OSCILLATORY CRITERIA FOR DIFFERENTIAL EQUATIONS WITH SEVERAL DEVIATING ARGUMENTS

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ABSTRACT

Consider the first-order delay differential equation

$$x'(t) + \sum_{i=1}^{m} p_i(t)x(\tau_i(t)) = 0, \quad t \geq 0, \tag{1.1}$$

where, for every $i \in \{1, \dots, m\}$, p_i is a continuous real-valued function in the interval $[0, \infty)$, and τ_i is a continuous real-valued function on $[0, \infty)$ such that

$$\tau_i(t) \leq t, \quad t \geq 0, \quad \text{and} \quad \lim_{t\to\infty} \tau_i(t) = \infty \tag{1.2}$$

and the (dual) advanced differential equation

$$x'(t) - \sum_{i=1}^{m} p_i(t)x(\sigma_i(t)) = 0, \quad t \geq 1, \tag{1.3}$$

where, for every $i \in \{1, \dots, m\}$, p_i is a continuous real-valued function in the interval $[1, \infty)$, and σ_i is a continuous real-valued function on $[1, \infty)$ such that

$$\sigma_i(t) \geq t, \quad t \geq 1. \tag{1.4}$$

Next, consider the discrete analogue difference equations

$$\Delta x(n) + \sum_{i=1}^{m} p_i(n)x(\tau_i(n)) = 0, \quad n \in \mathbb{N}_0, \tag{1.5}$$

where $\mathbb{N} \ni m \geq 2$, p_i, $1 \leq i \leq m$, are real sequences and $\{\tau_i(n)\}_{n\in\mathbb{N}_0}$, $1 \leq i \leq m$, are sequences of integers such that

$$\tau_i(n) \leq n - 1, \quad n \in \mathbb{N}_0, \quad \text{and} \quad \lim_{n\to\infty} \tau_i(n) = \infty, \quad 1 \leq i \leq m \tag{1.6}$$

and the (dual) advanced difference equation

$$\nabla x(n) - \sum_{i=1}^{m} p_i(n)x(\sigma_i(n)) = 0, \; n \in \mathbb{N} \tag{1.7}$$

where $\mathbb{N} \ni m \geq 2$, p_i, $1 \leq i \leq m$, are real sequences and $\{\sigma_i(n)\}_{n\in\mathbb{N}}$, $1 \leq i \leq m$, are sequences of integers such that

$$\sigma_i(n) \geq n + 1, \quad n \in \mathbb{N}, \quad 1 \leq i \leq m. \tag{1.8}$$

Here, as usual, Δ denotes the forward difference operator $\Delta x(n) = x(n+1) - x(n)$ and ∇ denotes the backward difference operator $\nabla x(n) = x(n) - x(n-1)$. Several oscillation conditions for the above equations are presented..