# A Topological Proof of a version of Artin's Induction Theorem

by

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We define an Euler characteristic $\chi(X, G)$, for a finite cell complex $X$ with a finite group $G$ acting cellularly on it. Then, each $K_i(X)$ (a complex vector space with basis the $i$-cells of $X$) is a representation of $G$, and we define the $\chi(X, G)$ to be the alternating sum of the representations $K_i(X)$, as an element of the representation ring $R(G)$ of $G$. By adapting the ordinary proof that $\sum_i (-1)^i \dim_{\mathbb{C}} K_i(X; \mathbb{C}) = \sum_i (-1)^i \dim_{\mathbb{C}} H_i(X; \mathbb{C})$, we prove that there is another definition of $\chi(X, G)$ with the alternating sum of the representations $H_i(X)$, again as elements of the representation ring $R(G)$.

We also give a formula for the character of $\chi(X, G)$, in terms of the ordinary Euler characteristic of $X^g$.

Finally, we prove a weaker version of Artin's induction theorem, stating that if $G$ is a group with an irreducible representation of dimension greater than 1, then every character of $G$, is a rational linear combination of characters induced up from proper subgroups.