FRANKL'S CONJECTURE HOLDS FOR SUBGROUP LATTICES

ALIREZA ABDOLLAHI, RUSS WOODROOFE, AND GJERGJI ZAIMI

The union-closed sets conjecture has been posed by Péter Frankl in 1979 and it is still open. A family of sets is said to be union-closed if the union of any two sets from the family remains in the family. The conjecture states that for any finite union-closed family of finite sets, other than the family consisting only of the empty set, there exists an element that belongs to at least half of the sets in the family.

Although stated above in terms of families of sets, Frankl's conjecture has also been formulated and studied as a question in lattice theory. The family of all subsets of a set S, ordered by set inclusion, forms a lattice in which the meet is represented by the set-theoretic intersection and the join is represented by the set-theoretic union; a lattice formed in this way is called a Boolean lattice. The lattice-theoretic version of Frankl's conjecture is that in any finite lattice there exists an element x that is not the join of any two smaller elements, and such that the number of elements greater than or equal to x totals at most half the lattice, with equality only if the lattice is a Boolean lattice. As Abe in 2000 shows, this statement about lattices is equivalent to the Frankl's conjecture for union-closed sets: each lattice can be translated into a union-closed set family, and each union-closed set family can be translated into a lattice, such that the truth of the Frankl's conjecture for the translated object implies the truth of the conjecture for the original object. This lattice-theoretic version of the conjecture is known to be true for several natural subclasses of lattices but remains open in the general case.

We show that the subgroup lattice of any finite group satisfies Frankl's conjecture.

Department of Mathematics, University of Isfahan, Isfahan, 81746-73441, Iran
E-mail address: a.abdollahi@math.ui.ac.ir
URL: http://sci.ui.ac.ir/~a.abdollahi

Department of Mathematics & Statistics, Mississippi State University, Starkville, MS 39762
E-mail address: rwoodroofe@math.msstate.edu
URL: http://rwoodroofe.math.msstate.edu/

E-mail address: gjergjiz@gmail.com