_t + \tilde{\varepsilon}_{2t+1} \quad (2b)$$

where the disturbance terms, $\varepsilon_{1\tau}$, $\varepsilon_{2\tau}$, $\tau \geq 1$, are unpredictable, zero-mean, variables; that is, $E_t[\tilde{\varepsilon}_{kt+\tau}] = 0$, $k = 1{,}2$ and $\tau \geq 1$.

Assumption (A3) places no restrictions on the variances and covariances of the disturbance terms. For example, the variances may be heteroscedastic.

The parameters of the process, ω and γ, are fixed and "known".[6] We restrict these parameters to be non-negative and less than one. The last condition implies that the unconditional means of x_t^a and v_t are zero.

Equation (2a) puts the coefficient associated with v_t equal to one without loss of generality. The issue is simply one of scaling. Further note that v_t is irrelevant in the dynamics if $v_0 = \varepsilon_{2\tau} = 0$, all $\tau \geq 1$. This special case is equivalent to $v_1 = v_2 = ... = 0$, and $\{x_\tau^a\}_\tau$ satisfies a regular autoregressive process.

Equation (2b) shows that the predictions $E_t[\tilde{v}_{t+\tau}]$, $\tau\geq 1$, depend at most on v_t, and not on x_t^a. We impose the independence because v_t should be thought of as summarizing value relevant events that have yet to have an impact on the financial statements. Such information bears upon future (abnormal) earnings independently of current and past (abnormal) earnings. The model also implies that realizations of v_t (or ε_{2t}) cannot "bypass" the financial statements. These realizations feed into the x_{t+1}^a, x_{t+2}^a,....sequence, and each realization of x_t^a, in turn, updates the date t book value via the recursive equation

$$y_t = x_t^a + R_f y_{t-1} - d_t \quad (3)$$

The expression shows that one infers y_t from the sequences $\{x_\tau^a\}_{\tau=1}^t$, $\{d_\tau\}_{\tau=1}^t$ and the initialization condition $y_0 = -d_0$. Since $\{\varepsilon_{1t},\varepsilon_{2t}\}_{\tau=1}^t$ determines $\{x_\tau^a\}_{\tau=1}^t$, it follows that the uncertainty resolution, $\{\varepsilon_{1t},\varepsilon_{2t}\}_{\tau=1}^t$, and the sequence of dividends, $\{d_\tau\}_{\tau=0}^t$, suffice to determine the book value, y_t. Thus one sees that the specification (2), combined with (3) or (A2a), works such that any uncertainty resolution through date t, $\{\varepsilon_{1t},\varepsilon_{2t}\}_{\tau=1}^t$, feeds gradually into current and future book values, y_t, y_{t+1},...

As part of assumption (A3), the model imposes the condition $\partial v_t/\partial d_t = 0$. This condition naturally makes sense if one thinks of v_t as capturing all non-accounting information used in the prediction of future abnormal earnings. Although a broad independence characterization of v_t exceeds the requirement of $\partial v_t/\partial d_t = 0$, it simplifies by avoiding irrelevant specification issues. We attach no significance to the possibility of relating v_t (or ε_{2t}) to current and past accounting data while maintaining $\partial v_t/\partial d_t = 0$.

Equation (2a) predicts next-period earnings in addition to next-period abnormal earnings:

$$E_t[\tilde{x}_{t+1}] = (R_f - 1)y_t + \omega x_t^a + v_t \tag{4}$$

Since the date t information set includes current book value, y_t, as well as x_t^a and v_t, equation (4) poses no problems. The earnings prediction issue becomes more complicated, however, if the focus shifts to periods beyond the next one. For $\tau \geq 2$, the model yields no predictions $E_t[\tilde{x}_{t+\tau}]$. To appreciate this point, consider the case $\tau=2$. Equation (4) shows that a prediction $E_t[\tilde{x}_{t+2}]$ requires a prediction $E_t[\tilde{y}_{t+1}]$ The last prediction, in turn, requires a prediction $E_t[\tilde{d}_{t+1}]$, which is not part of the model.

All elements in the sequence of expected dividends, $E_t[\tilde{d}_{t+1}]$, $E_t[\tilde{d}_{t+2}]$,...will remain unspecified, and the sequence can never be extracted from more basic assumptions (such as [A2a] and [A3]). The lack of a specific dividends process may seem puzzling because the model depends on the idea that the sequence of expected dividends ultimately determines value (i.e., [A1] applies). The paper resolves this apparent paradox.

Expressions for value and returns

Based on assumptions (A1), (A2a), and (A3), to derive the valuation function one uses (1) and evaluates $\sum R_f^{-\tau} E_t[\tilde{x}_{t+\tau}^a]$ given the dynamics (A3). The linearity in the specification leads, of course, to a linear solution:

$$P_t = y_t + \alpha_1 x_t^a + \alpha_2 v_t \tag{5}$$

where

$$\alpha_1 = \omega / (R_f - \omega) \geq 0$$
$$\alpha_2 = R_f / (R_f - \omega)(R_f - \gamma) > 0$$

Appendix 1 demonstrates this result.

Equation (5) implies that the market value equals the book value adjusted for (i) the current profitability as measured by abnormal earnings and (ii) other information that modifies the prediction of future profitability. One eliminates (ii) by restricting x_t^a to satisfy an autoregressive process. This special version of (A3) postulates that $v_t \equiv 0$, and thus abnormal earnings, alone, determine goodwill if and only if current abnormal earnings suffice in the prediction of future abnormal earnings.

A couple of observations concerning the valuation-coefficients, α_1 and α_2, help to understand the economics of the model. For $\omega > 0$, these two coefficients are positive simply because the predictions $E_t[\tilde{x}_{t+\tau}^a]$, any $\tau \geq 1$, relate positively to x_t^a and v_t. (The boundary case $\omega=0$ implies that $E_t[\tilde{x}_{t+\tau}^a]$ is independent of x_t^a, and thus P_t cannot depend on x_t^a either.) Further, the functions $\alpha_1(\omega)$ and $\alpha_2(\omega, \gamma)$ are increasing in their arguments. The property reflects that ω and γ act as persistence parameters in the (x_t^a, v_t) process; larger values of ω and γ make P_t more sensitive to (x_t^a, v_t) realizations.

Expression (5) combines with the dynamics (A3) to show how market returns depend on the realizations of unpredictable, "new", information. Straightforward manipulations yield (see Appendix 2):

$$(\tilde{P}_{t+1} + \tilde{d}_{t+1})/P_t = R_f + (1 + \alpha_1)\,\tilde{\varepsilon}_{1t+1}/P_t + \alpha_2\tilde{\epsilon}_{2t+1}/P_t \qquad (6)$$

Two sources of uncertainty, unexpected earnings (ε_{1t+1}) and unexpected innovations in "other information" (ϵ_{2t+1}), therefore explain returns. A term related to the unpredictability of dividends is absent, and yet expression (6) does not depend on narrow restrictions on the dividend policy, that is, d_t need not be a deterministic function of contemporanous earnings, book value, and other information (v_t). Irrelevance of dividend uncertainty in explaining returns is basic, and expression (6) suggests that the model embeds MM concepts. The claim is indeed valid; subsequent sections consider this important aspect of the model.

Examination of the response coefficients in expression (6) show that these are the same as those in the valuation function, except that the coefficient associated with unexpected earnings equals $1 + \alpha_1$, rather than α_1. The '1' in the $1 + \alpha_1$ expression is necessary because the valuation function (5) includes book value, which, in turn, traces from the clean surplus restriction and the valuation formula (1). A one-to-one correspondence between an incremental dollar of earnings/book value and market value occurs only if current abnormal earnings have the minimum, zero, persistence (i.e., when $\omega = \alpha_1 = 0$). Since an additional dollar of book value/earnings generally adds <u>more</u> than a dollar of market value, the coefficient associated with unexpected earnings makes intuitive sense.

The discussion indicates that assumptions (A1), (A2a), and (A3) lead to a cohesive, and perhaps appealing, model of value and returns. Even so, the extent to which the framework embeds accounting concepts needs to be addressed. The analysis so far has exploited the clean surplus assumption (A2a), but the dividends reduce book value assumption (A2b) remains unused. One can also question whether the dynamics (A3) has sufficient nuance to relate to any accounting concepts.

A subsequent section considers the possibility of deriving (A3) from basic accounting concepts, including (A2b). However, before doing so it will be helpful to analyze how earnings, in lieu of abnormal earnings, relate to value. The valuation function (5) and the information dynamics can be re-expressed to show how earnings and book values operate as the primary value indicators. This restated version of the model illuminates similarities as well as differences between earnings and book values.

A restatement of the model and two special cases

Using the definition of x_t^a, the valuation function (5) also equals $P_t = y_t + \alpha_1 x_t - \alpha_1(R_f - 1)y_{t-1} + \alpha_2 v_t$. If one further replaces y_{t-1} with the right-hand side of the clean surplus equation (A2a), then, after some simplifications,

$$P_t = k(\varphi x_t - d_t) + (1 - k)y_t + \alpha_2 v_t \qquad (7)$$

where

$$\varphi \equiv R_f/(R_f - 1)$$
$$k = (R_f - 1)\alpha_1 = (R_f - 1)\omega/(R_f - \omega)$$

The definition of φ and its place in (7) show that the parameter acts as earnings-multiplier. Concerning k, this coefficient has a one-to-one relation with ω (and α_1). Since $0 \leq \omega \leq 1$, k satisfies $0 \leq k \leq 1$, and for the boundary points one obtains $k(\omega = 0) = 0$, $k(\omega = 1) = 1$. However, for all interior points, note that $k \neq \omega$.

The form of (7) and the restrictions on k indicate that the valuation model can be viewed as a weighted average of an earnings model and a book value model.[7] The idea is valid. One relies on $\omega = k = 1$ and on $\omega = k = 0$ to identify the two supporting models in the weighting scheme.

To make the point we simplify (*w.l.o.g.*) by putting $v_t \equiv 0$. As the first special case, let $\omega = k = 1$:

$$P_t = \varphi x_t - d_t \tag{8}$$

and (A3) reduces to

$$\tilde{x}_{t+1} = R_f x_t - (R_f - 1)d_t + \tilde{\varepsilon}_{1t+1} \tag{9}$$

As the second special case, let $\omega = k = 0$:

$$P_t = y_t \tag{10}$$

and (A3) reduces to

$$\tilde{x}_{t+1} = (R_f - 1)y_t + \tilde{\varepsilon}_{1t+1} \tag{11}$$

In the first case earnings and dividends suffice to predict next-period expected earnings; these two variables accordingly determine value.[8] In the second case book value alone predicts earnings, and thus book value now suffices to determine the market value.

The expressions derived make it apparent that the valuation function (7) equals a weighted average of (8) and (10) with the weights k and $1 - k$. Less obvious, a weighted average of the two dynamic equations (9) and (11) leads to the more general dynamics (2a), except that now ω and $1 - \omega$ specify the weights. To verify this claim, the weighted average of (9) and (11) equals

$$\tilde{x}_{t+1} = \omega[R_f x_t - (R_f - 1)d_t + \tilde{\varepsilon}_{1t+1}] + (1 - \omega)[(R_f - 1)y_t + \tilde{\varepsilon}_{1t+1}]$$

which, after substituting $d_t = y_{t-1} + x_t - y_t$ into the expression, simplifies to

$$\tilde{x}^{a}_{t+1} = \omega x^{a}_t + \tilde{\varepsilon}_{1t+1}$$

Since this autoregressive specification implies valuation function (5) and thus (7) as well (with $v_t \equiv 0$), one concludes that a "pure" earnings/dividends model and a "pure" book value model combine as a weighted average to compose a more general earnings/dividends/book value model. This perspective poses no problems as long as one keeps in mind that there are two sets of weights, $(k, 1 - k)$ and $(\omega, 1 - \omega)$. Consistency in weights follows because the function $k(\omega)$ is increasing, and one naturally thinks of the persistence parameter ω as the exogenous factor that determines the relative importance of earnings as opposed to book value in valuation.

The two extreme cases appear asymmetric in the sense that the dividends show up explicitly in the first case ($\omega = 1$) only; dividend terms are present in (8) and (9) but absent in (10) and (11). However, this difference reflects the owners' equity accounting in (A2b), that is, dividends reduce book values while the current earnings remain unchanged. The next section discusses the point in detail.

Additional properties of the model

Assumptions (A1), (A2), and (A3) are deceiving in their simplicity. The setup allows for a straightforward derivation of the valuation function, but whether the analysis can be taken any further is less clear. One may also be concerned that the assumptions' simplicity makes undesirable implications likely. This section examines the model closely to show that, in fact, it embeds a number of subtle and meaningful properties. Most of these follow because the owners' equity accounting constructs combine appealingly with the abnormal earnings dynamics. Thus we suggest that the model provides an instructive starting point when one tries to understand how earnings, book values, and dividends, relate to market value.

The first two properties extract implications of (A2)—b as well as a—when applied to the information dynamics (A3).

$$\partial E_t[\tilde{x}_{t+1}]/\partial d_t = -(R_f - 1) \tag{P1}$$

The result depends on (A2b), $\partial v_t/\partial d_t = 0$, and expression (2a)[9,10]. Specifically, $\partial y_t/\partial d_t = -1$, $\partial x_t^a/\partial d_t = 0$, and $\partial v_t/\partial d_t = 0$, it follows that

$$\begin{aligned}\partial E_t[\tilde{x}_{t+1}]/\partial d_t &= \partial[(R_f - 1)y_t + \omega x_t^a + v_t]/\partial d_t \\ &= -(R_f - 1) + 0 + 0 = -(R_f - 1)\end{aligned}$$

Property (P1) shows that the payment of dividends reduces the next-period expected earnings by the risk-free rate, $R_f - 1$. The implication makes sense given the nature of accrual accounting and the availability of zero net present value activities; a firm can increase its dividends through incremental borrowings, but such borrowings incur an interest expense for the subsequent period.[11] Accrual accounting makes the interest expense dependent only on the amount borrowed and the interest rate; the debt servicing ("cash flow") schedule has <u>no</u> effect on the next period's interest expense. Property (P1) therefore imparts an accrual, "measurement," perspective on the behavior of expected earnings.

The next property of the model generalizes (P1) by expressing the effect of dividends on the expected earnings for two periods.

$$\partial E_t[\tilde{x}_{t+2} + \tilde{x}_{t+1} + \tilde{d}_{t+1}(R_f - 1)]/\partial d_t = -(R_f^2 - 1) \qquad \text{(P2)}$$

The proof of (P2) is similar to that of (P1).

(Since $x_{t+2} + d_{t+1}(R_f - 1) = x^a_{t+2} + (R_f - 1)(y_{t+1} + d_{t+1}) = x^a_{t+2} + (R_f - 1)(x^a_{t+1} + R_f y_t)$ and $x_{t+1} = x^a_{t+1} + (R_f - 1)y_t$, one obtains $E_t[\ .\ ] = E_t[\tilde{x}^a_{t+2} + R_f\tilde{x}^a_{t+1}] + (R_f^2 - 1)y_t$. Further, $\partial E_t[\tilde{x}^a_{t+2} + R_f\tilde{x}^a_{t+1}]/\partial d_t = 0$ due to [A2b] and $\partial v_t/\partial d_t = 0$, and $\partial y_t/\partial d_t = -1$ due to [A2b]. The result follows.)

Property (P2) shows the importance of earnings aggregation. That is, to evaluate the (marginal) effect of dividends on future expected earnings one must consider all sources of earnings, and these sources add without weights. Two kinds of earnings aggregation are present. First, the term $\tilde{x}_{t+2} + \tilde{x}_{t+1}$ requires earnings to add across periods. Second, date t+1 dividends generate earnings on private account for the period ending at t+2; this source of earnings, $\tilde{d}_{t+1}(R_f - 1)$, must be added to the firm's earnings for a correct evaluation of the effect of date t dividends on the next two periods' expected earnings. Date t+2 earnings clearly depend on date t+1 dividends. In sum, the property (P2) extends and generalizes (P1) by showing how the assumptions (A2) and (A3) jointly reflect earnings aggregation as well as accrual measurements.

The next property concerns implications of (A2b) on the valuation function (5) rather than the information dynamics (A3). Consistent with Modigliani and Miller (1961), the model satisfies dividend payment irrelevancy; that is, an additional dollar of date t dividends simply displaces a dollar of date t market value.

$$\partial P_t/\partial d_t = -1 \qquad \text{(P3)}$$

This result follows from (A2b) and (5) since $\partial y_t/\partial d_t = -1$ and x^a_t and v_t do not depend on d_t.

We emphasize that though the valuation function does not depend on (A2b), this assumption cannot be left out when one derives the value displacement property (P3). Its proof clearly requires $\partial y_t/\partial d_t = -1$ and $\partial x_t/\partial d_t = 0$. Nor can one argue that (A2b) is redundant, in that it follows from (A2a). To appreciate the point, note that an assumption of, say, $\partial y_t/\partial d_t = 0$ and $\partial x_t/\partial d_t = 1$ is consistent with (A2a), yet (5) and $\partial P_t/\partial d_t \neq -1$ are both met. One concludes that two features of owners' equity accounting—clean surplus and "dividends reduce book value.." —are essential to derive a basic MM property.

The model is obviously silent concerning the accounting principles that generate the accounting data. Nevertheless, the next property shows that the assumptions force the unspecified accounting measurements to satisfy certain broad properties. Based on the models general structure, book value seemingly works as a "rough" measure/estimate of value, whereas x_t^a and v_t augment book value as "correcting" information. This observation can be refined. Since the parameters in the dynamics satisfy $0 \leq \omega$, $\gamma < 1$, it follows that, over time, on average, x_t^a and v_t equal zero. Average goodwill therefore also equals zero over time. Stated somewhat differently, though the process $\{\tilde{P}_t - \tilde{y}_t\}_t$ allows for serial correlations, over sufficiently long periods the average realization approximates zero. We refer to this property as <u>unbiased</u> accounting with respect to book value. One can similarly address whether earnings, scaled by the multiplier φ, on average equals value (inclusive of dividends). The conclusion is affirmative. These aspects of the model are summarized formally as follows.

$$E_t[\tilde{P}_{t+\tau} - \tilde{y}_{t+\tau}] \to 0 \text{ as } \tau \to \infty; \tag{P4a}$$

$$E_t[\tilde{P}_{t+\tau} + \tilde{d}_{t+\tau} - \varphi x_{t+\tau}] \to 0 \text{ as } \tau \to \infty. \tag{P4b}$$

The proof of (P4a) follows directly from the valuation function (5) since (A3) implies $E_t[\tilde{x}^a_{t+\tau}]$ and $E_t[\tilde{v}_{t+\tau}] \to 0$ as $\tau \to \infty$. Similarly, (P4b) follows because $P_t + d_t = \varphi x_t + \frac{k-1}{R_f - 1} x_t^a + \alpha_2 v_t$.(To derive the last relation one solves for y_t in (5) and substitutes it into (7)). The use of $P_t + d_t$ rather than P_t in (b) makes sense since the date t dividends do not affect the difference $P_t + d_t - \varphi x_t$.

The unbiased properties of the accounting measures bear upon the prediction of next-period earnings. While the best prediction is obviously given by $\omega[R_f x_t - (R_f - 1)d_t] + (1-\omega)[(R_f - 1)y_t] + v_t$, unbiasedness in the accounting measures implies that both of the two basic accounting prediction components inside the brackets are, on average, unbiased predictors of next-period expected earnings. One expresses this idea more completely and rigorously as $E_t[\tilde{x}_{t+\tau+1} - (R_f\tilde{x}_{t+\tau} - (R_f - 1)\tilde{d}_{t+\tau})] \to 0$ and $E_t[\tilde{x}_{t+\tau+1} - (R_f - 1)\tilde{y}_{t+\tau}] \to 0$, $\tau \to \infty$. Since (A1) implies $E_t[\tilde{P}_{t+\tau+1} + \tilde{d}_{t+\tau+1} - R_f\tilde{P}_{t+\tau}] = 0$ for all $\tau \geq 0$, the proofs follow directly from (P4b) and (P4a), respectively.

Two properties discussed next deal with value as it relates to future expected realizations of accounting data. Of course, one such relation has already been developed, namely expression (1), which shows how value equals book value plus an adjustment for the present value of expected abnormal earnings. One may reasonably ask what happens if the focus shifts away from abnormal earnings to plain earnings. The answer is interesting because earnings aggregation, as opposed to discounting, applies.

Define

$$V_t^T \equiv E_t[\sum_{\tau=1}^{T} \tilde{x}_{t+\tau} + \sum_{\tau=1}^{T} (R_f^{T-\tau} - 1)\tilde{d}_{t+\tau}]/(R_f^T - 1)$$

then

$$\text{(a)}\quad V_t^T \to P_t \text{ as T} \to \infty, \tag{P5a}$$

and

(b) $V_t^T - P_t$ does not depend on the dividend policy. (P5b)

Appendices 3 and 4 demonstrate these results. Part (P5a) depends only on the clean surplus assumption (A2a) and, of course, (A1).[12] Part (P5b) also uses (A2b) and (A3).

Property (P5) has much in common with property (P2), as is suggested by the fact that (P2) evaluates the partial derivative of $(R_f^T - 1)V_t^T$ with respect to d_t for $T = 2$. Both (P2) and the market value approximation V_t^T depend on the aggregation of expected earnings without weights, and all sources of earnings must be considered: $\sum_{\tau=1}^{T} (R_f^{T-\tau} - 1)d_{t+\tau}$ equals earnings generated on private account by investing the dividends in a risk-free asset. Also like (P2), the aggregated expected earnings must be tied to a discount factor appropriate for the number of periods over which earnings are anticipated. The correctness of the scale factor $(R_f^T - 1)^{-1}$ becomes obvious by noting that the dividend displacement property $\partial V_t^T / \partial d_t = -1$ is simply equivalent to the generalized, T-periods version of (P2).

Approximation (P5a) would be of only modest interest if its error depended on the degree to which the dividend term dominates the aggregate earnings term. However, such dividend policy dependence can be ruled out. Given the assumptions (A2a) and (A3) it follows that V_t^T is linear in y_t, x_t^a, v_t:

$$V_t^T = y_t + \alpha_1(\mathrm{T})x_t^a + \alpha_2(\mathrm{T})v_t \tag{12}$$

and where the parameters $\alpha_1(\mathrm{T}) \to \alpha_1$ and $\alpha_2(\mathrm{T}) \to \alpha_2$ as $\mathrm{T} \to \infty$. The approximation V_t^T has the same general structure as P_t, except that the parameters in (12) generally deviate from the correct ones when $T < \infty$. Invoking (A2b), the solution implies that the dividend policy does not influence the evaluation of V_t^T.

One can actually express V_t^T in an alternative form that closely resembles the formula (1). On the basis of only (A1) and (A2a) it turns out that, for any T,

$$V_t^T = y_t + \varphi_{\mathrm{T}} \sum_{\tau=1}^{T} R_f^{-\tau} E_t[\tilde{x}_{t+\tau}^a]$$

where $\varphi_{\mathrm{T}} \equiv R_f^T/(R_f^T - 1)$. The above expression for V_t^T obviously approaches (1) as $T \to \infty$, and thus (P5a) follows. Concerning (P5b), if one again focuses on the last expression for V_t^T and compares it to (1), then the assumptions (A2b) and (A3) clearly imply that $\partial V_t^T / \partial d_t = \partial P_t / \partial d_t = -1$ since $\partial y_t / \partial d_t = -1$ and the two expected abnormal earnings terms do not depend on dividends. The analysis highlights the key feature of the dynamics (A3) when one also assumes (A2b): *these two assumptions ensure that current and future expected abnormal earnings cannot be influenced by a firm's decision concerning current or future expected div-*

idends. This statement goes to the heart of the model because the MM properties (P1), (P2), (P3) and (P5b) would otherwise be violated.

(P5a) concerns approximations, $P_t - V_t^T \approx 0$, but in some settings there may be no error for finite T. No error for T=1 is of particular interest since it provides a model illustration of the popular idea that value depends on capitalized next-period expected earnings. In fact, there are cases with no error for *all* $T \geq 1$. Consider the (A3) restrictions $(\omega,\gamma) = (1,0)$ or $= (0,1)$; one verifies without difficulty that for every T, $\alpha_1(T) = \alpha_1$ and $\alpha_2(T) = \alpha_2$. It follows that

$$P_t = V_t^1 \equiv (R_f - 1)^{-1}E_t[\tilde{x}_{t+1}]$$

and, more generally,

$$P_t = V_t^T, \text{ any } T \geq 1.$$

The possibility of $P_t = (R_f - 1)^{-1}E_t[\tilde{x}_{t+1}]$ points toward the close relationship between (P1) and (P3). Given that $P_t = V_t^1$, it follows immediately that $\partial E_t[\tilde{x}_{t+1}]/\partial d_t = -(R_f - 1)$ if and only if $\partial P_t/\partial d_t = -1$. One sees more generally that if value is expressed as

$$P_t = (R_f - 1)^{-1}E_t[\tilde{x}_{t+1}] + \text{error}_t$$

then (P1) and (P3) imply each other provided that $\partial\text{error}_t/\partial d_t = 0$. The observation concerning the error term is immediate from (P5b) (which depends on [A2b]).

The approximate value indicator V_t^T focuses on an expected <u>flow</u> of accounting value added over the period (t, t+T). One can similarly also focus on an expected <u>stock</u> of accounting value at t+T; this alternative approximate value indicator is, of course, based on the date t+T expected book value.

Define

$$W_t^T \equiv R_f^{-T}E_t[\tilde{y}_{t+T} + \sum_{\tau=1}^{T} R_f^{T-\tau}\tilde{d}_{t+\tau}]$$

then

$$\text{(a)}\quad W_t^T \rightarrow P_t \text{ as } T \rightarrow \infty, \qquad \text{(P6a)}$$

and

$$\text{(b)}\quad W_t^T - P_t \text{ does not depend on the dividend policy.} \qquad \text{(P6b)}$$

One interprets the expression inside the brackets as the date t+T book value plus the value that has cumulated on private account due to the payment of dividends. The dividend adjustment is similar to the one necessary for (P5), except that the term now represents a stock, rather than flow, of value. One sees immediately that the approximation error $W_t^T - P_t$ equals $R_f^{-T}E_t[\tilde{y}_{t+T} - \tilde{P}_{t+T}]$, which approaches zero as $T \rightarrow \infty$. Also similiar to (P5), $W_t^T - y_t$ is a linear function of (x_t^a, v_t) for each T, $\partial W_t^T/\partial d_t = -1$, and the dividend policy does not influence the evaluation of W_t^T or the approximation error.

A simple example verifies that V_t^T differs from W_t^T: $W_t^T = P_t$ for $T = 1$ if $(\omega,\gamma) = (0,0)$, but $V_t^T \neq P_t$ for this specification.[13] The example further illustrates that the approximation (P6) may work perfectly for T=1, even though $(x_t^a, v_t) \neq (0,0)$.

The final property deals with the relevance of current book value when one conceptualizes the expected long run, "permanent" earnings. To define such earnings, consider the expected earnings in the distant future if dividends are always put equal to earnings. One may then ask: What factor(s) determines these expected earnings? This question has a straightforward answer given the assumptions (A2a) (the clean surplus restriction) and (A3) (the information dynamics (2a) and (2b)).

$$\text{If } x_{t+\tau} = d_{t+\tau} \text{ for all } \tau \geq 1, \tag{P7}$$

then

$$E_t[\tilde{x}_{t+\tau}] = (R_f - 1)y_t \text{ as } \tau \rightarrow \infty.$$

(The above observation is due to Ramakrishnan [1990]; the proof is immediate[14] since $E_t[\tilde{x}^a_{t+\tau}] \rightarrow 0$ as $\tau \rightarrow \infty$, and $y_{t+\tau} = y_t$ for all $\tau \geq 1$.)

Hence, the current book value alone determines the earnings that can be expected in the long run if one eliminates any growth in earnings and book value due to changes in retained earnings (or capital stock). Alternatively, the model imposes an average book rate of return that equals the discount factor if one puts dividends equal to earnings. In the long run assets generate earnings, and, conversely, earnings cannot be expected without assets.

Assumptions sufficient to derive the abnormal earnings dynamics

Out of the three assumptions that formulate the model, assumption (A3), which concerns the abnormal earnings dynamics, may appear more abitrary or confining as compared to the first two. Though (A3) arguably provides a satisfactory specification as it leads to a model with many appealing properties, its "quality" and necessity deserves examination. Models of the prediction of earnings that go beyond (2a) can be entertained. The broad issue therefore concerns how earnings evolve over time.

To be more precise, because $E_t[\tilde{x}^a_{t+1}] = \omega x_t^a + v_t$ is equivalent to

$$E_t[\tilde{x}_{t+1}] = \omega R_f x_t + (1 - \omega)(R_f - 1)y_t - \omega(R_f - 1)d_t + v_t \tag{13}$$

one may hypothesize that the way $E_t[\tilde{x}_{t+1}]$ depends on (x_t,y_t,d_t) excludes other worthwhile prediction models. Expression (13) permits only one degree of freedom (the parameter ω), whereas a less restrictive linear approach permits three degrees of freedom, that is, one unrestricted coefficient for each of the variables (x_t,y_t,d_t). Hence the question arises whether one can use accounting constructs to show that the prediction of next-period earnings leads to (13), or equivalently, a weighted average of a pure earnings/dividends model (ω = 1) and a pure book value model (ω = 0).

To derive the one-degree-of-freedom model (13) one naturally tries to exploit previously derived "properties" as assumptions. The following result obtains: property (P2), derived from (A2) and (A3), used as an assumption in addition to (A2) and mild regularity conditions on the dynamics imply (13). That is, with these assumptions one retrieves (A3).
Proposition: Suppose the prediction of next-period earnings is given by a class of linear models

$$E_t[\tilde{x}_{t+1}] = \theta_1 x_t + \theta_2 y_t + \theta_3 d_t + v_t \tag{14}$$

Suppose further that

(i) $y_t = y_{t+1} + d_{t+1} - x_{t+1}$ (A2)
and $\partial y_t/\partial d_t = -1$; $\partial x_t/\partial d_t = 0$

(ii) $\partial E_t[\tilde{x}_{t+2} + \tilde{x}_{t+1} + \tilde{d}_{t+1}(R_f - 1)]/\partial d_t = -(R_f^2 - 1)$ (P2)

(iii) $E_t[\tilde{v}_{t+1}] = \gamma v_t$ and $\partial v_t/\partial d_t = 0$.

Then (14) reduces to (13), and, equivalently,

$$E_t[\tilde{x}^a_{t+1}] = \omega x^a_t + v_t$$

Proof: See Appendix 5.
The assumptions imply that the property (P1) holds, but one cannot replace (P2) with (P1). Thus (P2) is more powerful than (P1), and (P2) reflects accrual accounting as well as earnings aggregation.

The proposition provides a useful way of thinking about the valuation model. Assumptions (14) and (iii) serve as two broad (linear) restrictions on the stochastic behavior of accounting data and other information. If one additionally stipulates the following valuation/accounting constructs, then the entire valuation model follows.[15]

(a) The market value equals the present value of anticipated dividends (A1).
(b) Basic accounting constructs:
  A2. The clean surplus equation (A2a) and dividends reduce book value but not current earnings (A2b).
  P2. The penalty of paying dividends on future expected earnings reflects earnings aggregation.
(c) The behavior of other valuation relevant information, $E_t[\tilde{v}_{t+\tau}]$ does not depend on current or future dividends.

De-emphasizing the general linearity condition on the dynamics, the propositon shows that (b) and (c) imply (A3). It follows immediately that the building blocks (a), (b), and (c) lead to (A1), (A2), and (A3).

Though valuation theory starts from PVED (i.e., (a) or [A1]), it should be emphasized that the building blocks make no reference to the dividend policy or the elements in the sequence of anticipated dividends. These elements are of no particular interest, of course, since the owners' equity accounting constructs shift the analysis away from the distribution of wealth to the creation and recognition of wealth. Hence, the building blocks ensure the validity of the powerful formula (1) and combine it with a sequence of expected abnormal earnings conditioned on current abnormal earnings and other information.

Replacing dividends with abnormal earnings in present value evaluations condenses and streamlines the analysis, but it is not necessary *per se*. A more cumbersome approach introduces an explicit class of dividend policies so that the model incorporates a sequence of expected dividends. Applying this sequence to the PVED evaluation derives the valuation function (5) without the benefit of (1). Relatively simple mathematics follows if one specifies linear policies. Thus, consider the specification $\tilde{d}_{t+1} = \pi_1 x_t + \pi_2 y_t + \pi_3 d_t + \pi_4 v_t + u_{t+1}$, where π_1, π_2, π_3, π_4 are policy parameters, and u_{t+1} is a random disturbance term that possibly correlates with ε_{1t+1} and ε_{2t+1}. (One identifies $d_{t+1} = Kx_{t+1}$ as a special case.) Using A2 and A3 one can next infer the sequence $E_t[\tilde{d}_{t+\tau}]$, $\tau \geq 1$, and, further, evaluate PVED to show that (5) applies. Though the analysis is tedious, it remains linear and it demonstrates directly that the policy parameters π_1, π_2, π_3, π_4 do not influence value.[16] No complications arise provided that the parametric specifications satisfy the mild regularity condition $R_f^{-T} E_t[(\tilde{x}_{t+T}, \tilde{y}_{t+T}, \tilde{d}_{t+T})] \rightarrow 0$ as $T \rightarrow \infty$.

The proposition's emphasis on extracting implications of paying dividends leads to a final point. One identifies (or labels) the individual variables in the vector (x_t, y_t, d_t, v_t) depending on how these variables respond to marginal changes in dividends. Provided that assumptions (A2) and (A3) are met, observe that

$$\begin{aligned} \partial x_t/\partial d_t &= 0 \\ \partial y_t/\partial d_t &= -1 \\ \partial d_t/\partial d_t &= 1 \\ \partial v_t/\partial d_t &= 0. \end{aligned}$$

Further, to discriminate between x_t and v_t,

$$\partial E_t[\tilde{x}_{t+1}]/\partial d_t = -(R_f - 1)$$

whereas

$$\partial E_t[\tilde{v}_{t+1}]/\partial d_t = 0.$$

The conditions in the proposition therefore ensure that each of the variables in the vector (x_t, y_t, d_t, v_t) plays a distinct role. In addition, the labeling (x_t, y_t, d_t, v_t) = (earnings, book value, dividends, other information) is the only one that makes sense.

Some observations concerning risk

Since (A1) relies on the risk-free rate as a discount factor, the theory has been based on risk neutrality. This aspect naturally raises the issue of how one generalizes and modifies the analysis to incorporate the risk in the anticipated dividend sequence. Three approaches are possible; this section briefly discusses their relative strengths and limitations.

The most direct approach allowing for risk replaces the discount factor R_f with some factor, ρ, which adjusts R_f for risk. That is, $\rho = R_f +$ risk premium. A firm's cost-of-equity capital, or the expected market return, determines the parameter ρ. For example, CAPM implies that $\rho = R_f +$ beta x [expected return on the market portfolio - R_f]. This kind of modification obviously introduces no problems in analytical and technical terms.

This risk concept should be adequate in many empirical applications (or evaluations) of the model. In the usual fashion, one would infer ρ from a firm's estimated beta and the market's (average) risk premium for stocks. This concept of risk will also generally serve its purpose if one uses the model in "practical" investment analysis. For example, any implementation of the (approximate) intrinsic value formula based on future expected earnings (P6a) requires a discount factor to implement the formula. Simplicity is an obvious virtue in this case.

Though this approach is simple and perhaps useful for many practical purposes, it lacks theoretical appeal. An obvious theoretical drawback concerns the silence about from where the risk originates. It should presumably depend on the risk inherent in (abnormal) earnings, book values, dividends, and so on, but ρ tells us nothing about such matters. One also has to question, more generally, how the presence of risk, in any form, should modify the PVED formula and the formula (1). It is by no means obvious that risk can be captured properly by simply increasing the discount factor in PVED and (1). In sum, the "replacement procedure" is *ad hoc* and exogenous to the extreme.

A complete valuation theory identifies all risk adjustments as a function of economy inherent risk concepts. Modern finance theory (see Rubinstein 1976) provides such a general framework for risk by adjusting the numerators rather than the denominators in the formula (A1). This relativly abstract approach relies on generalized measure theory in lieu of the probablistic structure necessary to define the $E_t[\,.\,]$ operator. Thus the theory develops from an $E_t^*[\,.\,]$ operator, where the star indicates that the measure relates to the economy's underlying implicit (or event-contingent) price system (which works like ordinary probabilities; see Huang and Litzenberger 1988 or Ohlson 1990). Again, the analysis in this paper can be modified without any analytical complications. The "cost" of using this approach, however, is obviously heavy since it does not, by itself, lead to concrete implications.

As a third approach, one can add some structure to the general $E_t^*[\,.\,]$ framework. The work of Garman and Ohlson (1980) could potentially be exploited in this context. They model correlations between disturbance terms such as ε_{1t}, ε_{2t} and the implicit price system. In this scheme the expected market return is endogenous to reflect the risk in the information variables (x_t^a, v_t), and, indirectly, the dividend sequence as well. The theoretical and empirical usefulness of this approach, if any, in terms of the model in this paper remains to be worked out. An obvious limitation concerns the absence of a leverage concept in the model. It goes almost without saying that a satisfactory model of accounting data and market risk ought to separate the operating risk from the financial risk.

Concluding Remarks

At an elementary level, the current paper exploits two simple ideas. First, one can apply the clean surplus relation to shift the value analysis away from PVED to book value plus the present value of expected abnormal earnings. Second, an assumption that abnormal earnings satisfy a (modified) autoregressive process ensures analytical simplicity. These two ideas combine to yield a closed-form evaluation of the present value of expected abnormal earnings. Without violating the PVED precept, one obtains explicit and basic expressions relating value and return to accounting data. Given the popularity of research dealing with value/returns issues—especially in the empirical domain—these results are of interest in their own right. However, this paper proceeds by focusing on a third, more subtle, idea, which colloquially is expressed as "dividends are paid out of book value, and not out of current earnings." Adding this assumption to the one on abnormal earnings yields the key feature of the model, namely, the sequence of expected abnormal earnings depends on neither current dividends nor on the future dividend policy. In the spirit of Miller and Modigliani (1961), one obtains the fundamental value displacement property. Dividends reduce market value on a dollar-for-dollar basis because dividends (i) reduce book value similarly on a dollar-for-dollar basis but (ii) do not affect the expected abnormal earnings sequence. The analysis demonstrates further that the value displacement property intertwines closely with the idea that dividends reduce subsequent periods' expected earnings. The reason is simple yet easy to overlook: a firm's earnings must align with its net investment in assets, that is, its book value. Dividends today therefore reduce future earnings "via" a reduction of current book value. These model developments therefore show that the assumption on the abnormal earnings dynamics is not only analytically convenient, it also combines meaningfully with owners' equity accounting constructs.

Finally, the paper highlights the key role of accounting data when one tries to come to grips with an apparent paradox in neoclassical security valuation: the present value of expected dividends determines a firm's value, yet the prediction of the dividend *sequence* is basically irrelevant if the underlying dividend policy is irrelevant. (Who wants to predict next year's expected dividends when all dividend policies yield the same market value?) One resolves this paradox by elaborating on how dividends influence current and future accounting data realizations. With the "correct" accounting constructs — including clean surplus and "dividends are paid out of current book value but leave current earnings unchanged"— one conceptualizes a firm's value by predicting a variable sequence that does not depend on the dividend policy, that is, future abnormal earnings. This observation leads to the following summary conclusion: the theory developed converges on how accounting data depend on dividends as opposed to on how dividends depend on accounting data. In the author's view, the significance of this point cannot be overemphasized.

Appendix 1: Proof of the valuation function (5) given (A1), (A2a), and (A3).

Define the 2 x 2 matrix

$$P \equiv R_f^{-1} \begin{bmatrix} \omega & 1 \\ 0 & \gamma \end{bmatrix}$$

The information dynamics (A3) can be expressed as

$$(\tilde{x}^a_{t+1}, \tilde{v}_{t+1}) = R_f P(x^a_t, v_t) + (\tilde{\varepsilon}_{1t+1}, \tilde{\varepsilon}_{2t+1}),$$

and

$$R_f^{-\tau} E_t[\tilde{x}^a_{t+\tau}] = (1,0)P^{\tau}(x^a_t, v_t).$$

Given (A1) and (A2a) one can use (1) and combine it with the last expression:

$$P_t - y_t = \sum_{\tau=1}^{\infty} R_f^{-\tau} E_t[\tilde{x}^a_{t+\tau}] = (1,0)[P+P^2+...](x^a_t, v_t)$$

$$\equiv (\alpha_1, \alpha_2)(x^a_t, v_t).$$

The sum of the matrix series $P + P^2 + ...$ converges because the maximum characteristic root of P is less than one. Using routine algebra one shows that the sum of the series equals $P[I-P]^{-1}$. One obtains

$$(\alpha_1, \alpha_2) = (1,0)P[I-P]^{-1}$$

and, via explicit calculation,

$$\alpha_1 = \omega/(R_f - \omega)$$

$$\alpha_2 = R_f/(R_f - \omega)(R_f - \gamma).$$

Appendix 2: Proof of the return expression (6).
We derive the expression for $P_{t+1} + d_{t+1} - R_f P_t$ and subsequently divide by P_t.

$$P_{t+1} + d_{t+1} - R_f P_t = y_{t+1} + d_{t+1} + \alpha_1 x^a_{t+1} + \alpha_2 v_{t+1}$$
$$-R_f(y_t + \alpha_1 x^a_t + \alpha_2 v_t)$$

$$= (\alpha_1 + 1)x^a_{t+1} + \alpha_2 v_{t+1} - R_f \alpha_1 x^a_t - R_f \alpha_2 v_t.$$

Substituting $x^a_{t+1} = \omega x^a_t + v_t + \varepsilon_{1t+1}$ and $v_{t+1} = \gamma v_t + \varepsilon_{2t+1}$ into the last expression one obtains after some simplifications

$$P_{t+1} + d_{t+1} - R_f P_t = (\alpha_1 + 1)\varepsilon_{1t+1} + \alpha_2 \varepsilon_{2t+1} + \beta_1 x^a_t + \beta_2 v_t$$

where

$$\beta_1 \equiv (\alpha_1 + 1)\omega - \alpha_1 R_f$$

$$\beta_2 \equiv (\alpha_1 + 1) + \alpha_2 \gamma - \alpha_2 R_f.$$

Since $\alpha_1 = \omega/(R_f - \omega)$ and $\alpha_2 = R_f/(R_f - \gamma)(R_f - \omega)$ (See Appendix 1), one readily verifies that $\beta_1 = \beta_2 = 0$.

Thus the conclusion

$$(P_{t+1} + d_{t+1})/P_t = R_f + (\alpha_1 + 1)\varepsilon_{1t+1}/P_t + \alpha_2 \varepsilon_{2t+1}/P_t$$

follows.

Appendix 3: Proof of (P5a) given (A1) and (A2a).

Consider the recursive equation (4) implied by (A2a),

$$y_{t+1} = x^a_{t+1} + R_f y_t - d_{t+1}$$

By substituting recursively backwards from y_{T-1} to y_1 one obtains

$$y_T = \sum_{t=1}^{T} R_f^{T-t} x^a_t + R_f^T y_0 - \sum_{t=1}^{T} R_f^{T-t} d_t$$

Deducting y_0 and adding $\sum_{t=1}^{T} d_t$ on both sides of the equation yields

$$y_T - y_0 + \sum_{t=1}^{T} d_t = \sum_{t=1}^{T} x_t = \sum_{t=1}^{T} R_f^{T-t} x^a_t + (R_f^T - 1)y_0 - \sum_{t=1}^{T} (R_f^{T-t} - 1)d_t$$

The first equality follows because of (A2a), of course. The last equality can be reexpressed as

$$\Big(\sum_{t=1}^{T} x_t + \sum_{t=1}^{T} (R_f^{T-t} - 1)d_t\Big) / (R_f^T - 1) = y_0 + [R_f^T/(R_f^T - 1)] \sum_{t=1}^{T} R_f^{-t} x^a_t$$

Since $R_f^T/(R_f^T - 1) \to 1$ as $T \to \infty$ and $P_0 = y_0 + \sum_{t=1}^{\infty} R_f^{-t} E_0[\tilde{x}^a_t]$ given (A1) and (A2a), the result now follows by applying the $E_0[\ .\ ]$ operator.

Appendix 4: Proof of (P5b) given (A1), (A2a), and (A3).

Define the matrices

$$D \equiv \begin{bmatrix} \omega & 0 \\ 0 & \gamma \end{bmatrix} \text{ and } Q \equiv \begin{bmatrix} 1 & (\omega-\gamma)^{-1} \\ 0 & 1 \end{bmatrix}$$

Note next that

$$Q^{-1} \equiv \begin{bmatrix} 1 & (\omega-\gamma)^{-1} \\ 0 & 1 \end{bmatrix} \text{ and } QDQ^{-1} \equiv \begin{bmatrix} \omega & 1 \\ 0 & \gamma \end{bmatrix}$$

Hence, the expectation of x^a_{t+1} can be expressed as

$$E_t[\tilde{x}^a_{t+1}] = (1,0)\ QDQ^{-1}(x^a_t, v_t)$$

and thus

$$R_f^{-t}E_0[\tilde{x}_t] = (1,0)Q\begin{bmatrix} (\omega/R_f)^t & 0 \\ 0 & (\gamma/R_f)^t \end{bmatrix}Q^{-1}(x^a_0,\ v_0)$$

Since (see Appendix 3)

$$V_0^T = y_0 + \varphi_T \sum_{t=1}^{\infty} R_f^{-t} E_0[\tilde{x}^a_t]$$

where $\varphi_T \equiv R_f^T/(R_f^T - 1)$, it follows that

$$V_0^T = y_0 + \varphi_T\,(1,0)Q\begin{bmatrix} a_T & 0 \\ 0 & b_T \end{bmatrix}Q^{-1}(x^a_0, v_0)$$

$$a_T \equiv (\omega/R_f) + (\omega/R_f)^2 + \ldots + (\omega/R_f)^T = (\omega/(R_f - \omega))(1 - (\omega/R_f)^T)$$

$$b_T \equiv (\gamma/R_f) + (\gamma/R_f)^2 + \ldots + (\gamma/R_f)^T = (\gamma/(R_f - \gamma))(1 - (\gamma/R_f)^T)$$

A direct computation shows that

$$(1,0)Q\begin{bmatrix} a_T & 0 \\ 0 & b_T \end{bmatrix}Q^{-1} = (a_T, (\omega - \gamma)^{-1}(a_T - b_T))$$

After some simplifications it follows that

$$V_0^T = y_0 + \alpha_1^T x^a_0 + \alpha_2^T v_0$$

where

$$\alpha_1^T \equiv [\omega/(R - \omega)]\left[\frac{R_f^T - \omega^T}{R_f^T - 1}\right]$$

$$\alpha_2^T \equiv (\omega - \gamma)^{-1}\left\{\alpha_1^T - [\gamma/(R_f - \gamma)]\left[\frac{R_f - \gamma^T}{R_f^T - 1}\right]\right\}$$

As $T \to \infty$ the parameters converge toward their correct values:

$$\alpha_1^\infty = R_f/(R_f - \omega)$$

$$\alpha_2^\infty = (\omega - \gamma)^{-1}\left(\frac{\omega}{R_f - \omega} - \frac{\gamma}{R_f - \gamma}\right) = \frac{R_f}{(R_f - \omega)(R_f - \gamma)}$$

Thus, V_0^T does not depend on the dividend policy and $V_0^T \to P_0$ as $T \to \infty$

Appendix 5: Proof of proposition

Assumption (iii) ensures that it makes no difference whether v_t is zero or not. We accordingly put $v_t = 0$ to keep matters simple.

The expression $E_t[\tilde{x}_{t+2} + \tilde{x}_{t+1} + \tilde{d}_{t+1}(R_f - 1)]$ equals

$$E_t[(1 + \theta_1)\tilde{x}_{t+1} + \theta_2\tilde{y}_{t+1} + (\theta_3 + R_f - 1)\,\tilde{d}_{t+1}] \tag{15}$$

Further, since $y_{t+1} = y_t + x_{t+1} - d_{t+1}$ and $x_{t+1} = \theta_1 x_t + \theta_2 y_t + \theta_3 d_t + \tilde{\varepsilon}_{1t+1}$ it follows that the last expression (15) equals

$$E_t[(1 + \theta_1 + \theta_2)(\theta_1 x_t + \theta_2 y_t + \theta_3 d_t) + \theta_2 y_t + (\theta_3 - \theta_2 + R_f - 1)\tilde{d}_{t+1}]$$

Differentiating with respect to d_t it follows that (A2) and (P2)imply

$$-(1 + \theta_1 + \theta_2)(\theta_2 - \theta_3) - \theta_2 + (\theta_3 - \theta_2 + R_f - 1)\partial E_t[\tilde{d}_{t+1}]/\partial d_t = -(R_f^2 - 1)$$

Since the last equation must hold for all dividend policies, and $\partial E_t[\tilde{d}_{t+1}]/\partial d_t$ is possibly random, one obtains (P1). That is, $-\theta_2 + \theta_3 = -(R_f - 1)$. Further,

$$-(1 + \theta_1 + \theta_2)(\theta_2 - \theta_3) - \theta_2 = -(R_f^2 - 1)$$

Solving for θ_2 in the expression implies

$$\theta_2 = (R_f - 1)(1 - \theta_1/R_f)$$

Define $\omega = \theta_1/R_f$; it is now easily seen that

$$\theta_2 = (R_f - 1)(1 - \omega),\ \theta 1 = \omega R_f, \text{ and } -\theta_3 = (R_f - 1)\omega:$$

$$Et[\tilde{x}_{t+1}] = \omega R_f xt + (R_f - 1)(1 - \omega)yt - \omega(R_f - 1)dt.$$

After applying (A2a) and allowing for nonzero v_t, one obtains

$$E_t[x^a_{t+1}] = \omega x^a_t + v_t,$$

as asserted.

Endnotes

1 Generally accepted accounting principles (GAAP) violates the clean surplus relation for some kinds of transactions (e.g., the accounting for foreign currency translation and some prior period adjustments due to change in accounting principles). From the point of view of accounting theory the clean surplus attribute has an important standing; see, for example, Paton and Littleton (1940).

2 Rubinstein (1976) provides a general theoretical foundation for dividends capitalization under uncertainty.

3 The literature often refers to "abnormal earnings" as "residual income".

4 Relying only on A1 and A2a, one can also show that

$$P_{t+1} + d_{t+1} - R_f P_t = \sum_{\tau=1}^{\infty} R_f^{-(\tau-1)} \Delta E_{t+1}[\tilde{x}^a_{t+\tau}]$$

where $\Delta E_{t+1}[.] \equiv E_{t+1}[.] - E_t[.]$. The result parallels

$$P_{t+1} + d_{t+1} - R_f P_t = \sum_{\tau=1}^{\infty} R_f^{-(\tau-1)} \Delta E_{t+1}[\tilde{d}_{t+\tau}],$$

but, of course, $\tilde{x}^a_{t+\tau} \neq \tilde{d}_{t+\tau}$ generally.

5 For a savings account one has $x_t^a = 0$, and thus $P_t = y_t$. It is instructive to note that this setting does not imply that the dividends' (or earnings') sequence is certain since it may depend on random events. However, one obtains $\sum_{\tau}^{T} R_f^{-\tau} \tilde{d}_{t+\tau} \rightarrow P_t$ with probability one as $T \rightarrow \infty$. Ohlson (1991) discusses the point.

6 In loose terms, a firm's economic environment and its accounting principles determine the exogenous parameters ω and γ.

7 For a savings account one obtains $P_t = y_t = \varphi x_t - d_t$, $v_t = 0$, and (7) holds for every k. Since $x_t^a = 0$, for all t, neither ω nor k is identified.

8 Ohlson (1991) analyzes the case of $\omega = 1$ in detail.

9 Note that P1 reduces to $\partial x_{t+1}/\partial d_t = -(R_f - 1)$ for a savings account.

10 Ryan (1988) identifies the property P1 for the case of "ungarbled" earnings, which occurs when $\upsilon_t = 0$ and $\omega = 1$.

11 This argument is similar to that used by Modigliani and Miller (1958) to prove capital structure irrelevancy.

12 The approximation error is determined by $P_t - V_t^T = E_t[(\tilde{P}_{t+T} - \tilde{y}_{t+T}) - (P_t - y_t)]/(R_f^T - 1)$, so that $P_t = V_t^T$ if and only if $E_t[\tilde{P}_{t+T} - \tilde{y}_{t+T}] = P_t - y_t$. The proof, which depends only on A1 and A2a, is routine.

13 One shows without difficulty that $V_t^T = W_t^T$ if and only if $V_t^T = y_t$ or $W_t^T = y_t$. The observation depends only on A2a.

14 Notice that P7 does not work for $\omega = 1$; provided that $\gamma < 1$ but $\omega = 1$ one obtains $E_t[\tilde{x}_{t+\tau}] = x_t + (1-\gamma)^{-1} v_t$ as $\tau \rightarrow \infty$. P7 clearly requires ω, $\gamma < 1$. (If $\omega = \gamma = 1$ then $E_t[\tilde{x}_{t+\tau}]$ will generally not converge as $\tau \rightarrow \infty$.)

15 The Proposition is silent concerning the magnitudes of the parameters ω and γ. In a sense, the result is "disappointing" because one does not fully recover the model as developed since it stipulates $0 < \omega$, $\gamma < 1$. Not even convergence in $\sum_{\tau}^{\infty} R_f^{-\tau} E_t[\tilde{x}_{t+\tau}^a]$ is guaranteed (convergence occurs if $|\omega|, |\gamma| < R_f$). Provided that convergence is met, the cases $\omega \geq 1$ or $\gamma \geq 1$ still yield the properties P1, P2, P3, P5 and P6, but P4 and P7 will generally be violated as the latter properties require $|\omega|, |\gamma| < 1$. (I am indebted to Steve Penman and Froystein Gjesdal for directing my attention to this point.)

16 Ohlson (1989) develops this result using assumptions that are isomorphic to A1, A2, and A3 plus a linear dividends prediction equation.

References

Edwards, E.O., and P.W. Bell. *The Theory of and Measurement of Business Income.* University of California Press, 1961.

Garman, M., and J.A. Ohlson. Information and the Sequential Valuation of Assets in Arbitrage-Free Economies. *Journal of Accounting Research* (Autumn 1980), 420-440.

Huang, C., and R.H. Litzenberger. *Foundations for Financial Economics.* North Holland, 1988.

Miller, M., and F. Modigliani. Dividend Policy, Growth and the Valuation of Shares. *Journal of Business* (October 1961), 411-433.

Modigliani, F., and M. Miller. The Cost of Capital, Corporation Finance, and the Theory of Investment. *American Economic Review* (March 1958), 261-297.

Ohlson, J.A. Accounting Earnings, Book Value, and Dividends: The Theory of the Clean Surplus Equation (Part I). Unpublished paper. Columbia University, 1989.

Ohlson, J.A. A Synthesis of Security Valuation Theory and the Role of Dividends, Cash Flows, and Earnings. *Contemporary Accounting Research* (Spring 1990), 648-676.

Ohlson, J.A. The Theory of Value and Earnings, and An Introduction to the Ball-Brown Analysis. *Contemporary Accounting Research* (Fall 1991), 1-19.

Paton, W.A., and A.C. Littleton *An Introduction to CorporateAccounting Standards. American Accounting Association Monograph No.3*, 1940.

Peasnell, K.V. Some Formal Connections Between Economic Values and Yields and Accounting Numbers. *Journal of Business Finance and Accounting* (October 1982), 361-381.

Peasnell, K.V. On Capital Budgeting and Income Measurement. *Abacus* (June 1981), 52-67.

Ramakrishnan, R. What Matters from the Past-Price, Book Value or Earnings? Sufficient Statistics in Accounting. Unpublished memo. Columbia University, 1990.

Rubinstein, M. The Valuation of Uncertain Income Streams and the Pricing of Options. *Bell Journal of Economics* (Autumn 1976), 407-425.

Ryan, S. Structural Models of the Accounting Process and Earnings. Ph.D. Dissertation. Stanford University, 1988.